2.8 Inequalities

Question Paper

Course	DP IB Maths
Section	2. Functions
Topic	2.8 Inequalities
Difficulty	Hard

Time allowed: 70

Score: /56

Percentage: /100

Question 1a

(a)
Sketch the graph of the function $f(x) = x(x-2)(x-4)^2$.
Mark your sketch clearly with the x-coordinates of the x-axis intercepts.

[2 marks]

Question 1b

(b)
Write down the solution to the inequality $f(x) \leq 0$.

[1 mark]

Question 1c

(c)
Briefly explain how the graph shows that there are no real solutions to the inequality $f(x) + 10 \leq 0$.

[2 marks]

Question 2

Consider the function defined by $g(x) = e^{-|x|}$.

(i)
Sketch the graph of $y = g(x)$.
(ii)
Solve the inequality $g(x) \geq 0.5$ using exact values.

[4 marks]

Question 3a

Consider the function $f(x) = 6x^3 - 19x^2 + 16x - 4.$

(a)
Fully factorise $f(x)$.

[2 marks]

Question 3b

(b)
Solve
(i)
$f(x) < 0$
(ii)
$f(2x) < 0$
(iii)
$f(x-3) < 0$
(iv)
$|f(x-3)| \leq 0$

[4 marks]

Question 4

Find the values of k such that the equation $kx = kx^2 + k - 2$ has real solutions and the equation $(4k-3)x^2 + 2kx + 1 = 0$ has no real solutions.

[4 marks]

Question 5a

(a)

Giving answers to three significant figures, find the set of values of x that satisfy

$$|\sin(2x^\circ)| \geq 1 - \frac{x}{360}$$

for $0 \leq x \leq 360$.

[3 marks]

Question 5b

(b)

Explain how your answer to part (a) would differ if the domain of x was changed from $0 \leq x \leq 360$ to $x \in \mathbb{R}$.

[1 mark]

Question 6a

(a)

Write down the set of values of x for which $e^{2x} \leq e^{x}$.

[1 mark]

Question 6b

(b)

Find the set of values of x for which

(i) $e^{2(x-4)} \leq e^{x-4}$

(ii) $e^{5(x-4)} \geq e^{-(x-4)}$

[2 marks]

Question 6c

(c)

Find the set of values of x for which

(i)

$2\ln(x-4) \leq \ln(x-4)$

(ii)

$-\ln(x-4) \geq 5\ln(x-4).$

[2 marks]

Question 7a

Consider the functions

$$f(x) = \frac{a}{a-x} \text{ and } g(x) = \frac{2a}{x-2a}$$

where a is real constant such that $a \neq 0$.

(a) In terms of the constant a, find

(i) the values of x for which $f(x)$ and $g(x)$ are undefined,

(ii) the x-coordinate of any intersections between the graphs of $y = f(x)$ and $y = g(x)$.

[4 marks]

Question 7b

(b)

In the case $a > 0$, find the set of values of x in terms of a for which

(i) $f(x) > g(x)$,

(ii) $f(x) < g(x)$.

[3 marks]

Question 7c

(c)
Repeat questions (b) (i) and (ii) in the case $a < 0$.

[2 marks]

Question 8a

$f(x) = x^3 - 11x^2 + 40x - 48$

(a)
(i)
Show that $x = 3$ is a root of the function .
(ii)
Hence fully factorise $f(x)$.

[3 marks]

Question 8b

(b)
Solve the inequality $x^3 - 10x^2 + 32x - 32 \leq (x-4)^2$.

[3 marks]

Question 8c

(c)

Solve the inequality $(x+a)(x+b)^2 > 0$, where a and b are constants such that $a > b > 0$. Give your answers in terms of a and b.

[2 marks]

Question 9a

(a)

Consider the functions defined by $f(x) = x^2 - a$ and $g(x) = a - x^2$, where a is a positive constant. Solve the inequality $f(x) < g(x)$, giving your answer in terms of a.

[3 marks]

Question 9b

(b)

Consider the functions defined by $p(x) = x^3 - b$ and $q(x) = b - x^3$, where b is a real constant. Solve the inequality $p(x) < q(x)$, giving your answer in terms of b.

[2 marks]

Question 9c

(c)

Consider the functions defined by $h(x) = x^n - m$ and $j(x) = m - x^n$, where $m > 0$ and $n \in \mathbb{Z}^+$. Write down, in terms of m and n, the solution to the inequality $h(x) < j(x)$ when

(i)

n is even,

(ii)

n is odd.

[3 marks]

Question 10

Consider the functions defined by $f(x) = a + \ln bx$ and $g(x) = a - \ln bx$ where a and b are positive constants. Show that

$$f(x) < g(x) \text{ for } 0 < x < \frac{1}{b}. \ .$$

[3 marks]