IB Physics DP

YOUR NOTES

4. Waves

CONTENTS

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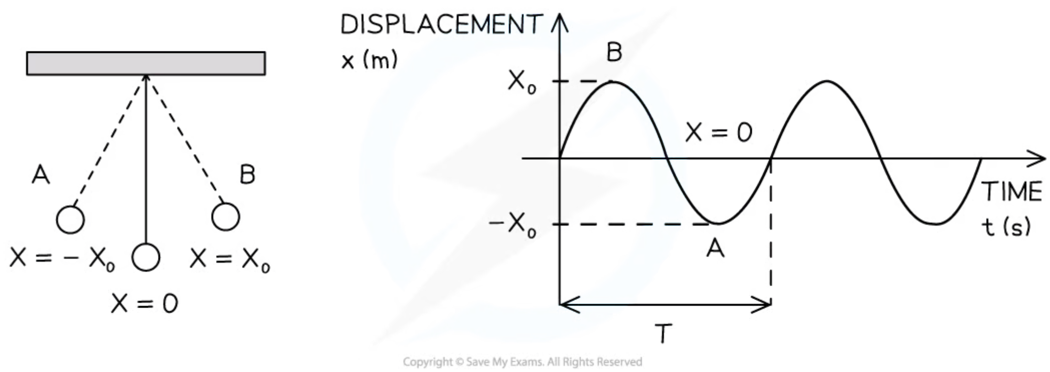
4.1 Oscillations

4.1.1 Properties of Oscillations

Describing Oscillations

- An **oscillation** is defined as follows:

  The repetitive variation with time *t* of the displacement *x* of an object about the equilibrium position (*x* = 0)



A pendulum oscillates between A and B. On a displacement-time graph, the oscillating motion of the pendulum is represented by a wave, with an amplitude equal to x_0

- **Displacement (*x*)** of a wave is the distance of a point on the wave from its equilibrium position
  - It is a vector quantity; it can be positive or negative and it is measured in metres (m)
- **Period (*T*)** or time period, is the **time interval** for one complete repetition and it is measured in seconds (s)
  - If the oscillations have a **constant period**, they are said to be **isochronous**

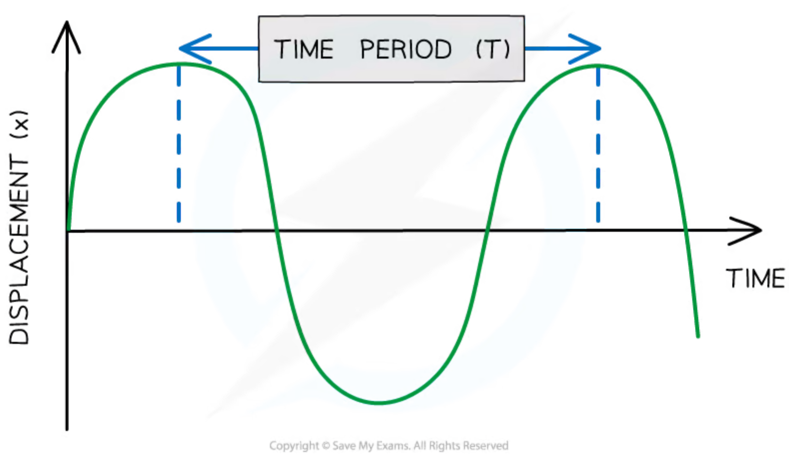


Diagram showing the time period of a wave

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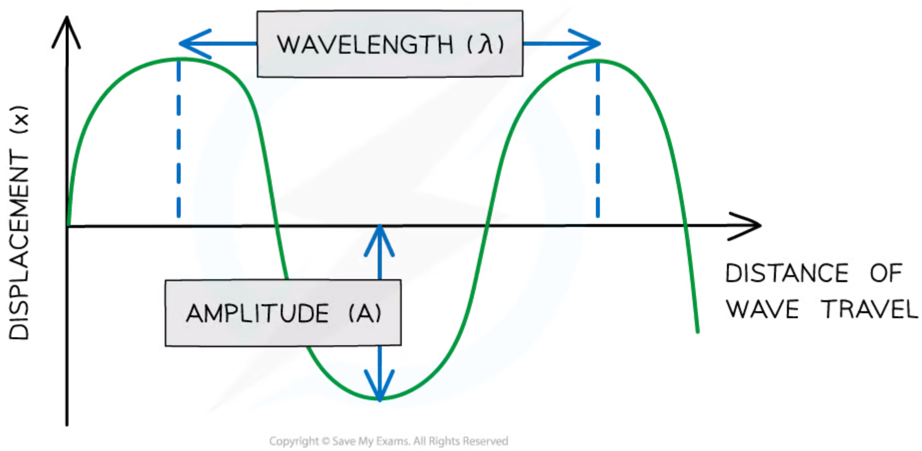
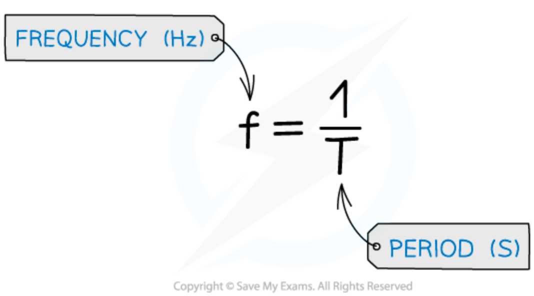
- **Amplitude (x_0)** is the **maximum value of the displacement** on either side of the equilibrium position is known as the **amplitude** of the oscillation
  - Amplitude is measured in metres (m)
- **Wavelength (λ)** is the **length of one complete oscillation** measured from the same point on two consecutive waves
  - Wavelength is measured in metres (m)



- **Frequency (f)** is the **number of oscillations per second** and it is measured in hertz (Hz)
  - Hz have the SI units of per second s^{-1}
- The frequency and the period of the oscillations are related by the following equation:



Phase & Phase Difference

- Phase is a useful way to think about waves
- The phase of a wave can be considered in terms of:
  - Wavelength
  - Degrees
  - Radians
- One complete oscillation is:
  - 1 wavelength
  - 360°
  - 2π radians

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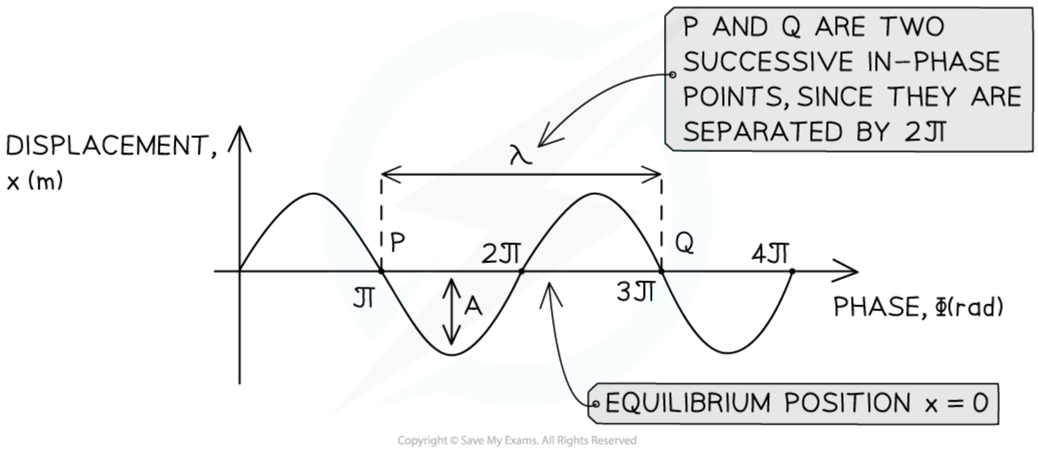
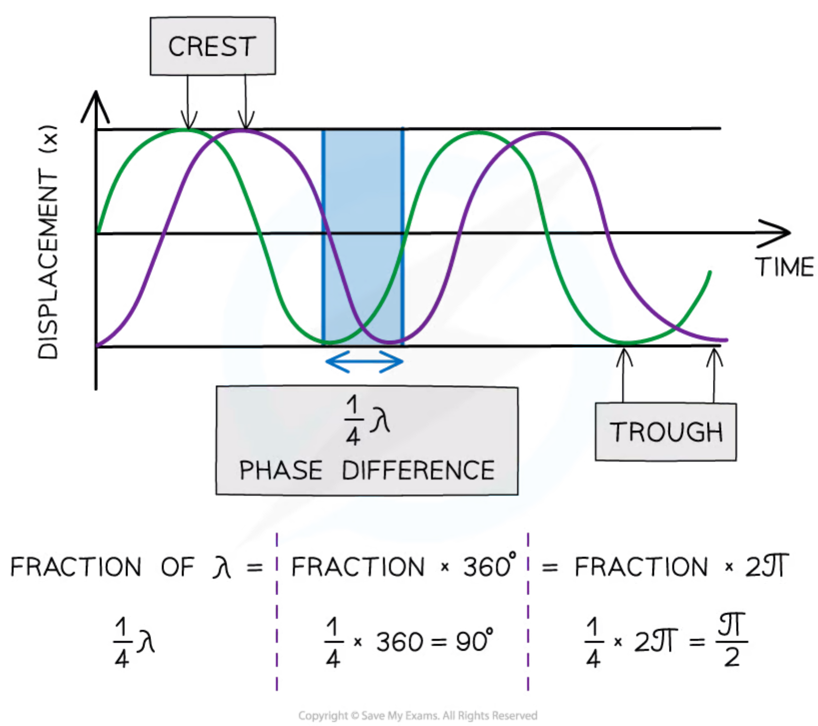


Wavelength λ and amplitude A of a travelling wave

- The **phase difference** between two waves is a measure of **how much a point or a wave is in front or behind another**
- This can be found from the relative position of the crests or troughs of two different waves of the same frequency
  - When the crests of each wave, or the troughs of each wave are aligned, the waves are **in phase**
  - When the crest of one wave aligns with the trough of another, they are in **antiphase**
- The diagram below shows the green wave **leads** the purple wave by ¼ λ



Two waves ¼ λ out of phase

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- In contrast, the purple wave is said to **lag** behind the green wave by ¼ λ
- Phase difference is measured in **fractions of a wavelength, degrees** or **radians**
- The phase difference can be calculated from two different points on the same wave or the same point on two different waves
- The phase difference between two points can be described as:
  - **In phase** is **360°** or 2π radians
  - **In anti-phase** is **180°** or π radians

Worked Example

A child on a swing performs 0.2 oscillations per second. Calculate the period of the child's oscillations.

Step 1: Write down the frequency of the child's oscillations

$$f = 0.2 \text{ Hz}$$

Step 2: Write down the relationship between the period *T* and the frequency *f*

$$T = \frac{1}{f}$$

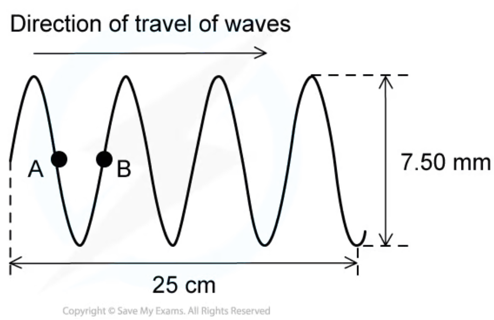
Step 3: Substitute the value of the frequency into the above equation and calculate the period

$$T = \frac{1}{0.2}$$

$$T = 5 \text{ s}$$

Worked Example

Plane waves on the surface of water at a particular instant are represented by the diagram below.



The waves have a frequency of 2.5 Hz. Determine:

a) The amplitude

b) The wavelength

c) The phase difference between points **A** and **B**

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A. THE AMPLITUDE

MAXIMUM DISPLACEMENT FROM THE EQUILIBRIUM POSITION

$7.50 \text{ mm} \div 2 = 3.75 \text{ mm}$

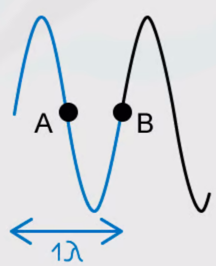
B. THE WAVELENGTH

DISTANCE BETWEEN POINTS ON SUCCESSIVE OSCILLATIONS OF THE WAVE THAT ARE IN PHASE

FROM DIAGRAM: $25\text{cm} = 3\frac{3}{4}$ WAVELENGTHS

$1\lambda = 25 \text{ cm} \div 3\frac{3}{4} = 6.67 \text{ cm}$

C. THE PHASE DIFFERENCE BETWEEN POINTS A AND B



POINTS A AND B HAVE $\frac{1}{2}\lambda$ DIFFERENCE $= \frac{1}{2} \times 360° = 180°$

Exam Tip

When labelling the wavelength and time period on a diagram:

- Make sure that your arrows go from the **very top** of a wave to the very top of the next one
- If your arrow is too short, you will lose marks
- The same goes for labelling amplitude, don't draw an arrow from the bottom to the top of the wave, this will lose you marks too.

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Simple Harmonic Oscillations

- Simple harmonic motion (**SHM**) is defined as follows:

  The motion of an object whose acceleration is directly proportional but opposite in direction to the object's displacement from a central equilibrium position

- An object is said to perform **simple harmonic oscillations** when all of the following apply:
  - The oscillations are isochronous
  - There is a central equilibrium point
  - The object's displacement, velocity and acceleration change continuously
  - There is a **restoring force** always directed towards the equilibrium point
  - The magnitude of the restoring force is proportional to the displacement

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Conditions for SHM

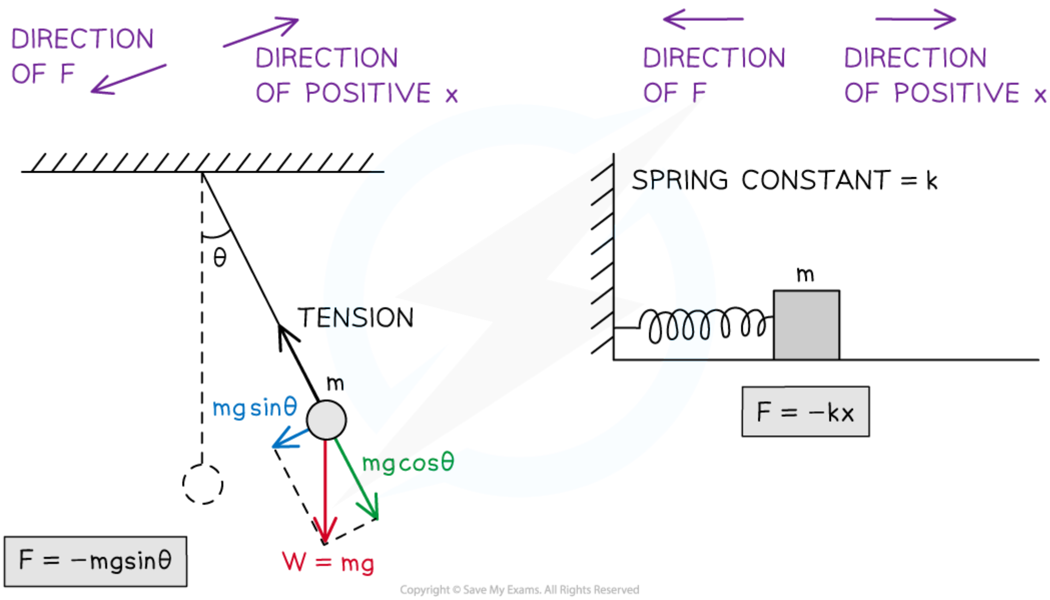
- The defining conditions of simple harmonic oscillations are that the **restoring force** and the **acceleration** must always be:
  - Directed towards the equilibrium position, and hence, is always in the **opposite** direction to the **displacement**
  - Directly **proportional** to the **displacement**

$$a \propto -x$$

- Where:
  - a = acceleration (m s^{-2})
  - x = displacement (m)

The Restoring Force

- One of the defining conditions of simple harmonic motion is the existence of a restoring force
- Examples of restoring forces are:
  - The component of the weight of a pendulum's bob that is parallel and opposite to the displacement of the bob
  - The force of a spring, whose magnitude is given by Hooke's law



For a pendulum, the restoring force is provided by the component of the bob's weight that is perpendicular to the tension in the pendulum's string. For a mass-spring system, the restoring force is provided by the force of the spring.

- For a mass-spring system in simple harmonic motion, the relationship between the restoring force and the displacement of the object can be written as follows:

$$F = -kx$$

- Where:

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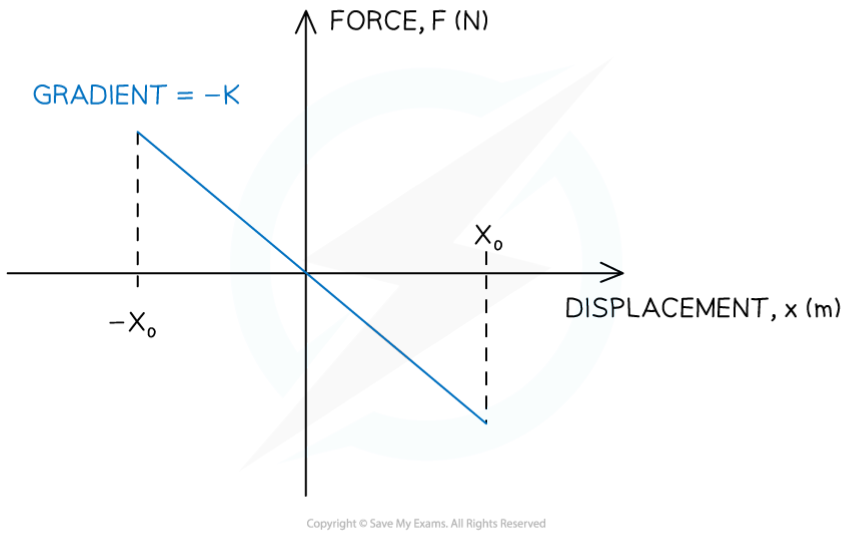
- F = restoring force (N)
- k = spring constant ($N\ m^{-1}$)
- x = displacement from the equilibrium position (m)



Graph of force against displacement for an object oscillating with SHM

- Force and displacement in SHM have a **linear** relationship where the gradient of the graph represents the constant
  - In this case, the spring constant k
- An object in SHM will also have a restoring force to return it to its equilibrium position
  - This restoring force will be directly proportional, but in the **opposite direction**, to the displacement of the object from its equilibrium position

Acceleration & Displacement

- According to Newton's Second Law, the net force on an object is directly proportional to the object's acceleration, $F \propto a$ for a constant mass

$$F = ma$$

- Where
  - F = force (N)
  - m = mass (kg)
  - a = acceleration ($m\ s^{-2}$)
- Since $F = ma$ (Newton's second law), and $F = -kx$ (Hooke's law), the equations can be set as equal to one another:

$$ma = -kx$$

- Rearranging to show the relationship between acceleration and displacement gives:

$$a = -\frac{k}{m}x$$

- This equation shows that

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- There is a **linear** relationship between the **acceleration** of the object moving with simple harmonic motion and its **displacement** from its equilibrium position

- The **minus** sign shows that when the mass on the spring is displaced to the **right**
  - The direction of the acceleration is to the **left** and vice versa
  - In other words, *a* and *x* are always in opposite directions to each other

- This equation shows acceleration is **directly proportional** but in the **opposite direction** to displacement for an object in SHM

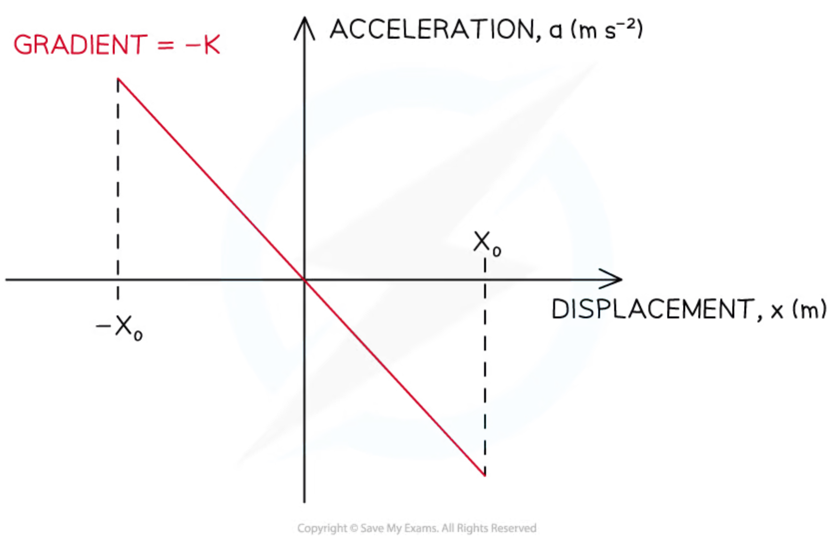
$$a \propto -x$$

- Therefore, it can be stated that:

$$a = -kx$$

- Where
  - a = acceleration
  - k = is a constant but in this instance **not** the spring constant
  - x = displacement

- Note that in physics, *k* is the standard letter used for an **undefined constant**



Graph of acceleration against displacement for an object oscillating with SHM

Worked Example

A pendulum's bob oscillates about a central equilibrium position. The amplitude of the oscillations is 4.0 cm. The maximum value of the bob's acceleration is 2.0 m s^{-2}.

Determine the magnitude of the bob's acceleration when the displacement from the equilibrium position is equal to 1.0 cm.

You may ignore energy losses.

Step 1: List the known quantities

- Amplitude of the oscillations, x_0 = 4.0 cm = 0.04 m
- Maximum acceleration, a = 2.0 m s^{-2}
- Displacement, x = 1.0 cm = 0.01 m

Remember to convert the amplitude of the oscillations and the displacement from centimetres (cm) into metres (m)

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Step 2: Recall the relationship between the maximum acceleration *a* and the displacement *x*

- The maximum acceleration a occurs at the position of maximum displacement $x = x_0$

$$a = -kx_0$$

Step 3: Rearrange the above equation to calculate the constant of proportionality *k*

$$k = -\frac{a}{x_0}$$

Step 4: Substitute the numbers into the above equation

$$k = -\frac{2.0}{0.04}$$

$$k = -50 \text{ s}^{-2}$$

Step 5: Use this value of *k* to calculate the acceleration *a'* when the displacement is *x* = 0.01 m

$$a' = -kx$$

$$a' = -(-50)\text{ s}^{-2} \times 0.01 \text{ m}$$

$$a' = 0.50 \text{ m s}^{-2}$$

4.1.3 SHM Graphs

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SHM Graphs

Displacement & Velocity

- The **displacement-time graph** for an object moving with SHM is a **sinusoidal curve** if:
  - The object starts to oscillate from the equilibrium position.
  - The equilibrium position is $x = 0$ at $t = 0$
- The **displacement-time graph** is a **cosine** curve if
  - The object starts to oscillate from the position of maximum displacement.
  - Maximum displacement is $x = x_0$ at $t = 0$
- The maxima and minima on the graph are the values of **maximum displacement** (**x_0**) of the oscillating object on either side of the equilibrium position

- The **velocity-time graph** is obtained by taking the **gradients** of tangents to all points on the **displacement-time** graph
- The **velocity-time graph** is a **cosine** curve if:
  - The object starts to oscillate from the equilibrium position when $x = 0$ at $t = 0$
  - The displacement-time graph is a sine curve
- The **velocity-time graph** is a **sine** curve if:
  - The object starts to oscillate from the position of maximum displacement when $x = x_0$ at $t = 0$
  - The displacement-time graph is a cosine curve
- The maxima and minima on the graph are the values of **maximum velocity** (**v_0**) of the oscillating object as it passes the equilibrium position
  - The difference in the sign of the velocity accounts for the different directions of the velocity vector as the object passes through the equilibrium position (i.e. from right to left or vice versa)

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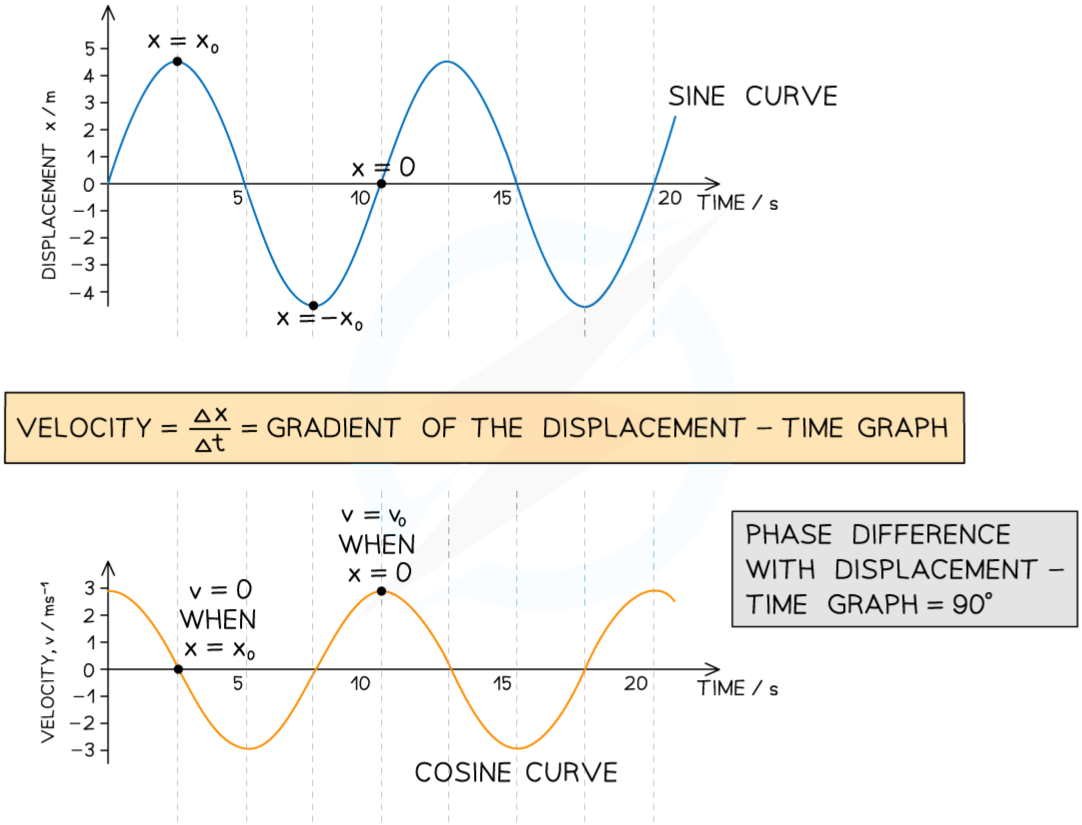

Displacement-time & velocity-time graphs for an object oscillating with SHM. The object starts oscillating from the equilibrium position (x = 0 and t = 0)

Acceleration

- The **acceleration-time graph** is obtained by taking the **gradients** of tangents to all points on the **velocity-time** graph
- The graph is a **negative sine** curve if:
  - The object starts to oscillate from the equilibrium position when $x = 0$ at $t = 0$
- The graph is a **negative cosine** curve if:
  - The object starts to oscillate from the position of maximum displacement when $x = x_0$ at $t = 0$
- The maxima and minima on the graph are the values of **maximum acceleration** (a_0) of the oscillating object at the positions of maximum displacement ($x = x_0$)
  - Once again, the difference in sign indicates a difference in the direction of the acceleration vector

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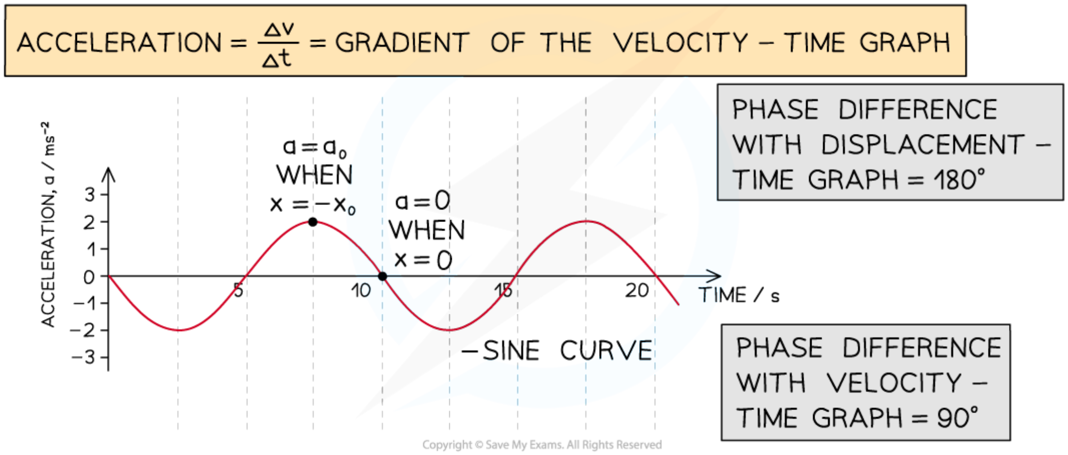
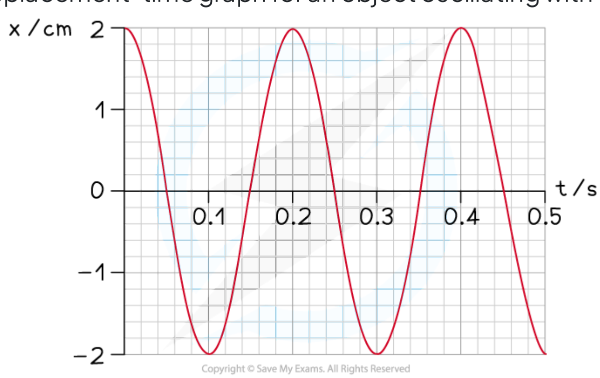


An acceleration-time graph for an object oscillating with SHM. The object starts oscillating from the equilibrium position (x = 0 and t = 0)

- Note that all graphs must have the **same period**
- The only two **differences between the graphs** are:
  - The shift in time - i.e. there is a phase difference of 90° between successive graphs
  - The amplitude of the wave form - i.e. the different amplitudes of the three graphs are the values of maximum displacement x_0, maximum velocity v_0 and maximum acceleration a_0 of the oscillating object

Worked Example

Below is the displacement-time graph for an object oscillating with SHM.



(i) Determine the period of the oscillations
(ii) Calculate the frequency of the oscillations
(iii) Mark a point on the graph where the velocity is zero, label this with "$v = 0$"
(iv) Mark a point on the graph where the velocity is maximum and positive, label this with "v_0"
(v) Mark a point on the graph where the acceleration is maximum and positive, label this with "a_0"
(vi) Determine the value of the maximum velocity v_0

(i) Identify the period *T* of the oscillating object on the graph

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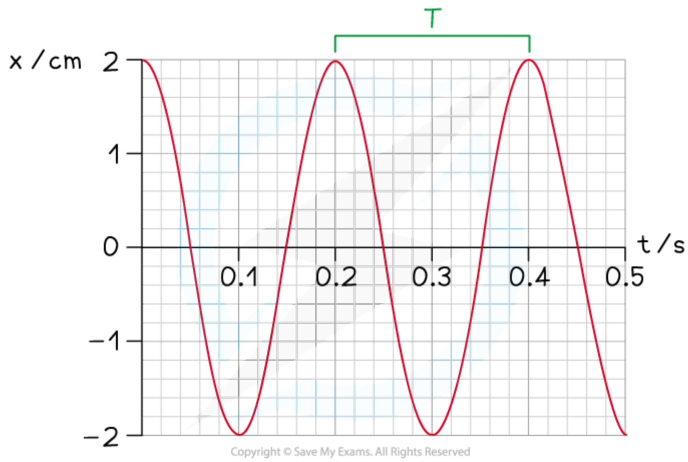
- Mark the time between any two identical points on the graph (e.g. two peaks)



$T = 0.20$ s

(ii) Calculate the frequency *f*

Step 1: Write down the relationship between frequency and period

$$f = \frac{1}{T}$$

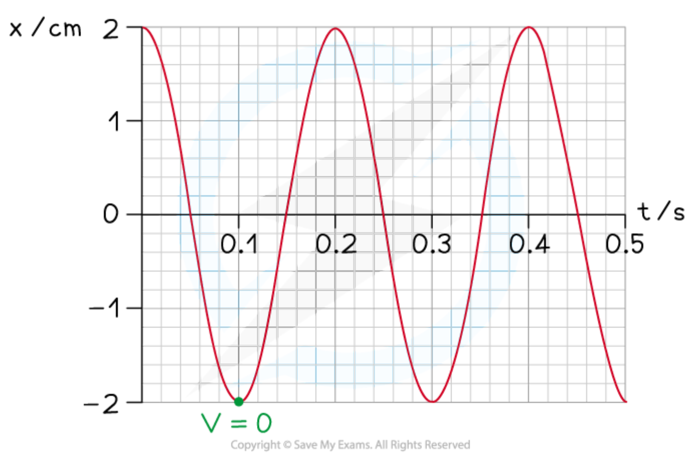
Step 2: Substitute the value of the period you have determined in part (i)

$$f = \frac{1}{0.20}$$

$f = 5$ Hz

(iii) Identify any position of zero velocity on the displacement-time graph and label this "v = 0"

- The velocity of an object oscillating with SHM is zero at the positions of maximum displacement $x = x_0$
- Hence, the velocity is zero at any minima or maxima on the displacement-time graph (e.g. at $t = 0.10$ s)



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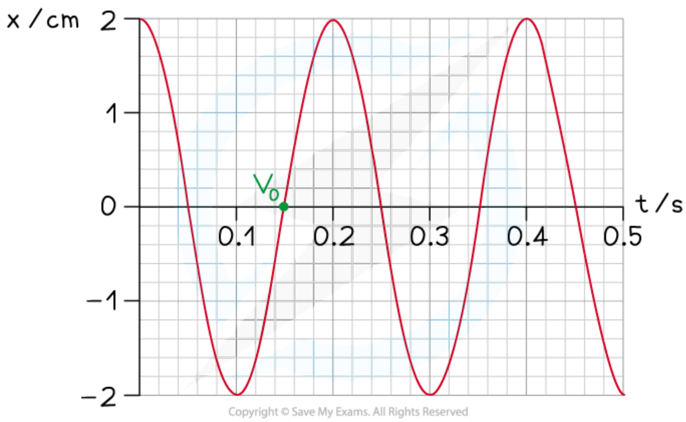
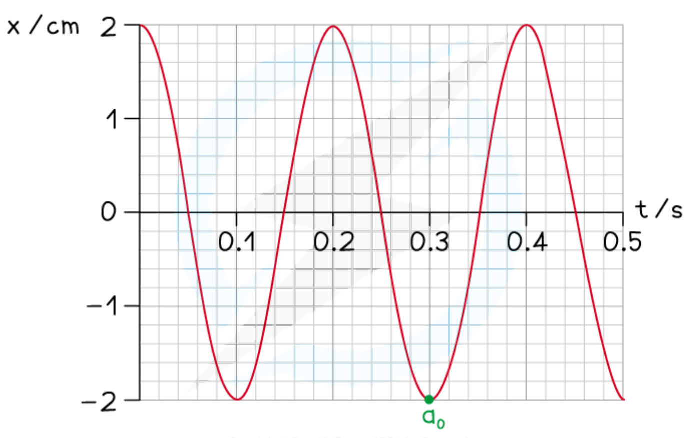
(iv) Identify any position of maximum positive velocity on the displacement-time graph and label this "v_0"

- An object oscillating with SHM has its maximum velocity at the equilibrium position (x = 0)
- Velocity is defined as the rate of change of displacement
- The velocity is the gradient of the tangent to a point of zero displacement
- The gradient must be positive (e.g. at t = 0.15 s)



(v) Identify any position of maximum positive acceleration on the displacement-time graph and label this "a_0"

- An object oscillating with SHM has its maximum acceleration at the positions of maximum displacement ($x = x_0$)
- Acceleration is proportional to negative displacement
- The acceleration is maximum and positive when the displacement is maximum and negative
- The acceleration is maximum and positive at any minima on the displacement-time graph (e.g. at t = 0.30 s)



(vi)

Step 1: Draw the tangent to the point of maximum positive velocity identified in Step 4 (i.e. at t = 0.15 s)

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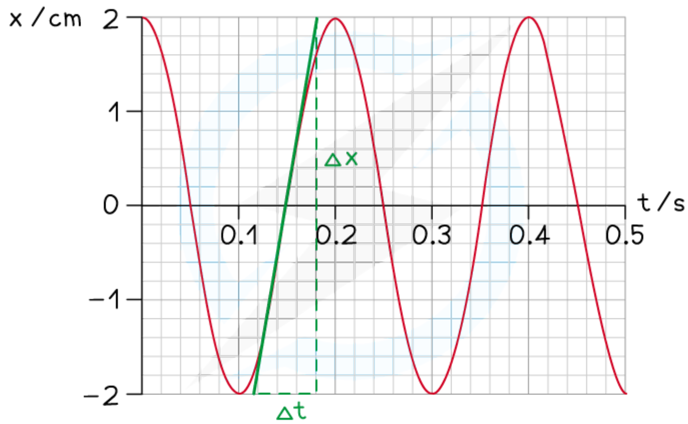

Step 2: Calculate the gradient of the tangent to get the value of the maximum velocity v_0 in centimetres per second (cm s^{-1})

$$Gradient = \frac{\Delta x}{\Delta t}$$

$$Gradient = \frac{4}{0.06}$$

Gradient = 67 cm s^{-1}

v_0 = 67 cm s^{-1}

4.1.4 Energy in SHM

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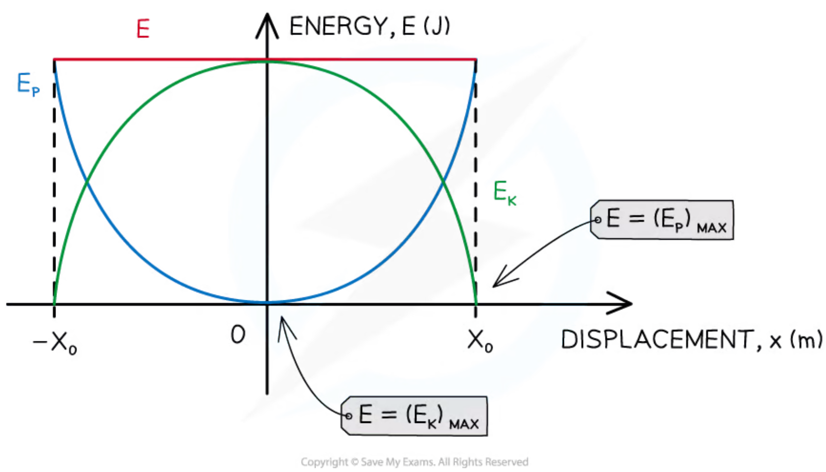
Energy in SHM

- The **total energy** of an object oscillating with SHM is the **sum of** its **potential energy** (gravitational or elastic) and **kinetic energy**

$$E = E_P + E_K$$

- Where:
  - E = total energy in joules (J)
  - E_P = potential energy in joules (J)
  - E_K = kinetic energy in joules (J)



Graph of total energy E, potential energy E_P and kinetic energy E_K of an object oscillating with SHM

- If energy losses due to friction or drag are zero or ignored, the **total energy *E*** of the system is **conserved**
- The **potential energy** store of the object is at a **maximum** at the point of maximum displacement from the equilibrium position
  - The point of maximum displacement is **amplitude** x_0
  - Kinetic energy is zero at amplitude
  - Potential energy is equal to the total energy of the system at this point
- Energy is **transferred** from the object's potential energy store to its kinetic energy store as the object moves from amplitude to the equilibrium position
  - The object has both potential and kinetic energy
  - The sum of the potential and kinetic energy is equal to the total energy of the system
  - The total energy of the system is conserved
- The **kinetic energy** store of the object is at a **maximum** at the **equilibrium position**
  - This is because velocity is at a maximum as the object passes through the equilibrium position
  - Kinetic energy is equal to the total energy of the system at this point

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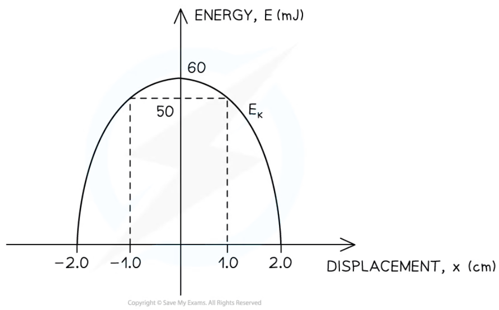
- Energy is **transferred** from the object's kinetic energy store to its potential energy store as the object moves from the equilibrium position to amplitude
  - The object has both potential and kinetic energy
  - The sum of the potential and kinetic energy is equal to the total energy of the system
  - The total energy of the system is conserved

Worked Example

The following graph shows the variation with displacement of the kinetic energy of an object of mass 0.50 kg oscillating with SHM. Energy losses can be neglected.



(i) Determine the total energy of the object
(ii) Determine the amplitude of the object's oscillations
(iii) Calculate the maximum velocity of the object in metres per second (m s^{-1})
(iv) Determine the potential energy of the object when the displacement is x = 1.0 cm

(i) Determine the total energy of the object by reading the maximum value of the kinetic energy from the graph

- From the graph, read the maximum value of the object's kinetic energy $(E_K)_{MAX}$ = 60 mJ
- Recall that, at the equilibrium position (x = 0), the total energy E is exactly equal to the maximum value of the kinetic energy $(E_K)_{MAX}$
- Since energy losses can be neglected, the total energy is constant

$$E = 60 \text{ mJ}$$

(ii) Read the amplitude of the object's oscillations from the graph

- The maximum displacement positions are the locations on either side of the equilibrium position where the kinetic energy is zero $E_K = 0$

$$x_0 = 2.0 \text{ cm}$$

(iii)

Step 1: Recall the equation for the kinetic energy E_K of an object in terms of its mass m and velocity v

$$E_K = \frac{1}{2}mv^2$$

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Step 2: Rearrange the above equation to calculate the velocity *v*

$$v = \sqrt{\frac{2E_K}{m}}$$

Step 3: Substitute the numbers into the equation to calculate the maximum velocity of the object

- Mass of the object, m = 0.50 kg
- You must convert the maximum kinetic energy must from millijoules (mJ) into joules (J)

$$E_K = 60 \text{ mJ} = 0.06 \text{ J}$$

$$v = \sqrt{\frac{2 \times 0.06}{0.50}}$$

$$v = 0.49 \text{ m s}^{-1}$$

(iv)

Step 1: Read the value of the kinetic energy E_K of the object when the displacement is x = 1.0 cm

$$E_K = 50 \text{ mJ}$$

Step 2: Write down the relationship between total energy E, kinetic energy E_K and potential energy E_P

$$E = E_P + E_K$$

Step 3: Rearrange the above equation to calculate the potential energy E_P

$$E_P = E - E_K$$

Step 4: Substitute the numbers in the above equation

- E_K = 50 mJ
- E = 60 mJ

$$E_P = 60 \text{ mJ} - 50 \text{ mJ}$$

$$E_P = 10 \text{ mJ}$$

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4.2 Travelling Waves

4.2.1 Properties of Waves

Properties of Waves

- **Travelling waves** are defined as follows:

  Oscillations that transfer energy from one place to another without transferring matter

- Energy is transferred by the waves, but matter is not
- The direction of the motion of the wave is the direction of the energy transfer
- Travelling waves can be of two types:
  - **Mechanical Waves**, which propagate through a medium and cannot take place in a vacuum
  - **Electromagnetic Waves**, which can travel through a vacuum
- Waves are generated by **oscillating sources**
  - These oscillations travel **away** from the source
- Oscillations can propagate through a **medium** (e.g. air, water) or in **vacuum** (i.e. no particles), depending on the type of wave
- The key properties of **travelling waves** are as follows:
- **Displacement** (x) of a wave is the distance of a point on the wave from its equilibrium position
  - It is a vector quantity; it can be positive or negative
  - Measured in metres (m)
- **Wavelength** (λ) is the length of one complete oscillation measured from same point on two consecutive waves
  - For example, two crests, or two troughs
  - Measured in metres (m)
- **Amplitude** (x_0) is the maximum displacement of an oscillating wave from its equilibrium position ($x = 0$)
  - Amplitude can be positive or negative depending on the direction of the displacement
  - Measured in metres (m)
- **Period** (T) is the time taken for a fixed point on the wave to undergo one complete oscillation
  - Measured in seconds (s)
- **Frequency** (f) is the number of full oscillations per second
  - Measured in Hertz (Hz)
- **Wave speed** (c) is the distance travelled by the wave per unit time
  - Measured in metres per second ($m\ s^{-1}$)

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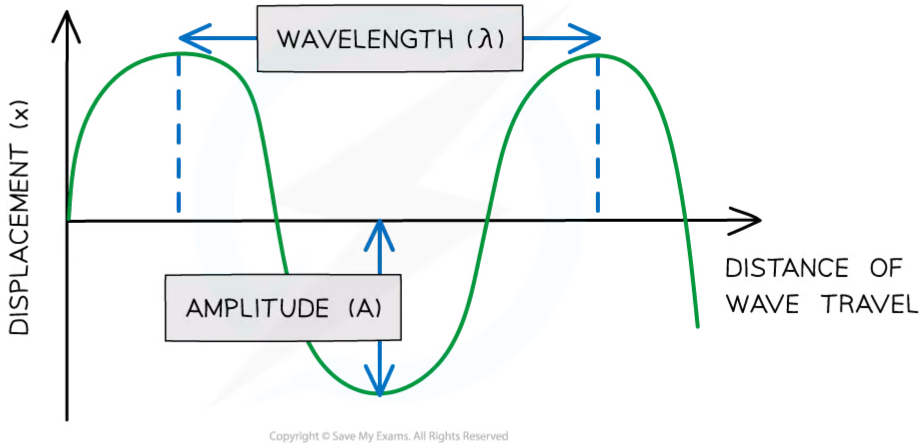
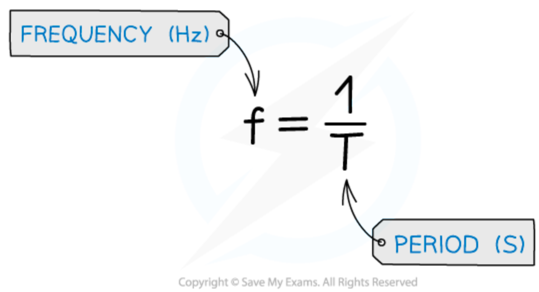
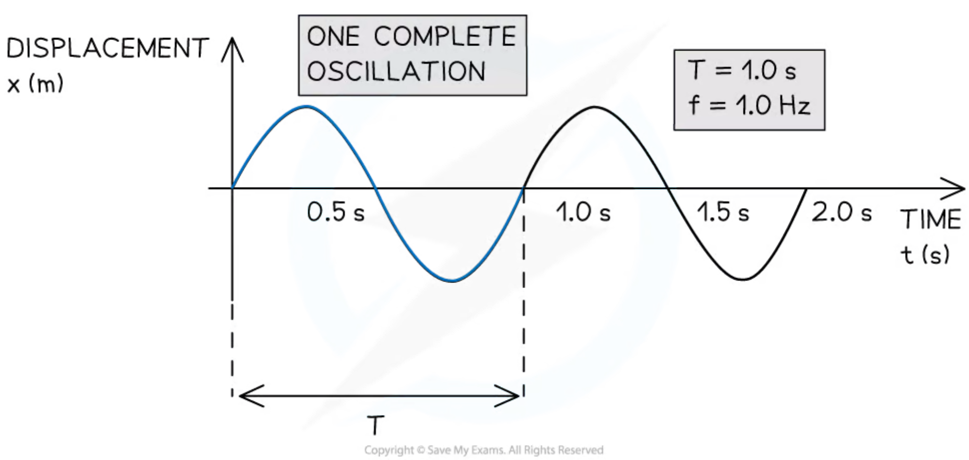


Diagram showing the amplitude and wavelength of a wave

- The frequency, *f*, and the period, *T*, of a travelling wave are related to each other by the equation:



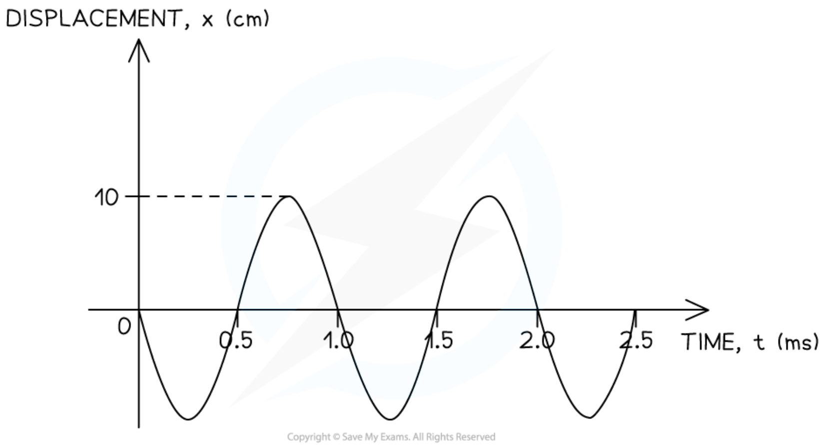


Period T and frequency f of a travelling wave

YOUR NOTES

Worked Example

The graph below shows a travelling wave.



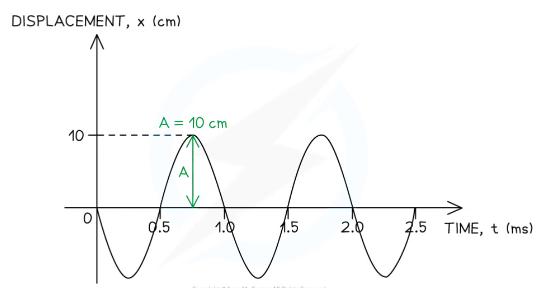
Determine:

(i) The amplitude *A* of the wave in metres (m)

(ii) The frequency *f* of the wave in hertz (Hz)

(i) Identify the amplitude *A* of the wave on the graph

- The amplitude is defined as the maximum displacement from the equilibrium position ($x = 0$)



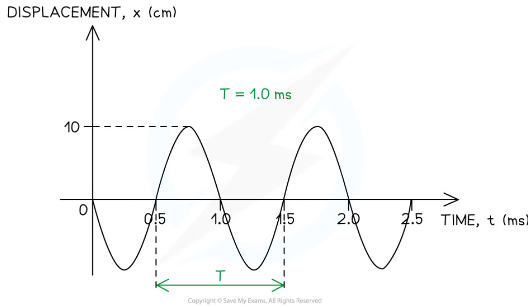
- The amplitude must be converted from centimetres (cm) into metres (m)

$$A = 0.1 \text{ m}$$

(ii) Calculate the frequency of the wave

Step 1: Identify the period *T* of the wave on the graph

- The period is defined as the time taken for one complete oscillation to occur



YOUR NOTES

- The period must be converted from milliseconds (ms) into seconds (s)

$$T = 1 \times 10^{-3}\ \text{s}$$

Step 2: Write down the relationship between the frequency *f* and the period *T*

$$f = \frac{1}{T}$$

Step 3: Substitute the value of the period determined in Step 1

$$f = \frac{1}{1 \times 10^{-3}}$$

$$f = 1000\ \text{Hz}$$

The Wave Equation

YOUR NOTES
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- The wave equation describes the relationship between the wave speed, the wavelength and the frequency of the wave

$$c = f\lambda$$

- Where
  - c = wave speed in metres per second ($m\ s^{-1}$)
  - f = frequency in hertz (Hz)
  - λ = wavelength in metres (m)

Deriving the Wave Equation

- The wave equation can be derived using the equation for speed

$$v = \frac{d}{t}$$

- Where
  - v = velocity or speed in metres per second ($m\ s^{-1}$)
  - d = distance travelled in metres (m)
  - t = time taken in seconds (s)
- When the source of a wave undergoes one complete oscillation, the travelling wave propagates forward by a **distance** equal to one wavelength λ
- The travelling wave covers this distance in the **time** it takes the source to complete one oscillation, the time period T

$$\text{wave speed} = \frac{\text{distance for one oscillation}}{\text{time taken for one oscillation}} = \frac{\lambda}{T}$$

- Therefore, the **wave speed c** is given by

$$c = \frac{\lambda}{T}$$

- The period T of a wave is given by:

$$T = \frac{1}{f}$$

- Therefore, combining these equations gives the wave equation

$$c = f\lambda$$

Worked Example

A travelling wave has a period of 1.0 μs and travels at a velocity of 100 cm s^{-1}.
Calculate the wavelength of the wave. Give your answer in metres (m).

YOUR NOTES
⬇

Step 1: Write down the known quantities

- Period, $T = 1.0\ \mu s = 1.0 \times 10^{-6}$ s
- Velocity, c = 100 cm s^{-1} = 1.0 m s^{-1}

Note the conversions:

- The period must be converted from microseconds (μs) into seconds (s)
- The velocity must be converted from cm s^{-1} into m s^{-1}

Step 2: Write down the relationship between the frequency *f* and the period *T*

$$f = \frac{1}{T}$$

Step 3: Substitute the value of the period into the above equation to calculate the frequency

$$f = \frac{1}{1 \times 10^{-6}}$$

$$f = 1.0 \times 10^{6} \text{ Hz}$$

Step 4: Write down the wave equation

$$c = f\lambda$$

Step 5: Rearrange the wave equation to calculate the wavelength λ

$$\lambda = \frac{c}{f}$$

Step 6: Substitute the numbers into the above equation

$$\lambda = \frac{1.0}{1 \times 10^{6}}$$

$$\lambda = 1 \times 10^{-6} \text{ m}$$

4.2.2 Transverse & Longitudinal Waves

YOUR NOTES
⬇

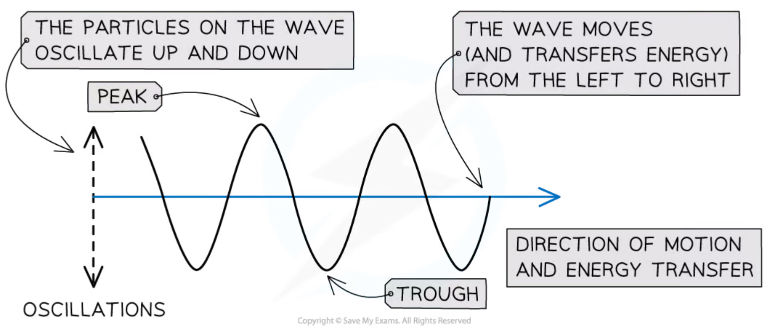
Transverse & Longitudinal Waves

- In mechanical waves, particles **oscillate** about fixed points
- There are two types of wave: **transverse** and **longitudinal**
- The type of wave can be determined by the direction of the oscillations in relation to the direction the wave is travelling

Transverse Waves

- Transverse waves are defined as follows:

> **A wave in which the particles oscillate perpendicular to the direction of motion and energy transfer**



A transverse wave travelling from left to right

- Transverse waves show areas of **peaks** and **troughs**
- Examples of transverse waves include:
  - Electromagnetic waves e.g. radio, visible light, UV
  - Vibrations on a guitar string
- Transverse waves do not need particles to propagate, and so they **can travel through a vacuum**

Longitudinal Waves

- Longitudinal waves are defined as follows:

> **A wave in which the particles oscillate parallel to the direction of motion and energy transfer**

YOUR NOTES
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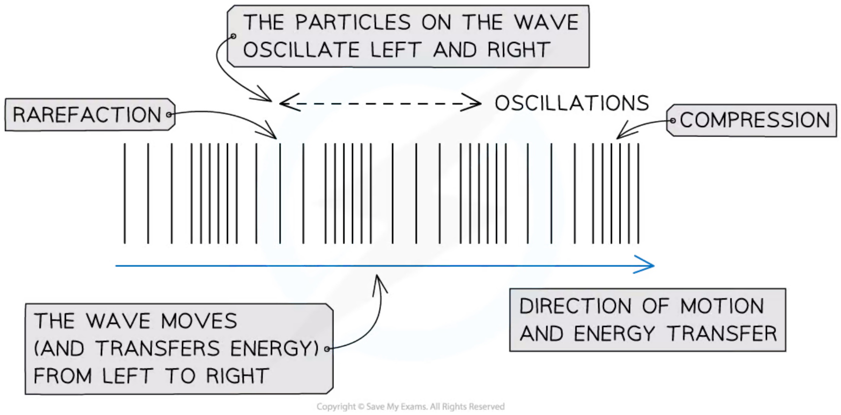
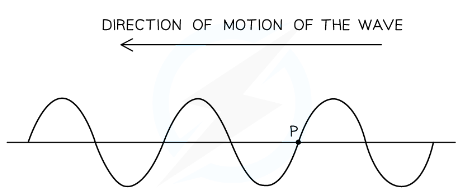


A longitudinal wave travelling from left to right

- As a longitudinal wave propagates, areas of low and high pressure can be observed:
  - A **rarefaction** is an area of low pressure, with the particles being further apart from each other
  - A **compression** is an area of high pressure, with the particles being closer to each other
- **Sound waves** are an example of longitudinal waves
- Longitudinal waves need particles to propagate, and so they **cannot travel through a vacuum**

Worked Example

The diagram below represents a transverse wave at time *t* = 0. The direction of motion of the wave is shown. Point **P** is a point on the wave. State in which direction point **P** will move immediately after the time shown.



Step 1: Determine the possible directions that point P can travel in

- In transverse waves, the particles oscillate perpendicular to the direction of motion
- This transverse wave travels from right to left
- Oscillations will either be up or down
- Hence point **P** will either move up or down

YOUR NOTES
⬇

Step 2: Determine the next direction of point P

- Since the wave is moving from right to left, a crest (i.e. a point of maximum displacement above the equilibrium position) will be approaching point P immediately after the time shown
- Point P will be moving upwards

Transverse & Longitudinal Graphs

YOUR NOTES
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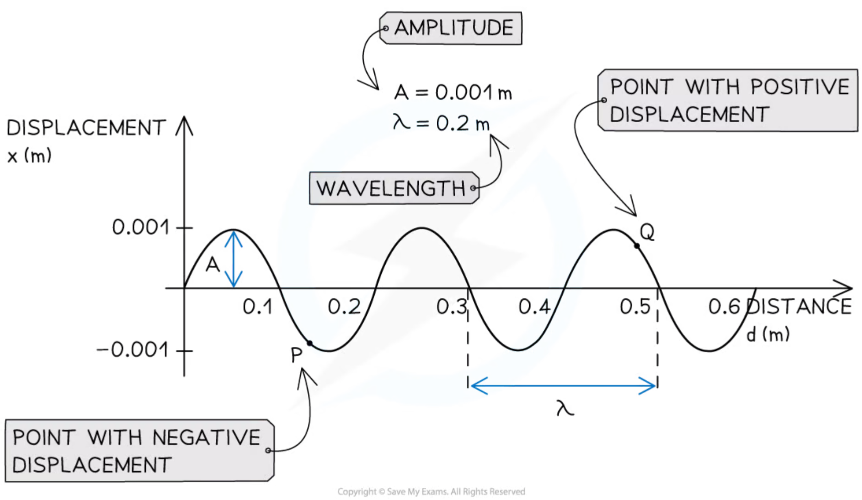
- Two different types of graphs can be used to represent a travelling wave:
  - **Displacement-distance graphs**, with displacement *x* (m) on the y-axis and distance *d* (m) on the x-axis
  - **Displacement-time graphs**, with displacement *x* (m) on the y-axis and time *t* (s) on the x-axis

- Both transverse and longitudinal waves can be shown on these graphs

Displacement-Distance Graphs

- A displacement-distance graph is also known as a **wave profile**
- It represents the **displacement** of many particles on the wave at a **fixed instant** in time (e.g. $t = 0$)
- A displacement-distance graph directly provides:
  - The **amplitude *A*** of the wave
  - The **wavelength *λ*** of the wave



An example of displacement-distance graph for a travelling wave

- In the displacement-distance graph of a **transverse wave** moving in the horizontal direction:
  - Particles with **positive displacement** are those moving **up**
  - Particles with **negative displacement** are those moving **down**

- In the displacement-distance graph of a **longitudinal wave** moving in the horizontal direction:
  - Particles with **positive displacement** are those moving to the **right**
  - Particles with **negative displacement** are those moving to the **left**

- The wavelength in a longitudinal wave can be figured out from the distance between two rarefactions (or two compressions)

YOUR NOTES
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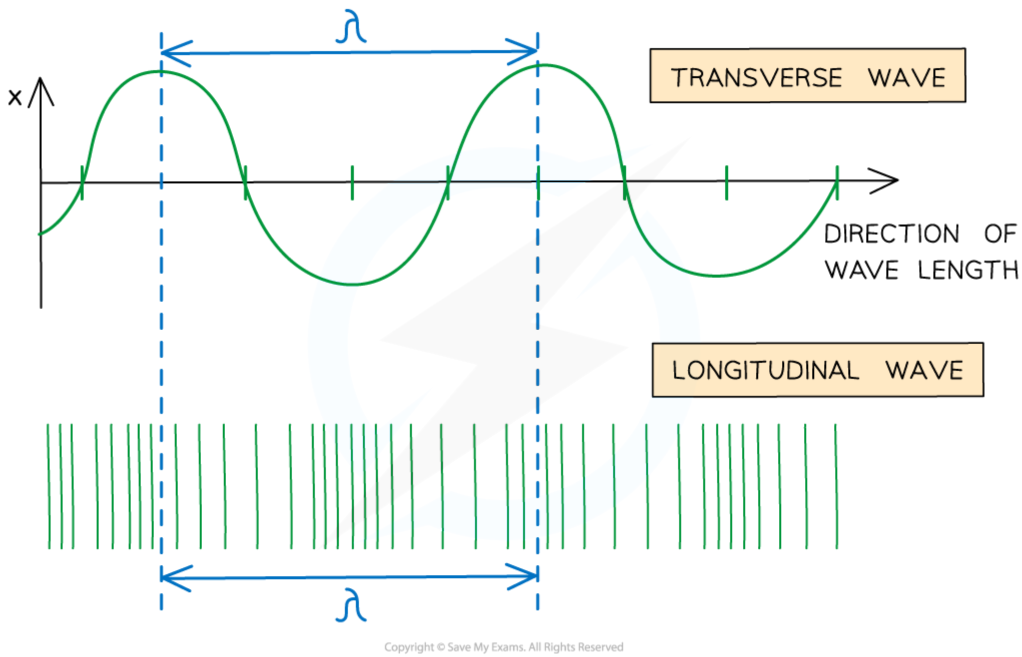
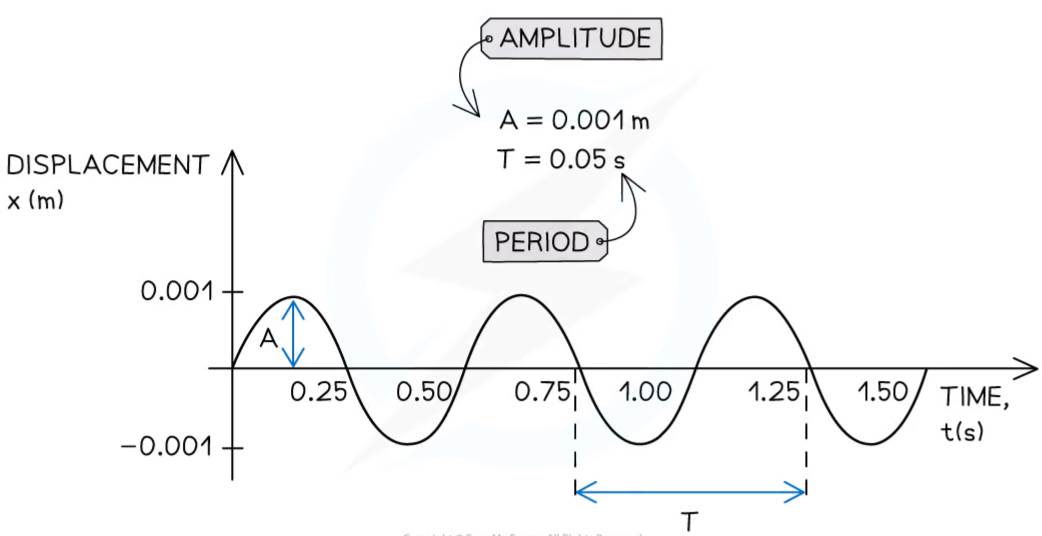


A wavelength on a longitudinal wave is the distance between two compressions or two rarefactions

Displacement-Time Graphs

- A displacement-time graphs represents the variation of the **displacement** of one particle with **time**
- A displacement-time graph directly provides:
  - The **amplitude** ***A*** of the wave
  - The **period** ***T*** of the wave



An example of displacement-time graph for a travelling wave

YOUR NOTES
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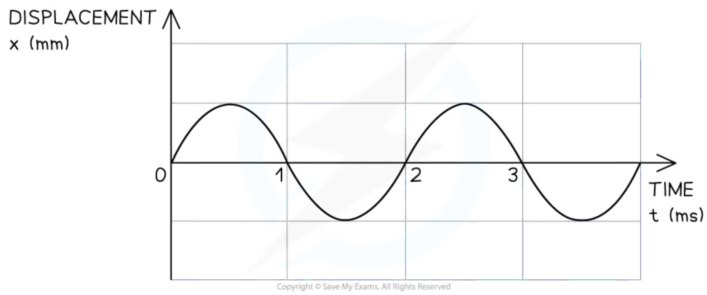
Worked Example

Below is the displacement-time graph for a light wave travelling at 3×10^8 m s^{-1}.



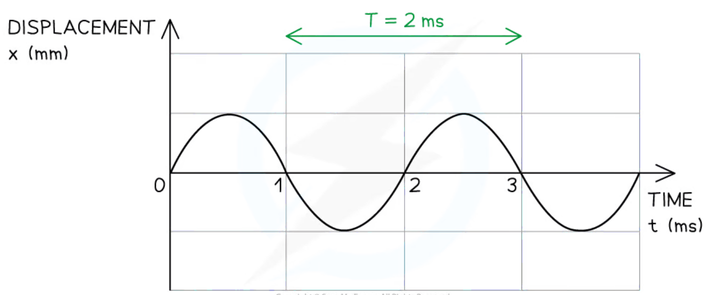
Determine:

(i) The period of the wave in seconds (s)

(ii) The wavelength of the wave in metres (m)

(i) Determine the period *T* directly from the displacement-time graph

- Recall that period is defined as the time taken for one complete oscillation
- Note that you must convert the period from milliseconds (ms) into seconds (s)



$$T = 2 \times 10^{-3}\ \text{s}$$

(ii) Determine the wavelength of the wave in metres

Step 1: Write down the relationship between frequency *f* and period *T*

$$f = \frac{1}{T}$$

Step 2: Substitute the value of the period determined in Step 1 into the above equation

$$f = \frac{1}{2 \times 10^{-3}}$$

$$f = 500\ \text{Hz}$$

Step 3: Write down the wave equation

$$c = f\lambda$$

Step 4: Rearrange the above equation to calculate the wavelength λ

$$\lambda = \frac{c}{f}$$

Step 5: Substitute the velocity c = 3×10^8 m s^{-1} and the frequency *f* calculated in Step 2

$$\lambda = \frac{3 \times 10^8}{500}$$

$$\lambda = 6 \times 10^5 \text{ m}$$

YOUR NOTES

Exam Tip

When approaching a question, pay attention to the label on the x-axis of the graph.

- The distance between two adjacent crests (or troughs) on a displacement-distance graph is equal to the wavelength λ of the wave
- The distance between two adjacent crests (or troughs) on a displacement-time graph is equal to the period *T* of the wave

Remember to look at the units of measure on both axes of the graph, and convert units if needed.

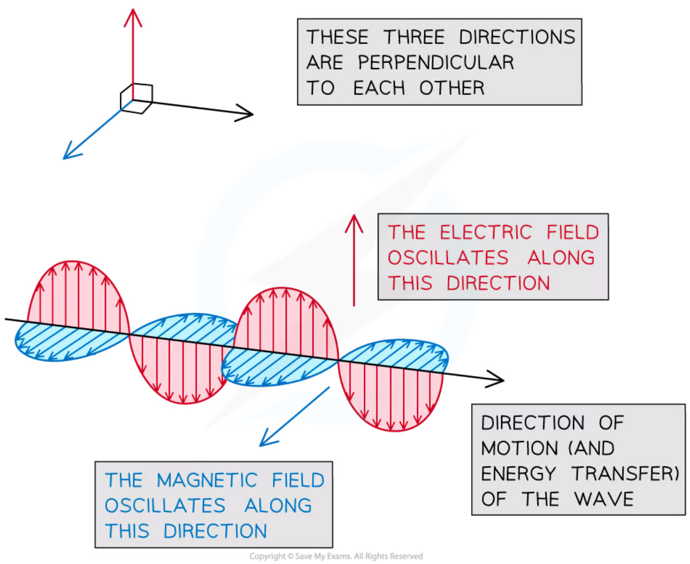
The speed of any electromagnetic wave is equal to the speed of light, c = 3×10^8 m s^{-1}

4.2.3 Electromagnetic Waves

YOUR NOTES

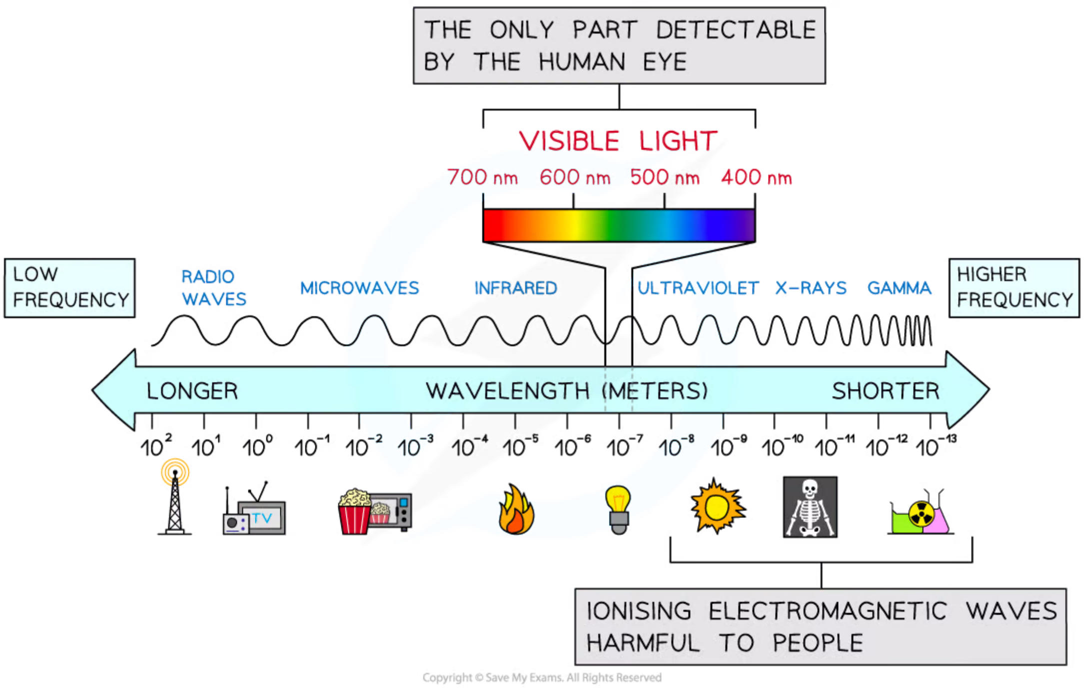
Electromagnetic Waves

- An electromagnetic wave is generated by the combined oscillation of an **electric** and a **magnetic field**
- These fields oscillate **perpendicular** to each other and to the direction of motion of the wave (i.e. the direction in which energy is transferred)



An electromagnetic wave is generated by the combined oscillation of an electric and a magnetic field

- Electromagnetic waves are **transverse waves** and, as such, they can travel through **vacuum**
- Regardless of their frequency, all electromagnetic waves travel at the **speed of light** $c = 3 \times 10^8$ m s^{-1} in vacuum
- Electromagnetic waves form a continuous **spectrum** based on their frequency



The electromagnetic spectrum

- Humans can only sense electromagnetic waves with wavelengths in the range 700 nm - 400 nm, which are the limits of the so-called **visible spectrum**
- Electromagnetic waves with longer and shorter wavelengths are invisible to the human eye

Electromagnetic Wave	Wavelength, λ (m)
Radio Waves	$> 10^{-3}$
Microwaves	$1 \times 10^{-3} - 2.5 \times 10^{-5}$
Infrared	$2.5 \times 10^{-5} - 7 \times 10^{-7}$
Visible Light	$7 \times 10^{-7} - 4 \times 10^{-7}$
Ultraviolet	$4 \times 10^{-7} - 1 \times 10^{-9}$
X-rays	$1 \times 10^{-9} - 1 \times 10^{-12}$
Gamma Rays	$< 10^{-12}$

- Knowing the wavelengths of electromagnetic waves, their frequencies can be calculated using the **wave equation**, and remembering that their speed in vacuum is always equal to the speed of light

YOUR NOTES
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Worked Example

The wavelength of blue light falls within the range 450 nm - 490 nm. Determine the range of frequencies of blue light.

Step 1: Write down the known quantities

- Note that you must convert the values of the wavelength from nanometres (nm) into metres (m)
  - λ_{lower} = 450 nm = 4.5×10^{-7} m
  - λ_{higher} = 490 nm = 4.9×10^{-7} m

Step 2: Remember that all electromagnetic waves travel at the speed of light in vacuum

- From the data booklet, $c = 3 \times 10^8$ m s^{-1}

Step 3: Write down the wave equation

$$c = f\lambda$$

Step 4: Rearrange the above equation to calculate the frequency *f*

$$f = \frac{c}{\lambda}$$

Step 5: Substitute the lower and higher values of the wavelength to calculate the limiting values of the frequency of blue light

- The lower frequency f_{lower} corresponds to the higher value of the wavelength λ_{higher}

$$f_{lower} = \frac{3 \times 10^8}{4.9 \times 10^{-7}}$$

$$= 6.1 \times 10^{14} \text{ Hz}$$

- The higher frequency f_{higher} corresponds to the lower value of the wavelength λ_{lower}

$$f_{higher} = \frac{3 \times 10^8}{4.5 \times 10^{-7}}$$

$$= 6.7 \times 10^{14} \text{ Hz}$$

Step 6: Write down the range of frequencies of blue light

$$f = 6.1 \times 10^{14} \text{ Hz} - 6.7 \times 10^{14} \text{ Hz}$$

Exam Tip

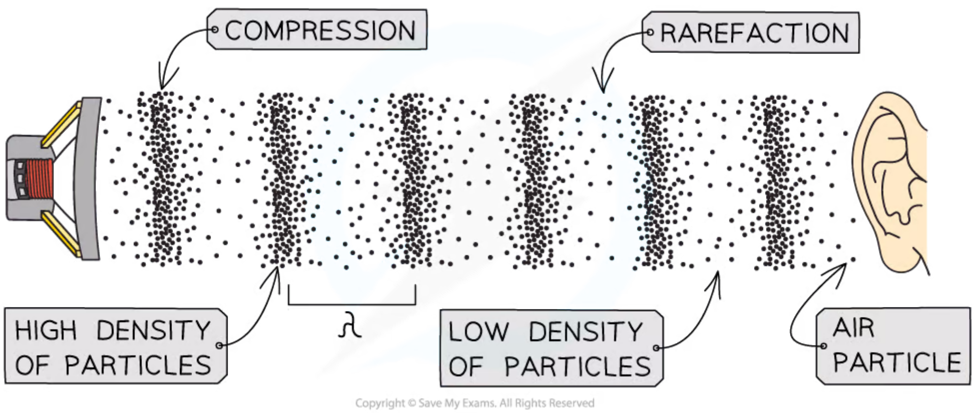
You must memorise all electromagnetic waves in the correct order, as well as the range of wavelengths for each type of wave. You must also remember that all electromagnetic waves travel at the speed of light in vacuum.

4.2.4 Sound Waves

YOUR NOTES ↓

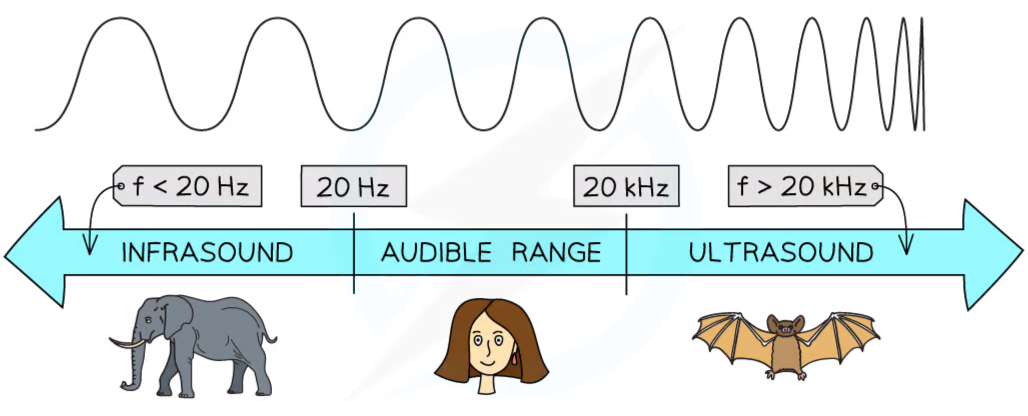
Sound Waves

- Sound waves are **longitudinal waves** and, as such, require a **medium** in which to propagate
- Sound waves are generated by oscillating sources, which produce a change in **density** of the surrounding medium
- The sound wave then travels with a series of **compressions** and **rarefactions**



A sound wave travelling through air

- Sound waves form a continuous **spectrum** based on their frequency



The spectrum of sound waves

- Humans can only hear sounds with frequencies in the range 20 Hz - 20 kHz, known as the **audible range**
- Sounds with frequencies below and above this range cannot be detected by the human ear

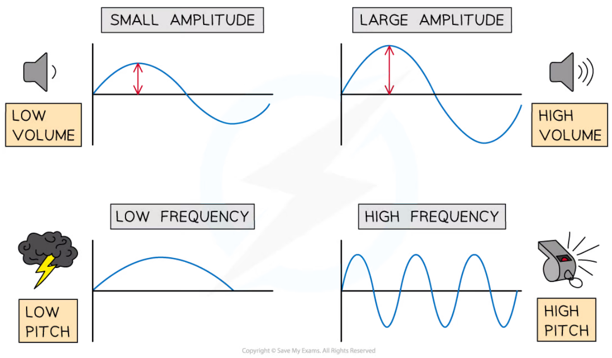
Pitch and Volume

- The frequency of a sound wave is related to its **pitch**
  - Sounds with a **high** pitch have a **high** frequency (or short wavelength)

YOUR NOTES
⬇

- Sounds with a **low** pitch have a **low** frequency (or long wavelength)

- The amplitude of a sound wave is related to its **volume**
  - Sounds with a **large** amplitude have a **high** volume
  - Sounds with a **small** amplitude have a **low** volume



Pitch and amplitude of sound

Speed of Sound

- Sound waves travel at a speed of about 340 m s^{-1} in air at room temperature
  - The higher the air temperature, the greater the speed of sound
  - The is because the average kinetic energy of the particles is higher

- Sound travels the **fastest** through **solids**, since solid particles are closely packed and can pass the oscillations onto their neighbours much faster
- Sound travels the **slowest** in **gases**, since gas particles are spread out and less efficient in transferring the oscillations to their neighbours

Echo

- Sound waves **reflect** off hard surfaces
- This phenomenon is known as **echo**
- Echo can be used to obtain an experimental value of the speed of sound. This is calculated using the equation

$$v = 2 \times \frac{d}{t}$$

- Where:
  - v = speed of sound in metres per second (m s^{-1})
  - d = distance between the sound source and the hard surface (m)
  - t = time taken to travel from the source to the hard surface and back (s)

Measuring the Speed of Sound Experimentally - Fast Timer

- The speed of sound can be measured using a fast timer (one which can measure to the nearest millisecond or even microsecond)
- Two microphones separated 1 m apart are connected to a fast timer
  - The first microphone triggers the timer to start
  - The second microphone triggers the timer to stop

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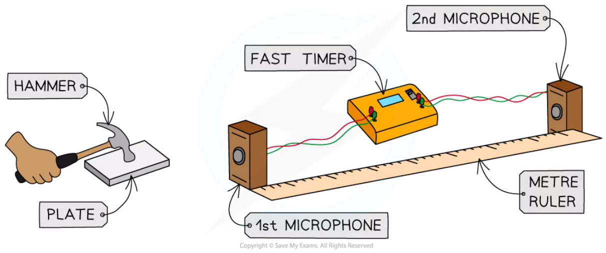
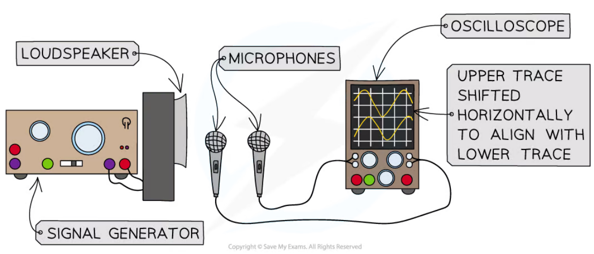


- A hammer is made to strike a plate
- The sound waves from the plate travel to the two microphones triggering the first and then the second
  - The time delay will be around 3.2 ms
- The speed of the waves can be calculated by rearranging the equation: *distance = speed × time*

Measuring the Speed of Sound Experimentally - Double Beam Oscilloscope

- Two microphones are connected to the input of a double beam oscilloscope
- A signal generator is connected to a loudspeaker and set to a frequency between 500 Hz and 2.0 kHz
  - One of the microphones is close to the loud speaker
  - The other microphone is 1 m away



- There will be two traces that appear on the screen
- The traces are compared as the second microphone is moved back and forth in line with the first microphone and the speaker
- Use a ruler to measure the distance that the second microphone needs to move for the traces to be in phase then out and phase and back in phase again
  - This distance is equal to the wavelength of the wave
- The speed of the waves are therefore calculated using $c = f\lambda$

Worked Example

A person stands 50 m from a wall. The person claps their hands repeatedly, and changes the clapping frequency until the echoes are synchronised with the claps. A mobile phone application measures the time between the claps, which is $t = 0.30$ s. Determine the speed of sound.

YOUR NOTES
⬇

Step 1: Write down the known quantities

- Distance between the person and the wall, d = 50 m
- Time between the claps, t = 0.30 s

Step 2: Write down the "echo equation"

$$v = 2 \times \frac{d}{t}$$

Step 3: Substitute the numbers into the above equation and calculate the speed of sound *v*

$$v = \frac{2 \times 50}{0.30}$$

$$v = 330 \text{ m s}^{-1}$$

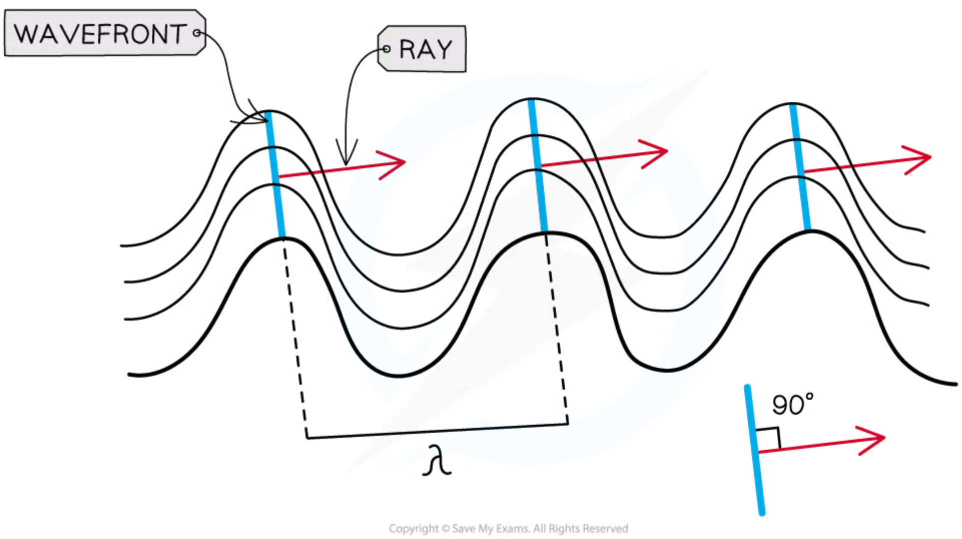
YOUR NOTES
⬇

4.3 Wave Characteristics

4.3.1 Wavefronts

Wavefronts

- Waves can be represented graphically in two different ways:
  - **Wavefronts** - lines joining all the points that oscillate in phase and are perpendicular to the direction of motion (and energy transfer)
  - **Rays** - lines showing the direction of motion (and energy transfer) of the wave that are perpendicular to the wavefront



Wavefronts and rays for transverse waves travelling in a horizontal plane

- The distance between successive wavefronts is equal to the wavelength of the waves

- Ripple tanks are used a common experiment to demonstrate diffraction of water waves

YOUR NOTES
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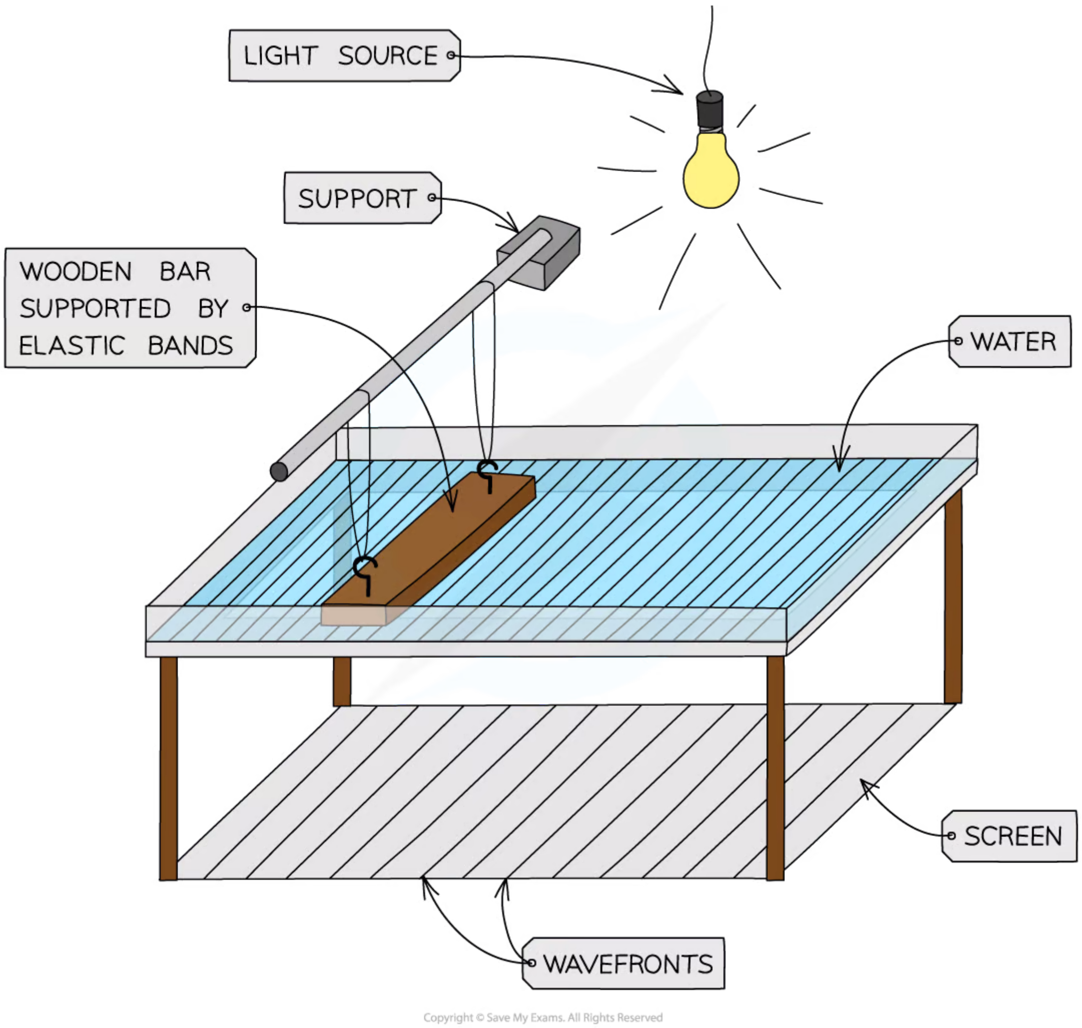

Wave effects may all be demonstrated using a ripple tank

- The diagram below shows how the wavelengths differ with frequency in a ripple tank
  - The **higher** the frequency, the **shorter** the wavelength
  - The **lower** the frequency, the **longer** the wavelength

YOUR NOTES
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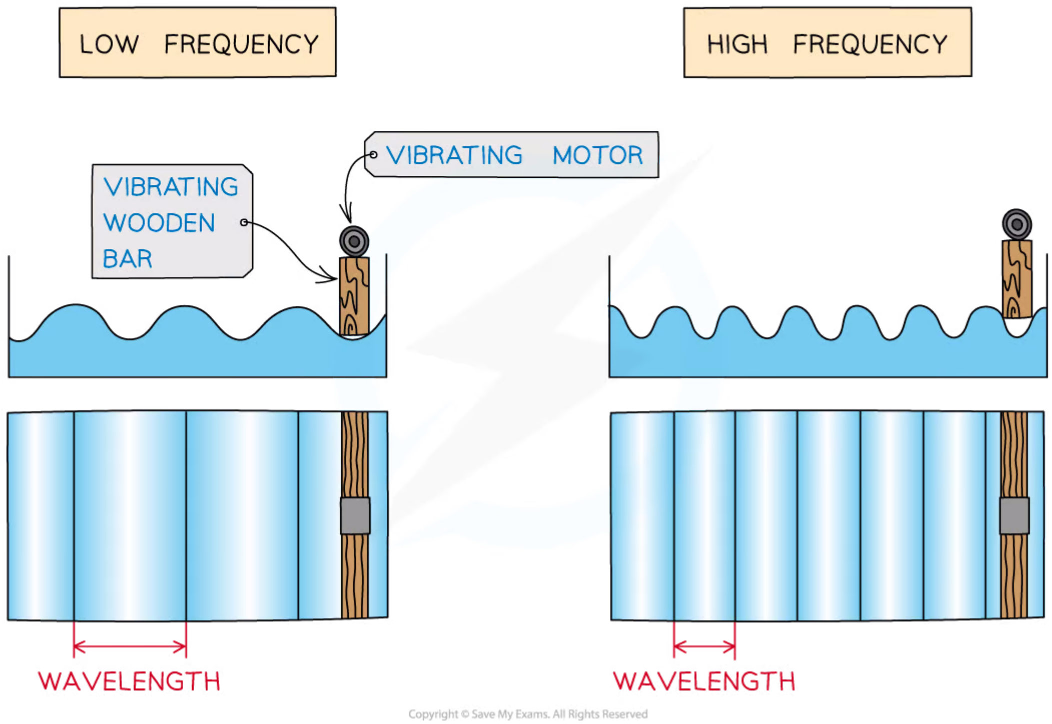

Ripple tank patterns for low and high frequency vibration

4.3.2 Amplitude & Intensity

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Amplitude & Intensity

Intensity

- The **intensity** of a wave is defined as follows:

  Power per unit area

- Intensity is measured in $\mathbf{W\,m^{-2}}$
- Power is defined as:

  The rate of energy transfer

- Therefore, intensity can also be defined as:

  The rate of energy transfer per unit area

- For spherical waves being emitted by a point source equally in all directions, the intensity follows an inverse square law with distance from the point source

$$I \propto \frac{1}{r^2}$$

- Where:
  - I = intensity of the wave in watts per metre squared ($W\,m^{-2}$)
  - r = distance from the point source in metres (m)
- For spherical waves being emitted by a point source equally in all directions, the intensity at the surface of a sphere is calculated using:

$$I = \frac{P}{4\pi r^2}$$

- Where:
  - P = power in watts (W)
  - r = radius of sphere in metres (m)
- For spherical waves being emitted by a point source equally in all directions, the power is a constant, therefore the relationship can be expressed as:

$$I \propto \frac{1}{r^2}$$

- This is an example of an inverse square law relationship

$$I \propto x^{-2}$$

YOUR NOTES
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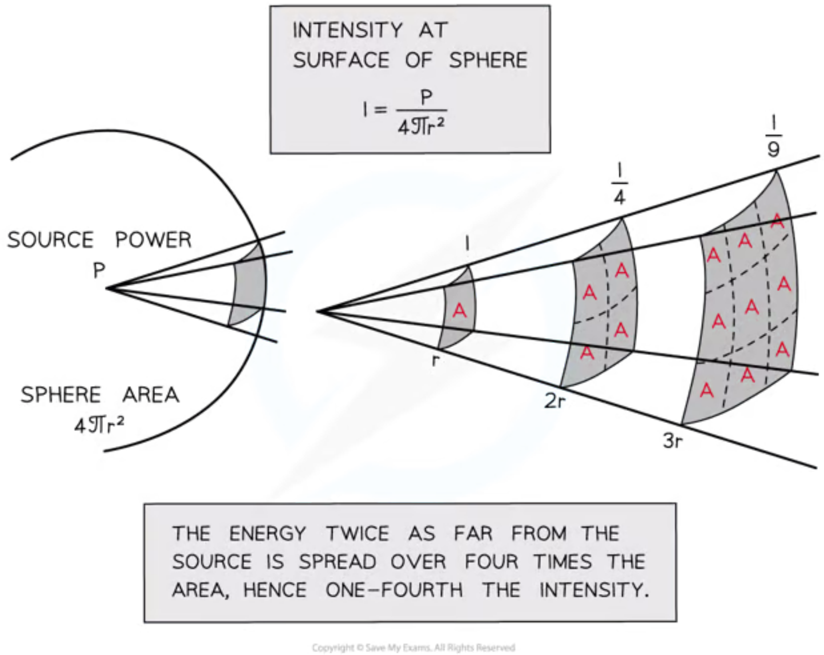

Intensity decreases by the inverse square law

Intensity Variation with Amplitude

- By definition, the intensity of a wave (its power per unit area) is proportional to the **energy transferred** by the wave
- The intensity of a wave at a particular point is related to the amplitude of the wave at that point
- The energy transferred by a wave is proportional to the square of the amplitude
- Therefore, the **intensity** of a wave is proportional to the **square** of the **amplitude**

$$I \propto A^2$$

- Where:
  - I = intensity of the wave in W m^{-2}
  - A = amplitude of the wave in metres (m)

Worked Example

A person stands 10 m away from a loudspeaker. The sound produced by the loudspeaker is very loud, so the person moves 20 m away from it.

State the effect of this change on the intensity and the amplitude of the sound waves heard by the person.

Step 1: Write down the known quantities

- Original distance, r_1 = 10 m
- New distance, r_2 = 20 m

YOUR NOTES
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Step 2: Write down the relationship between the intensity of a wave and the distance from the point source producing the wave

$$I \propto \frac{1}{r^2}$$

Step 2: State the new intensity

- **Since the distance doubles ($r_2 = 2r_1$), the intensity is reduced by a factor four**

$$I_2 = \frac{1}{4} I_1$$

Step 3: Write down the relationship between the intensity of a wave and its amplitude

$$I \propto A^2$$

Step 4: State the new amplitude

- **Since the intensity is reduced by a factor four, the amplitude decreases by half**

$$A_2 = \frac{1}{2} A_1$$

4.3.3 Superposition

YOUR NOTES
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Superposition

- The **principle of superposition** states that:

  When two or more waves meet, the resultant displacement is the vector sum of the displacements of the individual waves

- The principle of superposition applies to both transverse and longitudinal waves
- **Interference** occurs whenever two or more waves superpose
- For a clear stationary interference pattern, the waves must be of the same:
  - Type
  - Amplitude
  - Frequency
- They must also have a constant phase difference

Constructive & Destructive Interference

- **Constructive interference** occurs when the waves superpose and have **displacements in the same direction** (both positive or both negative)
- **Destructive interference** occurs when the waves superimpose and have **displacements in opposite directions** (one positive and one negative)
- When two waves with the same amplitude meet at a point, they can:
  - Be in phase and interfere **constructively**, so that the displacement of the resultant wave is **double** the displacement of each individual wave
  - Be in **anti-phase** and interfere destructively, so that the displacement of the resultant wave is equal to zero

YOUR NOTES
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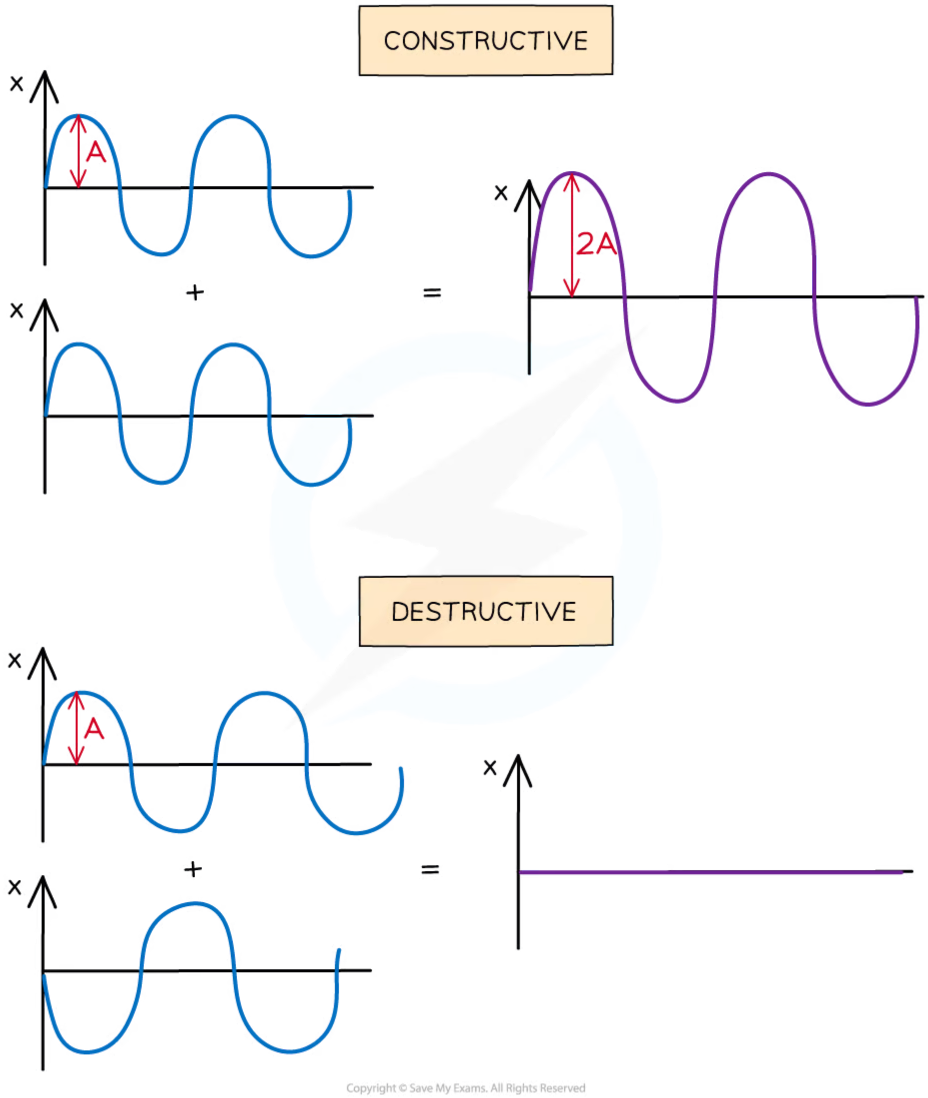
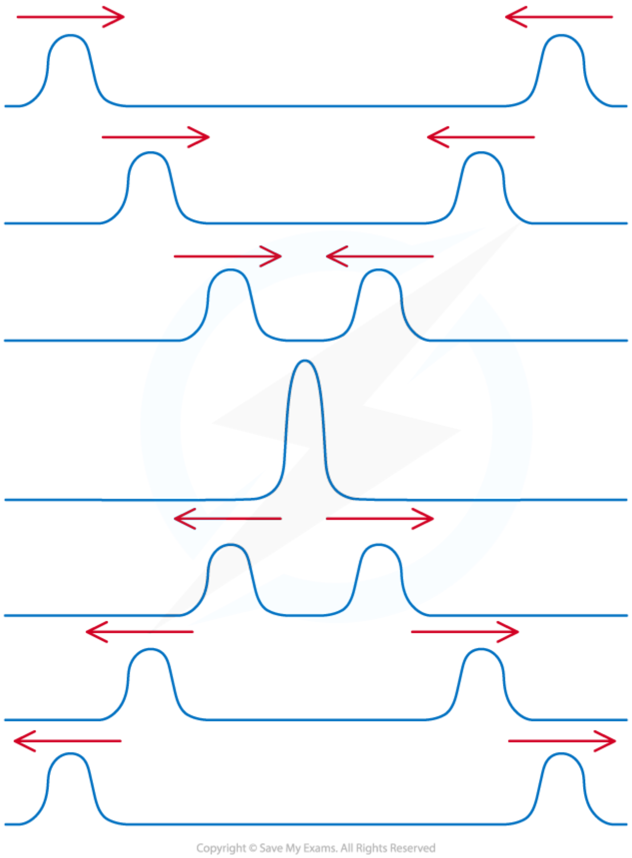


Waves in superposition can undergo constructive or destructive interference

- Superposition occurs for any two waves or pulses that overlap, and can result in a mix of constructive and destructive interference
  - For example, the peak of one wave superposes with the peak of another wave with a smaller displacement
  - The resultant peak will have a displacement that is in the middle of the displacement of both waves
- Superposition can also be demonstrated with two pulses
  - When the pulses meet, the resultant displacement is the algebraic sum of the displacement of the individual pulses
  - After the pulses have interacted, they then carry on as normal



YOUR NOTES
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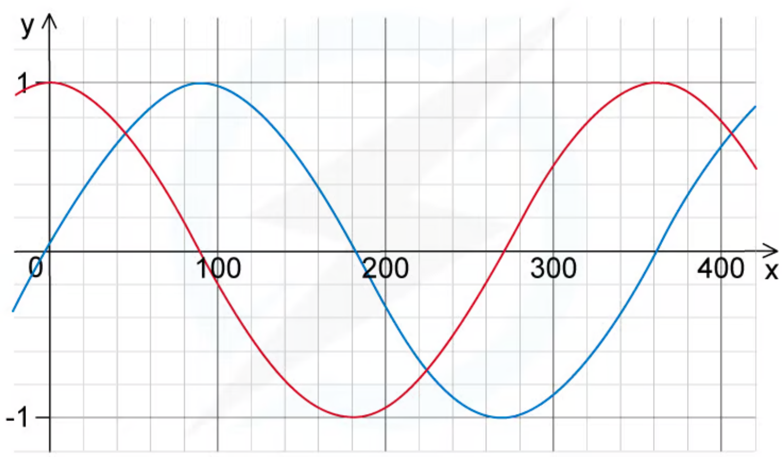
Worked Example

Two overlapping waves of the same types travel in the same direction. The variation with x and y displacement of the wave is shown in the figure below.



Use the principle of superposition to sketch the resultant wave.

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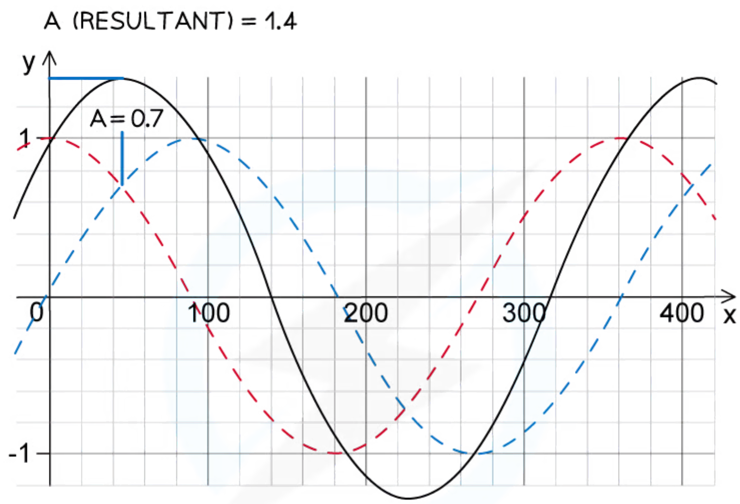
THE GRAPH OF THE SUPERPOSITION OF BOTH WAVES IS SHOWN IN BLACK BELOW:



TO PLOT THE CORRECT AMPLITUDE AT EACH POINT, SUM THE AMPLITUDE OF BOTH GRAPHS AT THAT POINT.

e.g. AT POINT A – EACH GRAPH HAS A VALUE OF 0.7. THEREFORE THE SAME POINT WITH THE RESULTANT SUPERPOSITION IS 0.7 x 2 = 1.4

YOUR NOTES
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4.3.4 Polarisation

Polarisation

- Transverse waves can oscillate in any plane perpendicular to the direction of motion (and energy transfer) of the wave
- Such waves are said to be **unpolarised**
- When a transverse wave is **polarised**, its **electric field** is only allowed to oscillate in **one fixed plane** perpendicular to the direction of motion of the wave
  - A transverse wave can be vertically polarised, horizontally polarised, or polarised in any direction in between

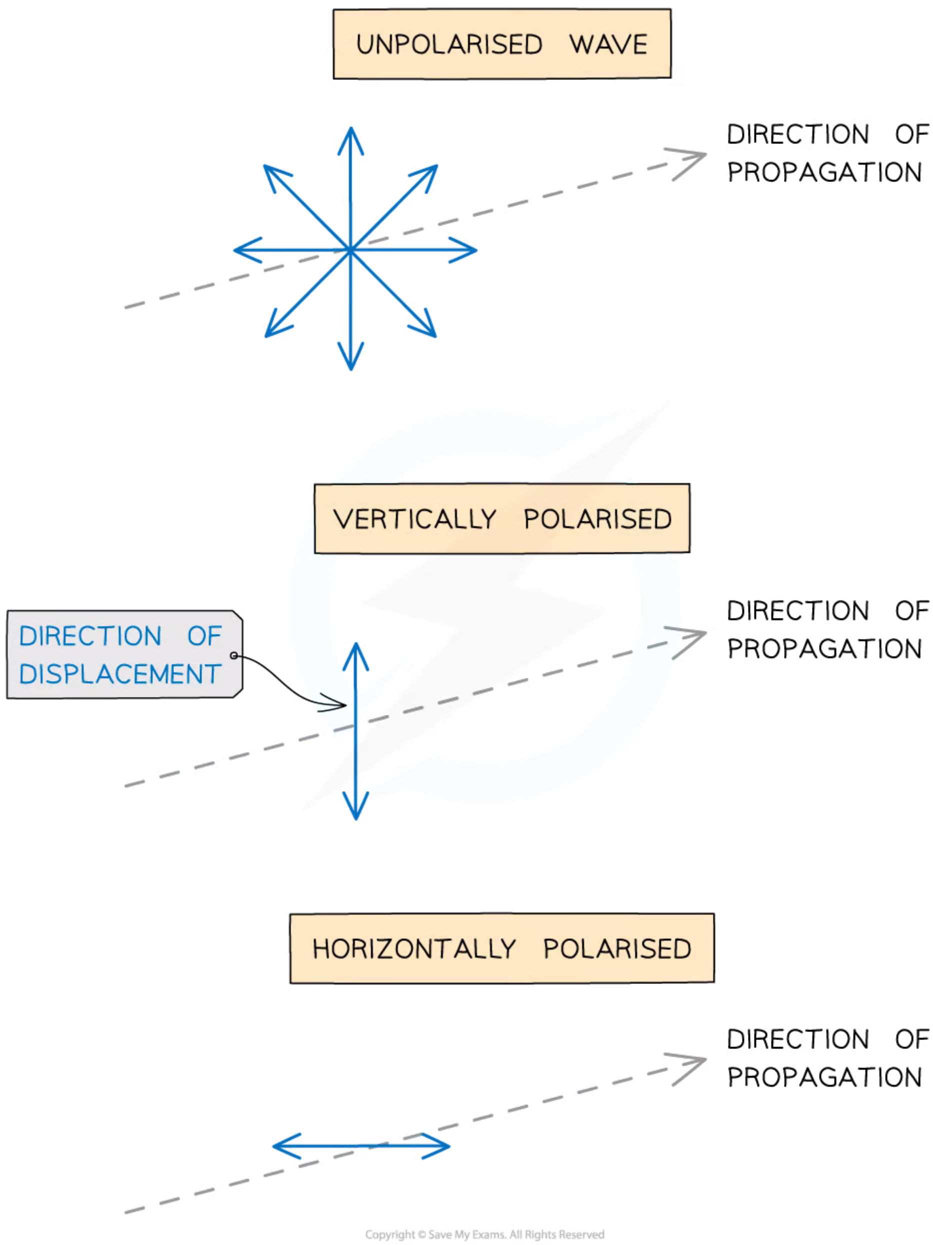

Diagram showing the displacement of unpolarised and polarised transverse waves

YOUR NOTES

- Since longitudinal waves oscillate in the same direction as the direction of motion of the wave, **polarisation of longitudinal waves cannot occur**
- Methods of polarisation include polarising filters and reflection from a non-metallic plane surface

Polarising Filters

- Light waves can be polarised by making them pass through a **polarising filter** called a **polariser**
- The filter imposes its plane of polarisation on the incident light wave
- A polariser with a vertical **transmission axis** only allows vertical oscillations to be transmitted through the filter (**A**)
- If vertically polarised light is incident on a filter with a horizontal transmission axis, no transmission occurs (**B**), and the wave is blocked completely

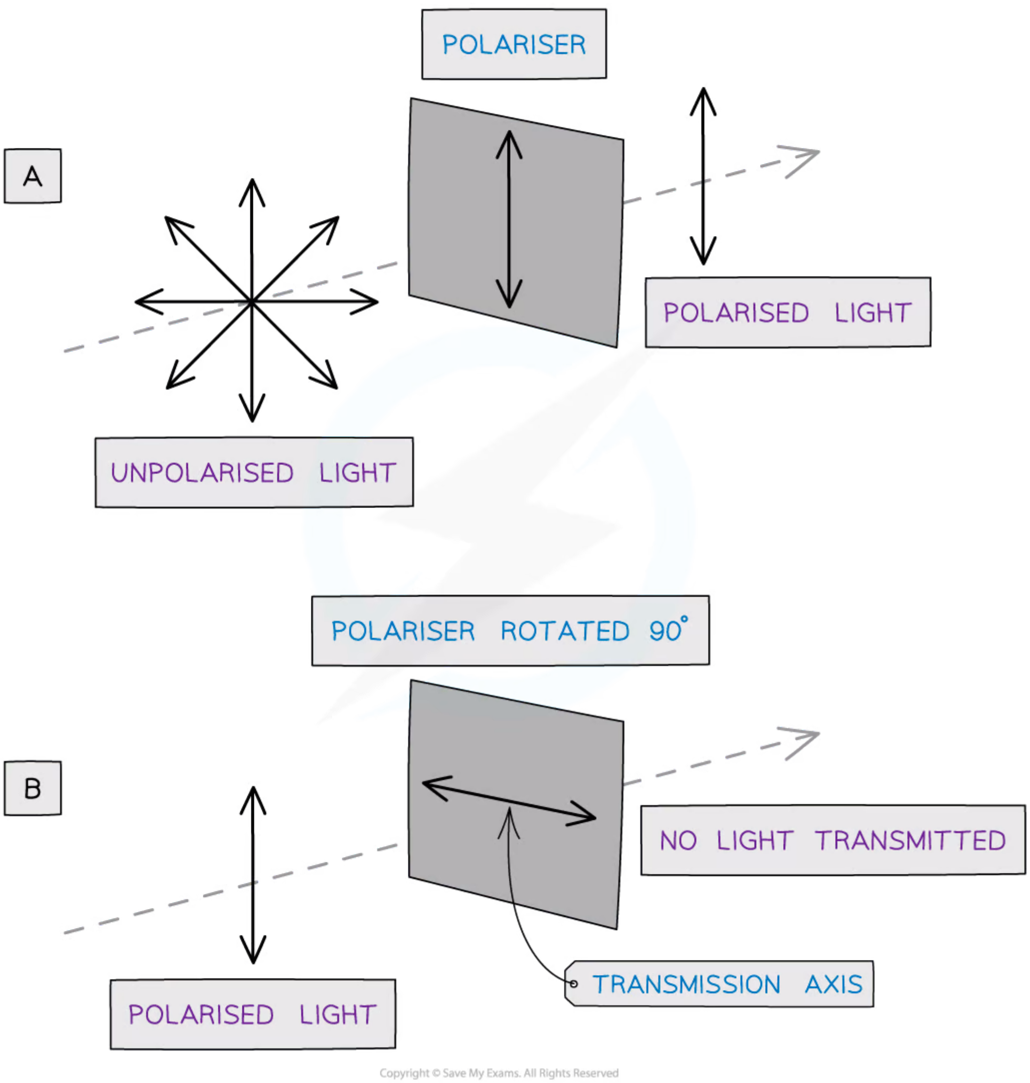

Diagram showing an unpolarised and polarised wave travelling through polarisers

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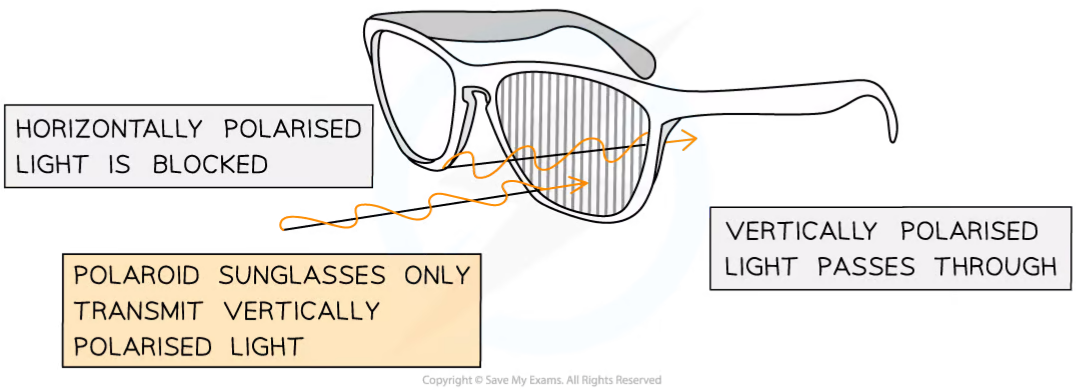
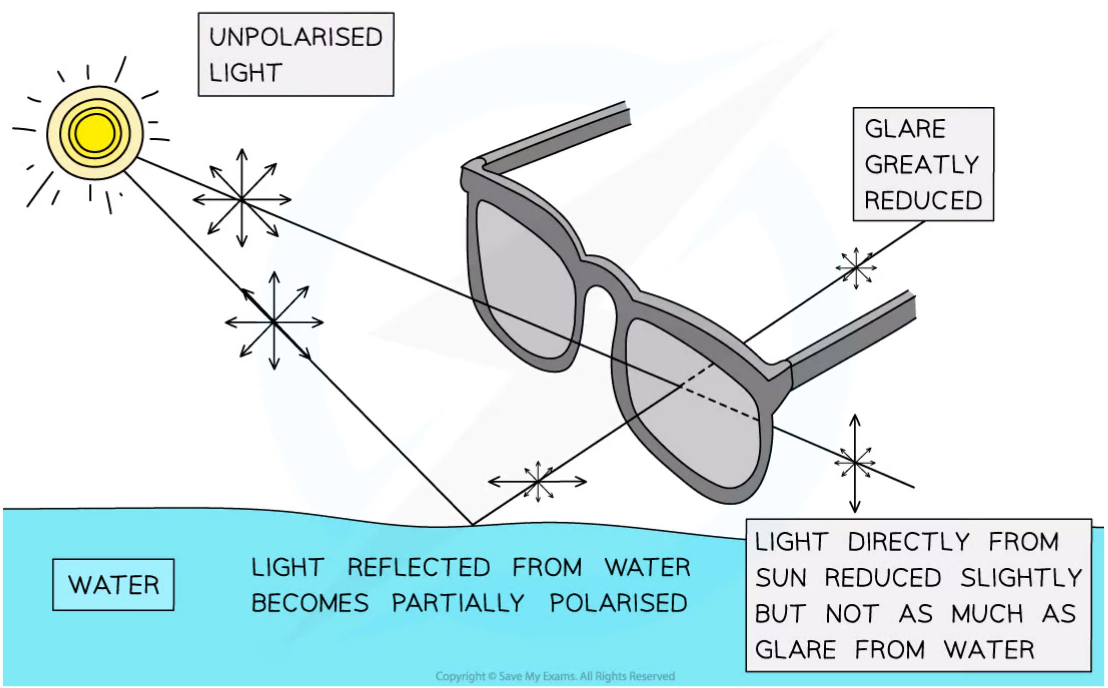
Polarisation via Reflection

- When unpolarised light reflects from a smooth non-metallic surface, **partial plane polarisation** always occurs
- Reflected light is polarised in a **plane parallel to the reflecting surface**
  - This means if the surface is horizontal, a proportion of the reflected light will oscillate more in the horizontal plane than the vertical plane
- **Polarising sunglasses** use this property of reflection in order to reduce the glare coming from a reflective surface (e.g. water)



Polaroid sunglasses contain vertically oriented polarising filters which block out any horizontally polarised light



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When sunlight reflects off a horizontal reflective surface (e.g. water) the light becomes horizontally polarised. This is where polaroid sunglasses come in useful with their vertically aligned filter

- As a result, objects under the surface of the water can be viewed more clearly

YOUR NOTES
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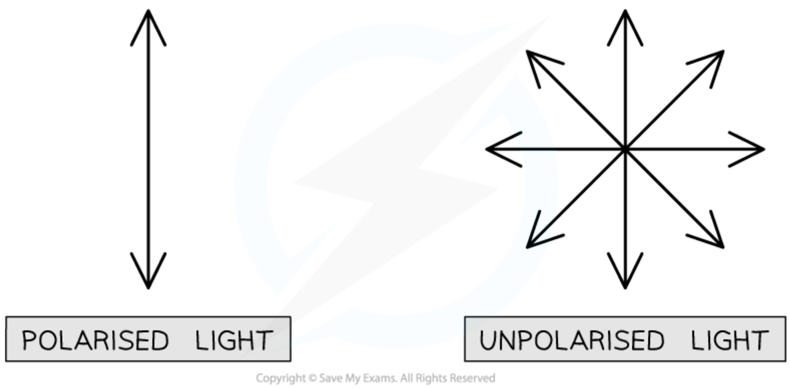
Polarised, Reflected & Transmitted Beams

- Beams can be **polarised**, **reflected** or **transmitted**
- When beams are polarised, the oscillations of the waves are made to oscillate only in one plane
  - This affects the intensity of the waves
- Diagrams demonstrating polarisation will include a double-headed arrow showing the plane of polarisation of the wave



- When beams are reflected, they bounce back in the direction that they have come in by the same angle
- When beams are transmitted, they travel straight through the medium
  - In both these cases, the light can still be polarised
- **Plane** polarisation is when the direction of the vibrations stays constant over time, and the vibrations are 100 % restricted in that direction
- **Partial** polarisation is when there is some restriction to the direction of the vibrations but not 100 %
- This can be seen when an unpolarised light beam travels from air to glass
  - The light is initially unpolarised when incident on the glass
  - Some of the beam is reflected, partially polarising it
  - Some of the beam is transmitted and refracted, also partially polarising it

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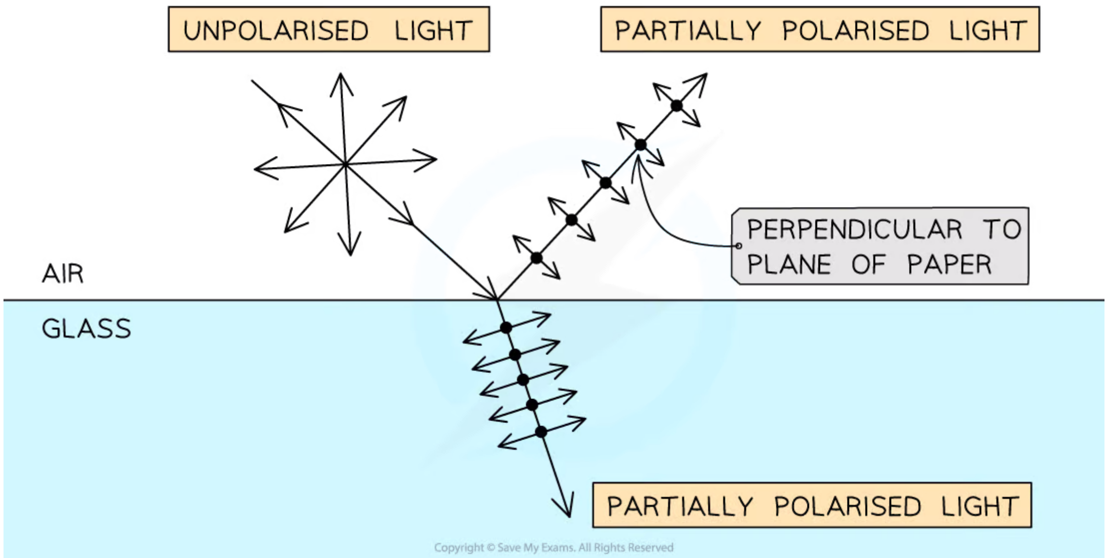
UNPOLARISED LIGHT
PARTIALLY POLARISED LIGHT
PERPENDICULAR TO PLANE OF PAPER
AIR
GLASS
PARTIALLY POLARISED LIGHT
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4.3.5 Malus's Law

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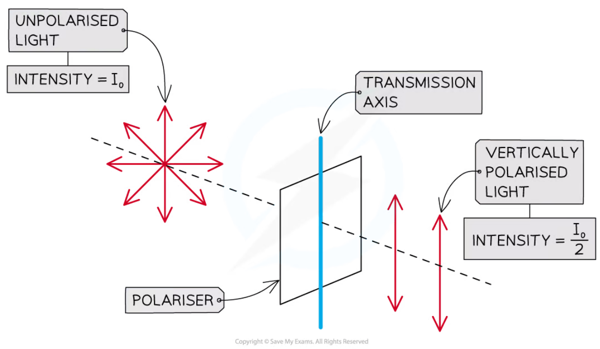
Malus's Law

Intensity of Polarised Light

- The intensity of unpolarised light is reduced as a result of polarisation
- If unpolarised light of intensity I_0 passes through a polariser, the **intensity of the transmitted polarised light falls by a half**



The intensity of polarised light transmitted by a polariser is half the intensity of the unpolarised light incident on it

Intensity of Analysed Light

- The first filter that the unpolarised light goes through is the **polariser**
- A second polarising filter placed after the first one is known as an **analyser**
  - If the analyser has the **same orientation** as the polariser, the light transmitted by the analyser has the **same intensity** as the light incident on it
  - If they have a different orientation, we must use **Malus's Law** to determine the intensity of the transmitted light

YOUR NOTES
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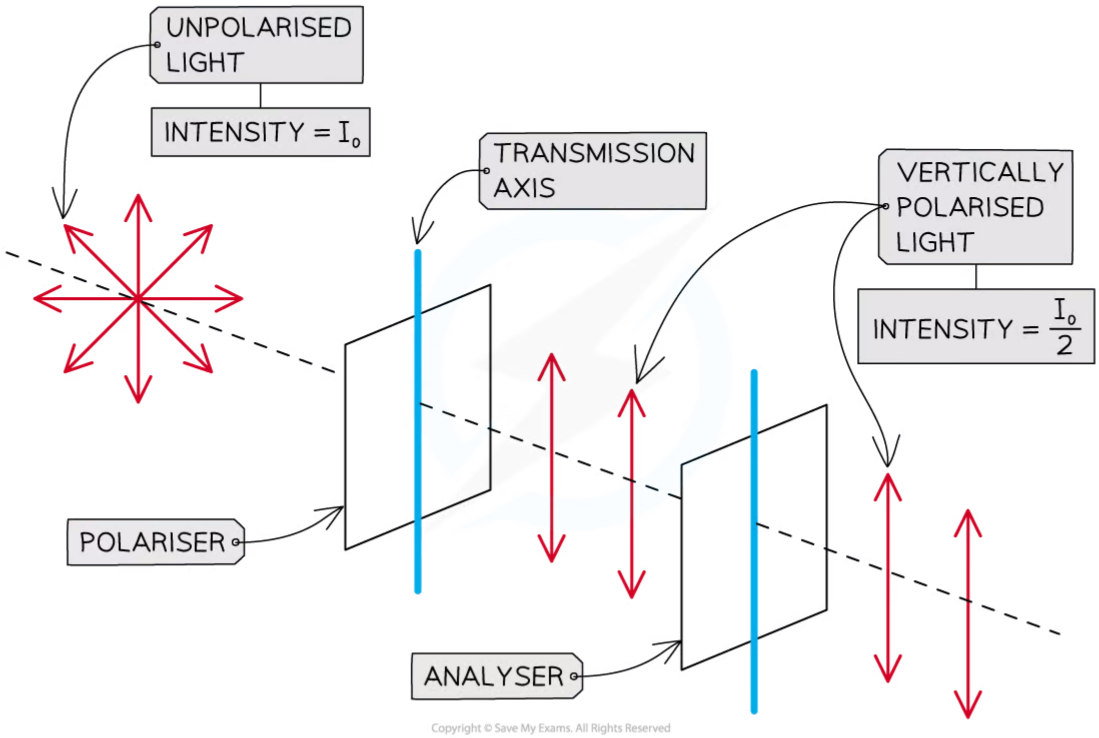
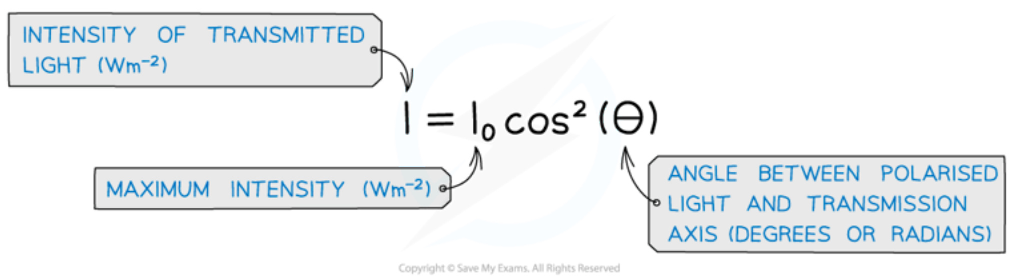


When the polariser and the analyser have the same orientation (i.e. parallel transmission axes), the intensity of analysed light is the same as the intensity of polarised light

- Malus's Law states that if the analyser is rotated by an angle θ **with respect to the polariser**, the intensity of the light transmitted by the analyser is



- If the polariser and the analyser have the **same orientation**, light transmitted by the analyser has the **same intensity** as light that is incident upon it, since cos(0) = 1
  - If vertically polarised light with intensity $\frac{I_0}{2}$ is incident on an analyser with a vertical transmission axis, all of the light will be transmitted through the analyser
  - The intensity will not decrease between the polariser and the analyser

- If the analyser is rotated by **90°** with respect to the polariser (θ = 90°), the intensity of the light transmitted by the analyser will be **zero**, since cos(90°) = 0

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- If vertically polarised light is incident on an analyser with a horizontal transmission axis, none of the light will be transmitted through the analyser
- In this instance, all the light will be absorbed

Table of transmitted intensity when vertically polarised light is incident upon an analyser

Angle of transmission axis θ / degrees	Direction of transmission axis	$\cos^2\theta$	Transmitted intensity I / W m−2	Max or min light intensity transmitted
0	↕	1	I_0	Max
180				
90	↔	0	0	Min
270				

- If the analyser has **any other orientation** with respect to the polarised light incident upon it, then **Malus's Law** is used to determine the intensity of the analysed light
  - The polarised light incident on the analyser will have an intensity $\frac{I_0}{2}$
  - The analysed light will have an intensity $\frac{I_0}{2}\cos^2\theta$

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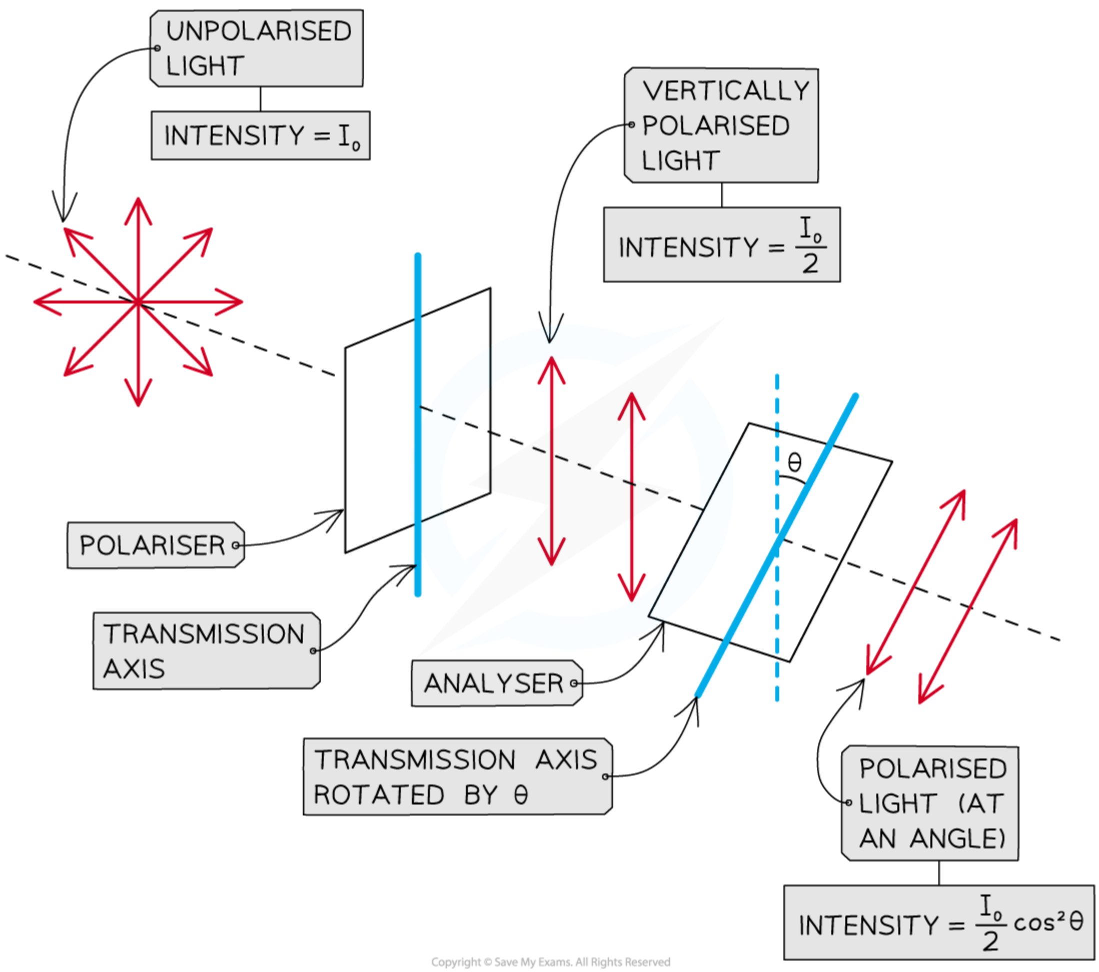

When the analyser is rotated with respect to the polariser by an angle, the intensity of analysed light variates with cos$^2\theta$

- The resulting graph of the light intensity with angle, as the analyser is rotated through 360°, looks as follows:

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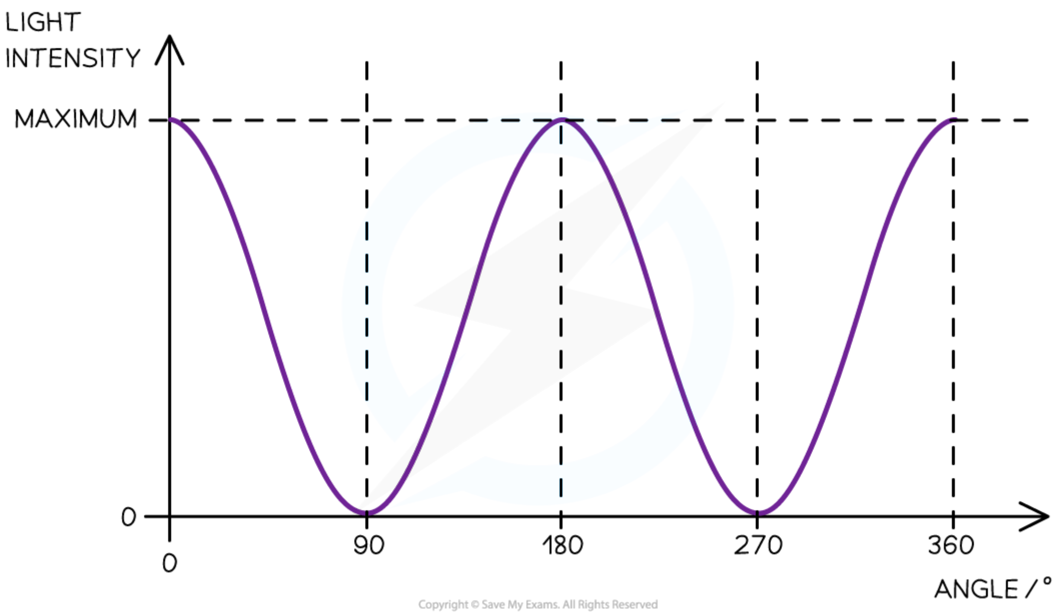

Graph showing how the intensity of the analysed light beam varies with the angle between the transmission axes of the polariser and analyser

- The maximum light intensity I of the graph is still **half** of the intensity from the unpolarised light (I_0)

$$I_{polarised} = \frac{1}{2} I_{unpolarised}$$

- The two extremes of maximum light intensity and minimum light intensity depend on the orientations of just one of the polarisers
- If an unpolarised light source is placed in front of two identical polarising filters, **A** and **B**, with their transmission axes **parallel**:
  - Filter **A** will polarise the light in a certain axis
  - All of the polarised light will pass through filter **B** unaffected
  - In this case, the **maximum** intensity of light is transmitted

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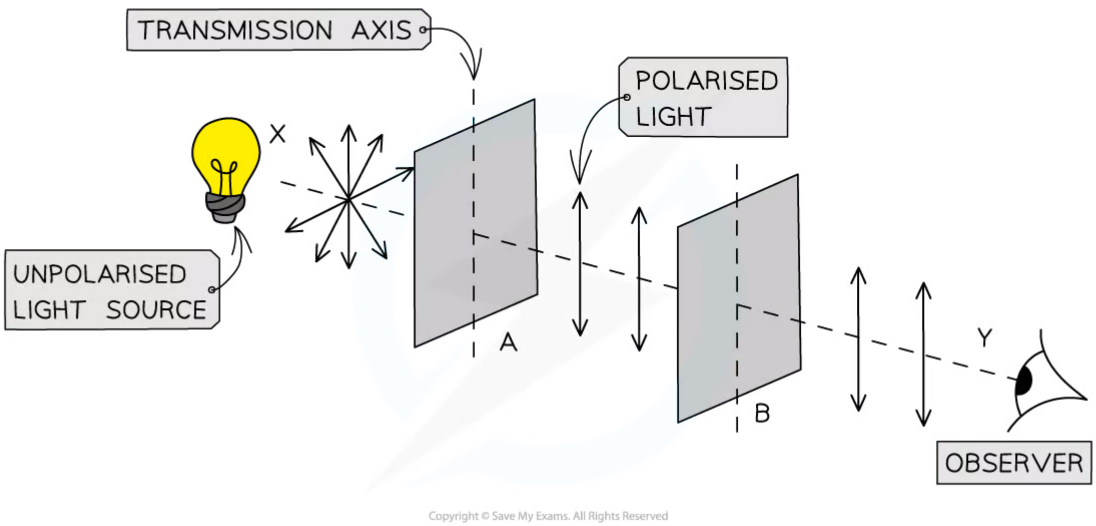
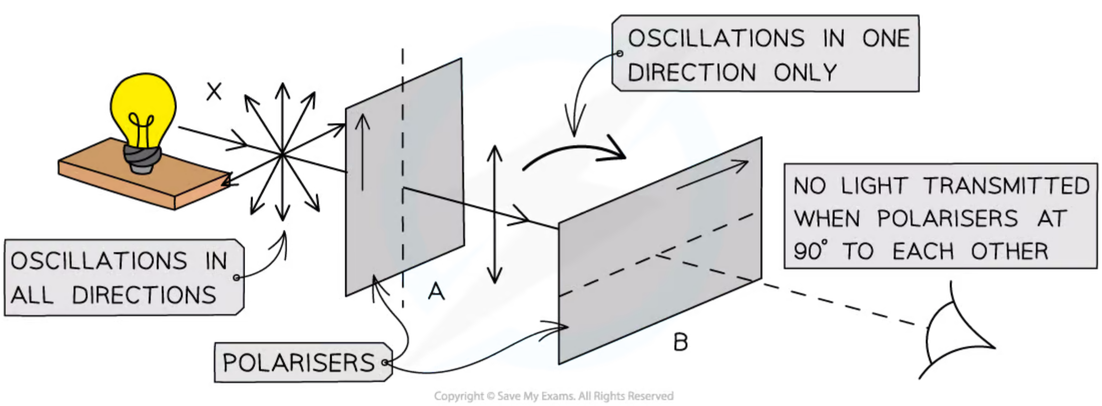


When both polarisers have the same transmission axis, the intensity of the transmitted light is at its maximum

- As the polarising filter **B** is rotated anticlockwise, the intensity of the light observed changes periodically depending on the angle **B** is rotated through
- When **A** and **B** have their transmission axes **perpendicular** to each other:
  - Filter **A** will polarise the light in a certain axis
  - This time none of the polarised light will pass through filter **B**
  - In this case, the **minimum** intensity of light is transmitted



When one of the polarisers is rotated through 90°, the intensity of the transmitted light drops to zero

Worked Example

Unpolarised light of intensity I_0 is incident on a polariser. The transmitted polarised light is then incident on an analyser. The transmission axis of the analyser makes an angle of 45° with the transmission axis of the polariser.

Determine the intensity of light transmitted by the analyser.

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Step 1: Write down the known quantities

- Intensity of unpolarised light = I_0
- Angle of rotation of analyser with respect to polariser, $\theta = 45°$

Step 2: Write down Malus's law

$$I = \frac{I_0}{2}\cos^2\theta$$

Step 3: Substitute the value of the angle $\theta = 45°$

$$I = \frac{I_0}{2} \times \cos^2(45°)$$

$$I = \frac{I_0}{2} \times \frac{1}{2}$$

$$I = \frac{I_0}{4}$$

- The intensity of light transmitted by the analyser is **a quarter** the intensity of unpolarised light
- (and one half the intensity of light transmitted by the polariser)

Exam Tip

Remember that the unpolarised light coming through will always halve in intensity when it becomes polarised through an polariser. Only **then** should you use Malus' law to find the intensity of the light after it has passed through the analyser. Therefore, the I and I_0 in Malus' law are the intensities of light that are already polarised.

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4.4 Wave Behaviour

4.4.1 Reflection, Refraction & Transmission

Reflection, Refraction & Transmission

- When waves arrive at a boundary between two materials, they can be:
  - Reflected
  - Refracted
  - Transmitted
  - Absorbed

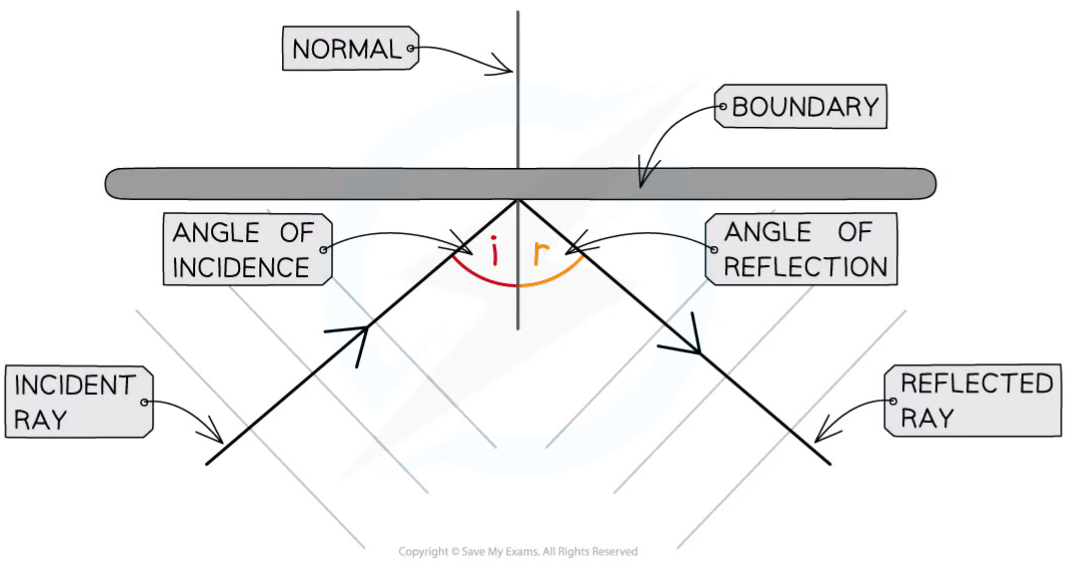
Reflection

- Reflection occurs when:

  A wave hits a boundary between two media and does not pass through, but instead bounces back to the original medium

- The **law of reflection** states:

  The angle of incidence, *i* = The angle of reflection, *r*



Reflection of a wave at a boundary

- When a wave is reflected, some of it may also be **absorbed** by the medium, **transmitted** through the medium, or polarised
- At a boundary between two media, the **incident** ray is the ray that travels **towards** the boundary

Refraction

- Refraction occurs when:

  A wave changes speed and direction at the boundary between two media

YOUR NOTES
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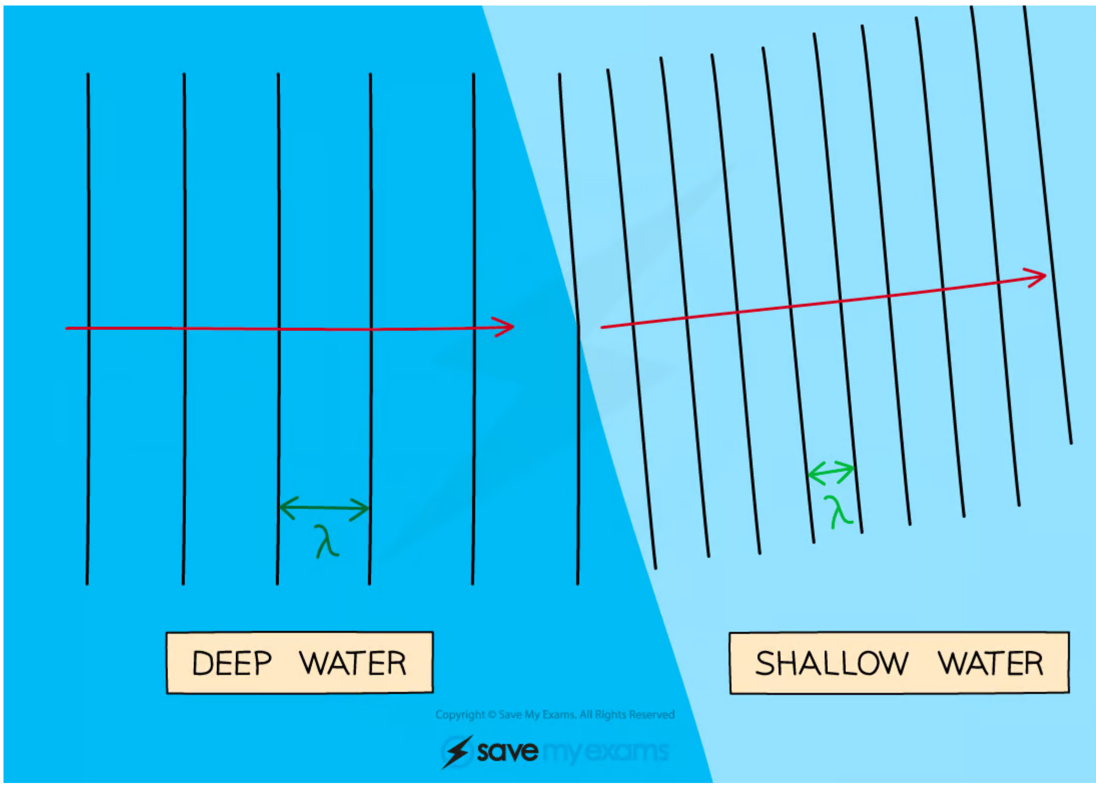
- This is due to the **density** of the media
  - If the medium is more dense, the wave slows down
  - If the medium is less dense, the wave speeds up
- When a wave refracts, its speed and wavelength change, but its frequency remains the same
  - This is noticeable by the fact that the **colour** of the wave does **not change**
- Both transverse and longitudinal waves can refract
- An example of water waves refracting is when they travel from deeper to shallower water
  - The wavelength of the waves decrease in the shallower water



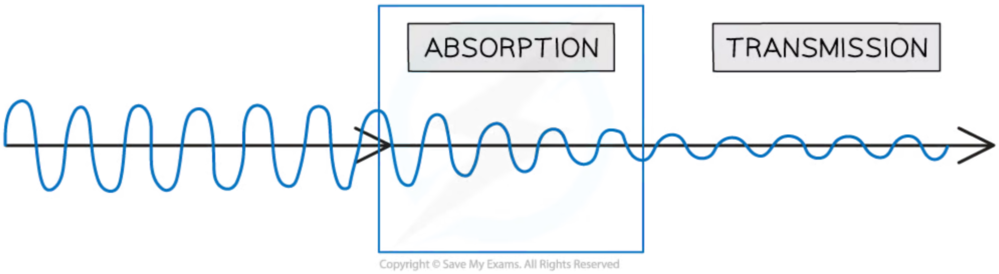
Water waves being refracted at the boundary between deep and shallow water

Transmission

- Transmission occurs when:

  A wave passes through a substance

- **Refraction** is a type of transmission
  - Transmission is the more general term of a wave appearing on the opposite side of a boundary (the opposite of reflection)
  - Refraction is specifically the **change in direction** of a wave when it crosses a boundary between two materials that have a different density
- When passing through a material, waves are usually partially **absorbed**
- The transmitted wave will have a lower amplitude if some absorption has occurred



When a wave passes through a boundary it may be absorbed and transmitted

4.4.2 Reflection

YOUR NOTES

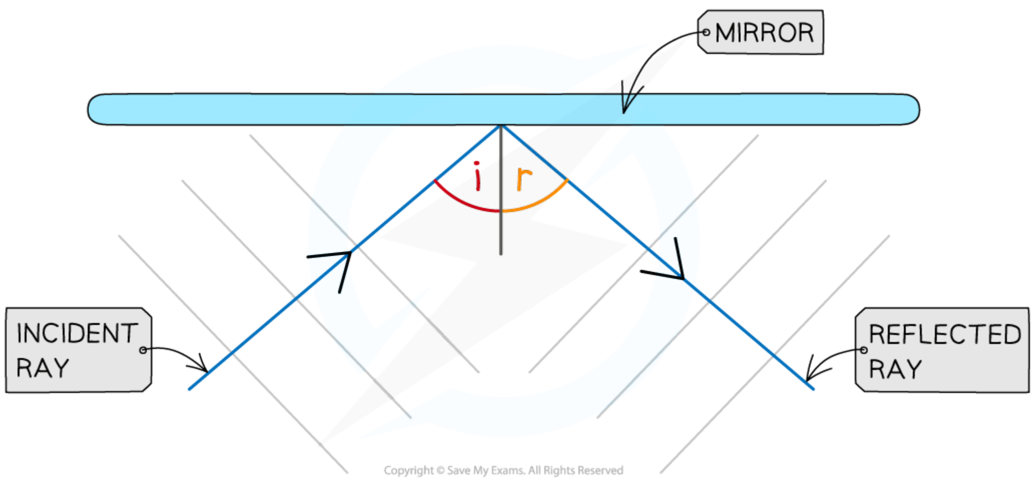
Reflection

- When light hits a smooth plane surface, most of it gets **reflected**
- Very smooth reflective surfaces (e.g. mirrors) are known as **specular plane reflectors**
- For these surfaces, the **law of reflection** applies:

The angle of reflection is equal to the angle of incidence

$$i = r$$

- Where:
  - The **angle of incidence** (*i*) is the angle between the **incident ray** and the normal
  - The **angle of reflection** (*r*) is the angle between the **reflected ray** and the normal



A light ray being reflected by a mirror

- The wavelength of the reflected ray is the same as that of the incident ray

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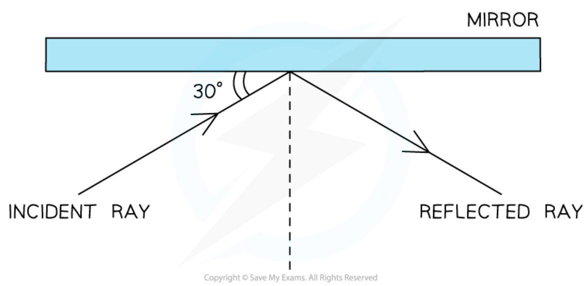
Worked Example

A light ray is reflected by a mirror as shown in the diagram below.

a) State the angle of reflection

b) Add the incident and reflected wavefronts to the diagram



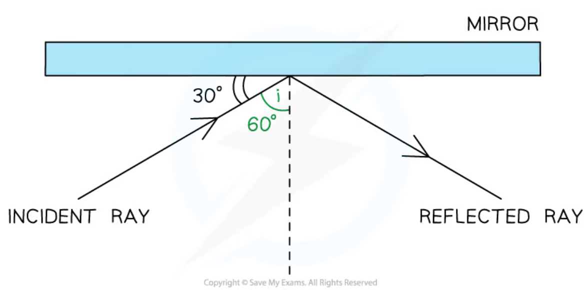
a) State the angle of reflection

Step 1: Recall the definition of the angle of incidence (*i*)

- The angle of incidence is the angle between the incident ray and the normal
- This is not the 30° angle marked in the diagram

$$i = 90° - 30°$$

$$i = 60°$$



Step 2: Recall the law of reflection

The angle of reflection is equal to the angle of incidence

Step 3: State the angle of reflection (*r*)

$$r = 60°$$

b) Add the incident and reflected wavefronts to the diagram

Step 1: Recall that wavefronts and rays are perpendicular to each other

- Add at least three equally spaced wavefronts all perpendicular to the incident ray
- Add at least three equally spaced wavefronts all perpendicular to the reflected ray

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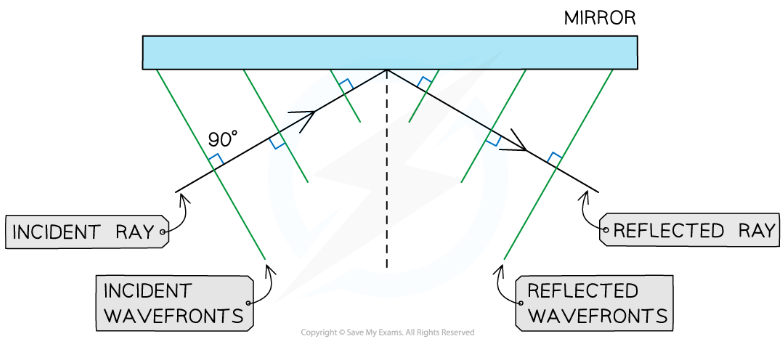

Exam Tip

When asked to complete or construct reflection ray diagrams, remember to add:

- arrows on rays to distinguish between incident and reflected rays
- labels to distinguish between incident and reflected wavefronts

In most cases, when dealing with light, you will just need to draw rays. However, you might still be required to draw wavefronts sometimes. Always use a ruler or a straight edge and a shape pencil for the ray diagrams.

Remember that the distance between each wavefront represents the wavelength of the wave. If you intend on the ray not changing wavelength (like in reflection) then make sure the incident **and** reflected wavefronts are all equally spaced by the same amount.

4.4.3 Refraction

YOUR NOTES
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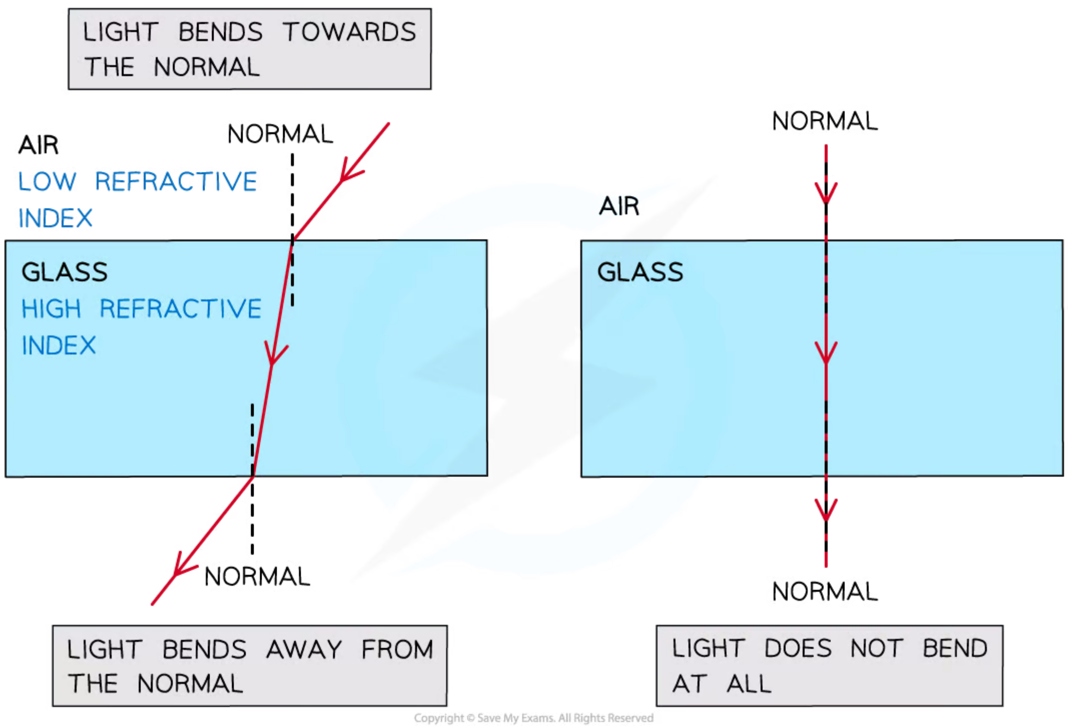
Refraction

- When light crosses the boundary between two media with different optical densities, it **refracts**
- At the boundary, the light undergoes a **change in direction**
- The change in direction depends on which media the light rays pass between:
  - From air to glass (less dense to more dense): light bends **towards** the normal
    - The angle of refraction < the angle of incidence
  - From glass to air (more dense to less dense): light bends **away** from the normal
    - The angle of refraction > the angle of incidence
  - When passing along the normal (perpendicular) the light **does not bend** at all
    - This would be described as **transmission**



Refraction of light through a glass block

- When light passes from a less dense medium to a more dense medium, such as from air to glass, the refracted light has a **lower speed** and a **shorter wavelength** than the incident light
- When light passes from a more dense medium to a less dense medium, such as from glass to air, the refracted light has a **higher speed** and a **longer wavelength** than the incident light
- When more than one boundary is shown in a refraction ray diagram, such as light passing through a glass block, the incident ray at the second boundary is the refracted ray from the first boundary

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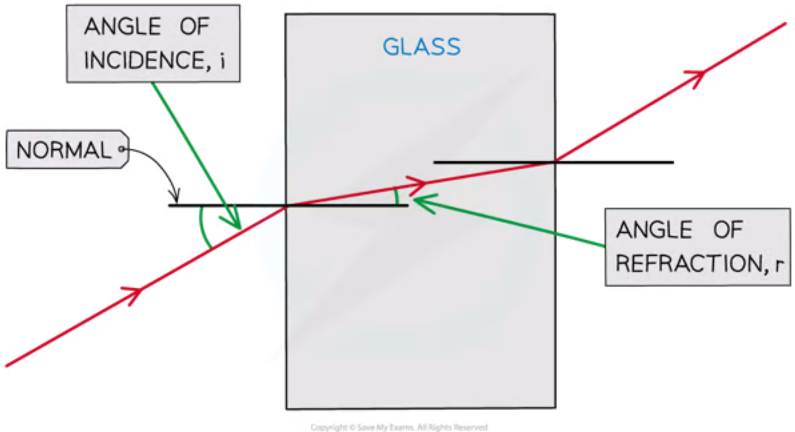


A light ray being refracted at the air-glass boundary

- Together with refraction, **reflection** might also occur
  - When light travels from a less optically dense medium into an optically denser medium, **both reflection and refraction always occur**
  - Light has to be reflected from the object to the eye in order for objects to be visible
  - Some of the energy in the incident light is reflected back, while some is **transmitted**

Absolute Refractive Index

- Transparent media have different optical densities
- Light is **transmitted** through these media at different speeds
- The **absolute refractive index**, ***n***, of a transparent medium is a measure of its optical density, and can be calculated as follows:

$$n = \frac{c}{v}$$

- Where:
  - n = absolute refractive index of the medium
  - c = speed of light in vacuum in metres per second (m s^{-1})
  - v = speed of light in the medium in metres per second (m s^{-1})

- The value of the speed of light in a vacuum is $c = 3.00 \times 10^8$ m s^{-1}, as given in the data booklet
- Note that, being a ratio, the absolute refractive index is a **dimensionless** quantity
  - This means that it has no units
- The refractive index of **air** can be assumed to be ***n* = 1**
- Because the speed of light will always be faster than the speed of light in a medium, the refractive index of **any other transparent medium** is ***n* > 1**

Snell's Law

- Snell's Law is:

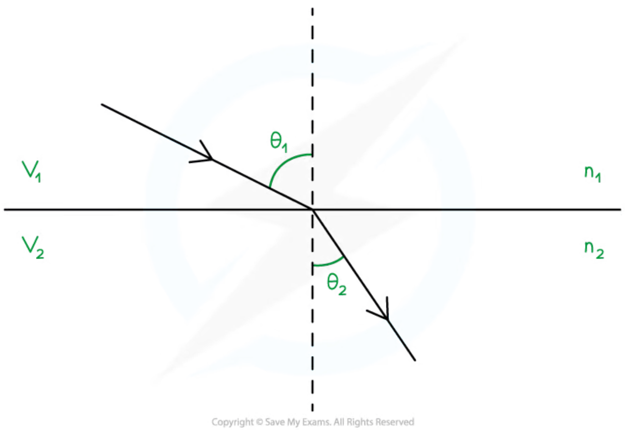
$$\frac{n_1}{n_2} = \frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1}$$

YOUR NOTES
⬇

- Where:
  - n = absolute refractive index
  - θ = angles of incidence and refraction
  - v = speed of light in medium

- Snell's Law describes the angle at which light meets the boundary, and the angle at which light leaves the boundary, so that the light travels through the media in the least amount of time

- Light can travel through medium 1 at a speed of v_1 due to the optical density n_1 of that medium
  - Light will approach the boundary at angle θ_1
  - This is the angle of incidence
- Light can travel through medium 2 at a speed of v_2 due to the optical density n_2 of that medium
  - Light will leave the boundary at angle θ_2
  - This is the angle of refraction



Snell's Law

- To illustrate this, there is a classic thought experiment that uses Fermat's Principle of Light
  - Fermat's Principle of Light states that light will travel between two points along the path that will take the least amount of time

- A life guard on a beach sees a swimmer in need of rescue. They can run at 5 m s^{-1} on the sand and they can swim at 2 m s^{-1} in the water. What is the fastest path to take?
  - The life guard could run on the sand straight to the water and then swim to the person
  - The life guard could run on the sand until they are parallel to the person, and then swim directly out to them
  - The life guard could run and then swim diagonally in a path that is a straight line from his position in the sand to the position of the swimmer in the water
  - Or, the life guard could run diagonally, but so that more of the distance covered is in the sand than in the water
    - Some permutation of this answer is where the fastest path will be found

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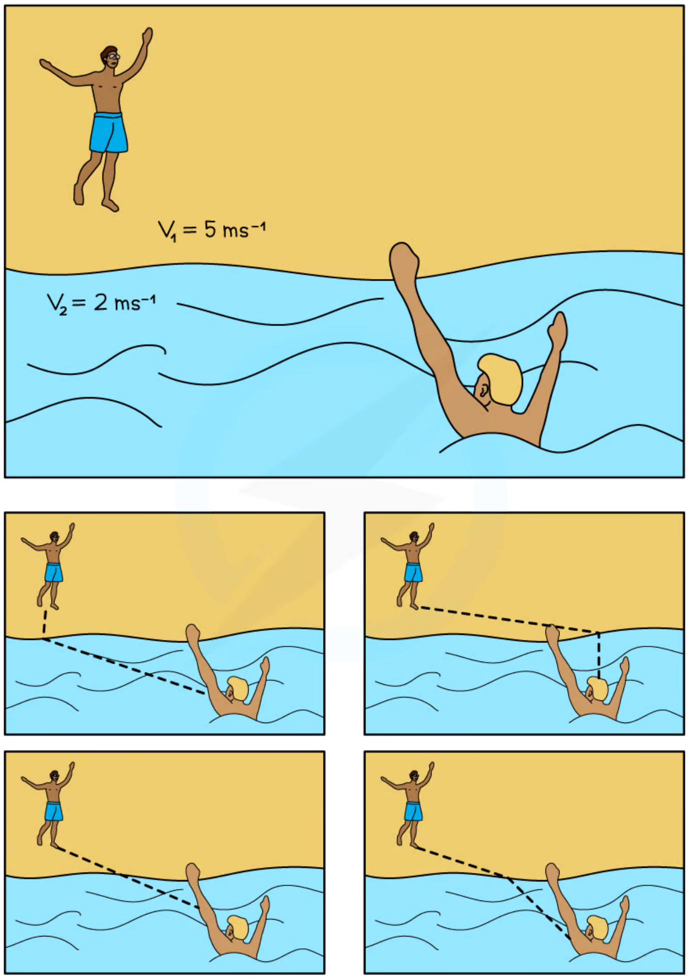
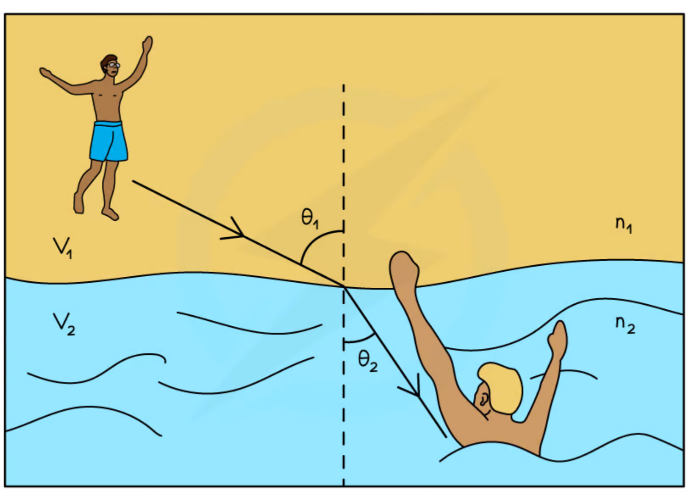


Thought Experiment: A life guard needs to find the fastest path to a swimmer in trouble

- Using this thought experiment, it can be seen that Snell's Law emerges



Snell's Law in the context of the thought experiment

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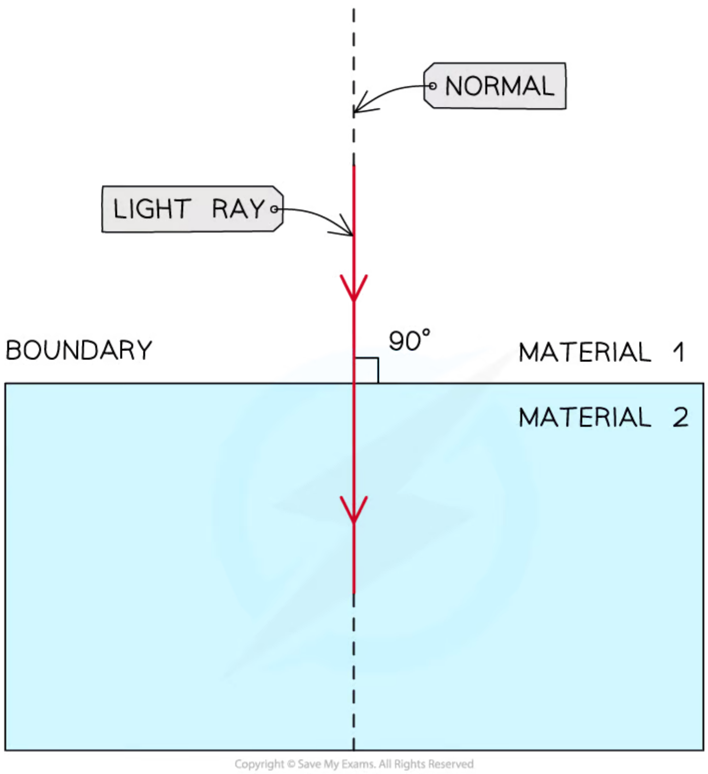
- Snell's Law can also be given as:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

- This form of Snell's Law can often be more convenient to use

- When the incident ray is perpendicular to the surface of the boundary, its **speed** changes, but its **direction** does not
  - Using Snell's Law to explain this
    - The angle of incidence is zero, $\theta_1 = 0$, therefore, $n_1 \sin 0 = n_2 \sin \theta_2$
    - $\sin 0 = 0$
    - Anything multiplied by zero is zero, therefore, $0 = n_2 \sin \theta_2$
    - Hence, the refracted angle θ_2 must also be zero
    - So, there is transmission without refraction



Light travelling along the normal to the boundary between material 1 and material 2

Critical Angle and Total Internal Reflection

- According to Snell's law, when light travels from an optically denser medium into a less optically dense medium, its speed increases and it bends away from the normal
  - The angle of refraction is greater than the angle of incidence
  - A small amount of light is also reflected back into the optically denser medium

- As the angle of incidence increases, the angle of refraction eventually reaches 90°
- This is known as the **critical angle θ_c**:

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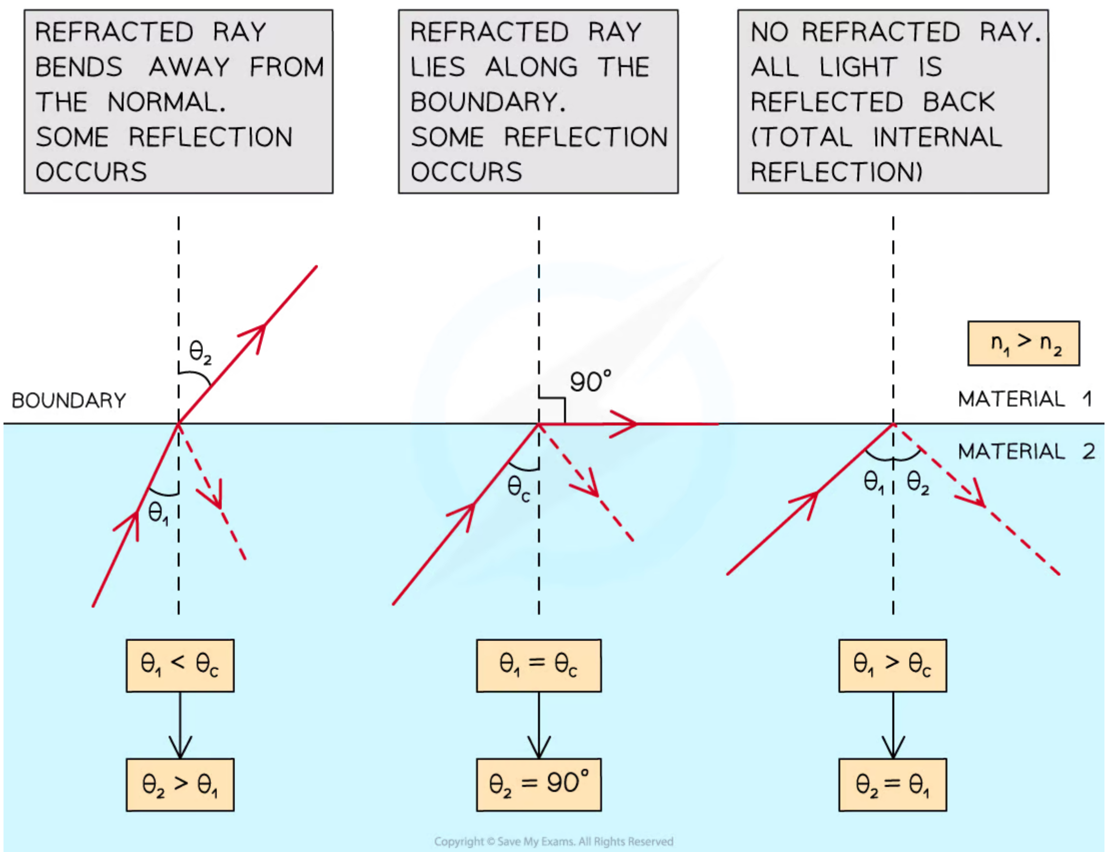
The angle of incidence which results in an angle of refraction of 90°, after which point, total internal reflection occurs

- The critical angle is important because this is the point at which **no light enters the new medium**
- For light travelling from an optically denser material 1 into a less optically dense material 2
  - If the angle of incidence is smaller than the critical angle ($\theta_1 < \theta_c$), the refracted ray bends away from the normal, and the angle of refraction is greater than the angle of incidence ($\theta_2 > \theta_1$)
  - If the angle of incidence is equal to the critical angle ($\theta_1 = \theta_c$), the refracted ray lies along the boundary between the two materials, and the angle of refraction is equal to 90° ($\theta_2 = 90°$)
  - If the angle of incidence is greater than the critical angle ($\theta_1 > \theta_c$), there is no refracted ray, and all light is reflected back into material 1 ($\theta_2 = \theta_1$)
    - This is known as **total internal reflection**



Light travelling from the optically denser material 1 into the less optically dense material 2 at different angles of incidence

- The critical angle of material 1 can be calculated as follows:

$$\sin\theta_c = \frac{n_2}{n_1}$$

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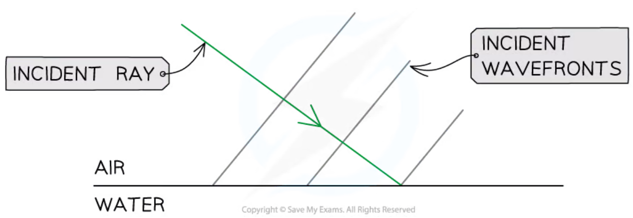
- Where:
  - θ_c = critical angle of material 1 (°)
  - n_1 = absolute refractive index of material 1
  - n_2 = absolute refractive index of material 2

Worked Example

Wavefronts travel from air to water as shown. Add the refracted wavefronts to the diagram.

Step 1: Add the incident ray to mark the direction of the incident waves

- The incident ray must be perpendicular to all wavefronts
- Remember to add an arrow pointing towards the air-water boundary



Step 2: Add the normal at the point of incidence

- Mark the angle of incidence (i) between the normal and the incident ray

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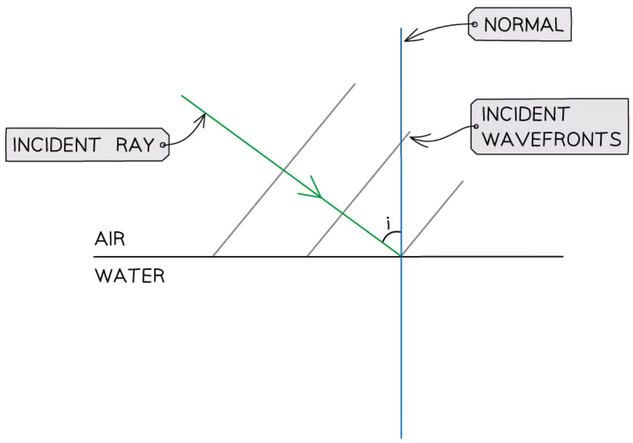
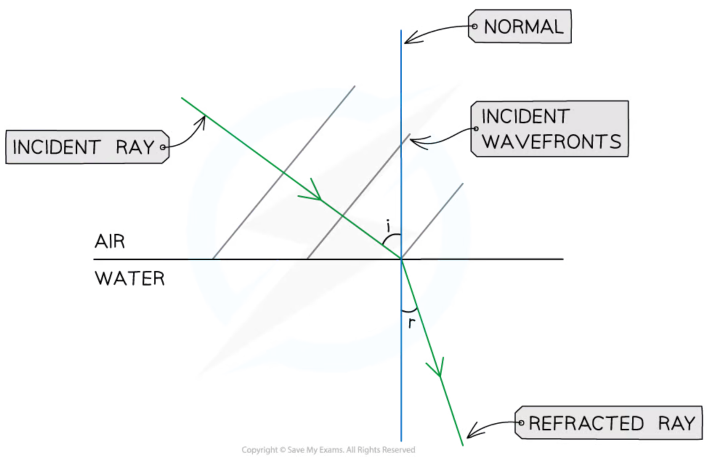


Step 3: Draw the refracted ray into the water

- Water is optically denser than air
- The refracted ray must bend towards the normal
- Mark the angle of refraction (r) between the normal and the refracted ray
- r < i, by eye



Step 4: Add three equally spaced wavefronts, all perpendicular to the refracted ray

- The refracted wavefronts must be closer to each other than the incident wavefronts, since:
  - The speed *v* of the waves decreases in water
  - The frequency *f* of the waves stays the same
  - The wavelength λ of the waves in water is shorter than the wavelength of the waves in air $\lambda_W < \lambda_A$, since $v = f\lambda$

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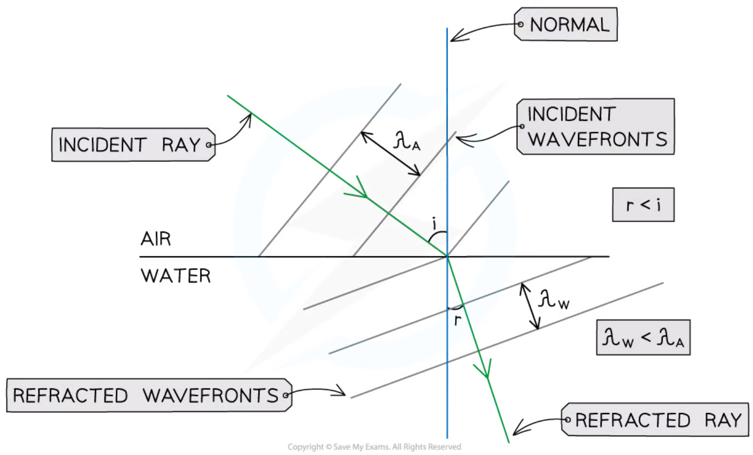

Worked Example

Light travels from a material with refractive index 1.2 into air. Determine the critical angle of the material.

Step 1: Write down the known quantities

- $n_1 = 1.2$
- $n_2 = 1.0$

Step 2: Write down the equation for the critical angle θ_c

$$\sin\theta_c = \frac{n_2}{n_1}$$

Step 3: Substitute the numbers into the above equation

$$\sin\theta_c = \frac{1.0}{1.2}$$

$$\sin\theta_c = 0.83$$

Step 4: Calculate θ_c by taking $\sin^{-1}$ of the above equation

$$\theta_c = \sin^{-1}0.83$$

$$\theta_c = 56°$$

Worked Example

Light travels from air into glass. Determine the speed of light in glass.

- Refractive index of air, $n_1 = 1.00$
- Refractive index of glass, $n_2 = 1.50$

Step 1: Write down the known quantities

- $n_1 = 1.00$

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- $n_2 = 1.50$
- From the data booklet, $c = 3 \times 10^8$ m s^{-1} (speed of light in air)

Step 2: Write down the relationship between the refractive indices of air and glass and the speeds of light in air (v_1) and glass (v_2)

$$\frac{n_1}{n_2} = \frac{v_2}{v_1}$$

Step 3: Rearrange the above equation to calculate v_2

$$v_2 = \frac{n_1}{n_2} v_1$$

Step 4: Substitute the numbers into the above equation

$$v_2 = \frac{1.00}{1.50} \times \left(3 \times 10^8\right)$$

$$v_2 = 2 \times 10^8 \text{ m s}^{-1}$$

Exam Tip

Always double-check if your calculations for the refractive index are greater than 1. Otherwise, something has definitely gone wrong in your calculation! The refractive index of air might not be given in the question. Always assume that $n_{air} = 1$

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Determining Refractive Index

Aim of the Experiment

- To investigate the refraction of light by a perspex block

Equipment

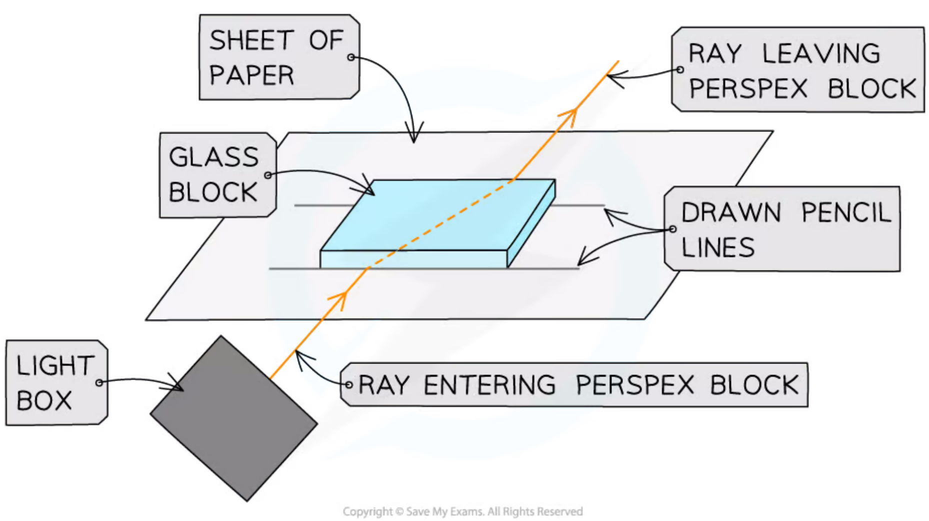
- Ray Box - to provide a narrow beam of light to refract through the perspex box
- Protractor - to measure the light beam angles
- Sheet of paper - to mark with lines for angle measurement
- Pencil - to make perpendicular line and angle lines on paper
- Ruler - to draw straight lines on the paper
- Perspex block - to refract the light beam

- Resolution of measuring equipment:
  - Protractor = 1°
  - Ruler = 1 mm

Variables

- Dependent variable = angle of refraction , *r*
- Control variables:
  - Use of the same perspex block
  - Width of the light beam
  - Same frequency / wavelength of the light

Method



Apparatus to investigate refraction

1. Place the glass block on a sheet of paper, and carefully draw around the block using a pencil
2. Switch on the ray box and direct a beam of light at the side face of the block
3. Mark on the paper:

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- A point on the ray close to the ray box
- The point where the ray enters the block
- The point where the ray exits the block
- A point on the exit light ray which is a distance of about 5 cm away from the block

4. Draw a dashed line normal (at right angles) to the outline of the block where the points are
5. Remove the block and join the points marked with three straight lines
6. Replace the block within its outline and repeat the above process for a ray striking the block at a different angle

- An example of the data collection table is shown below:

Angle of incidence, I	Angle of refraction, r
30°	
45°	
60°	
90°	

Analysis of Results

- *i* and *r* are always measured from the **normal**
- For light rays entering perspex block, the light ray refracts **towards** the central line:

i > r

- For light rays exiting the perspex block, the light ray refracts **away** from the central line:

i < r

- When the angle of incidence is 90° to the perspex block, the light ray does **not** refract, it passes straight through the block:

i = r

- If the experiment was carried out correctly, the angles should follow the pattern, as shown below:

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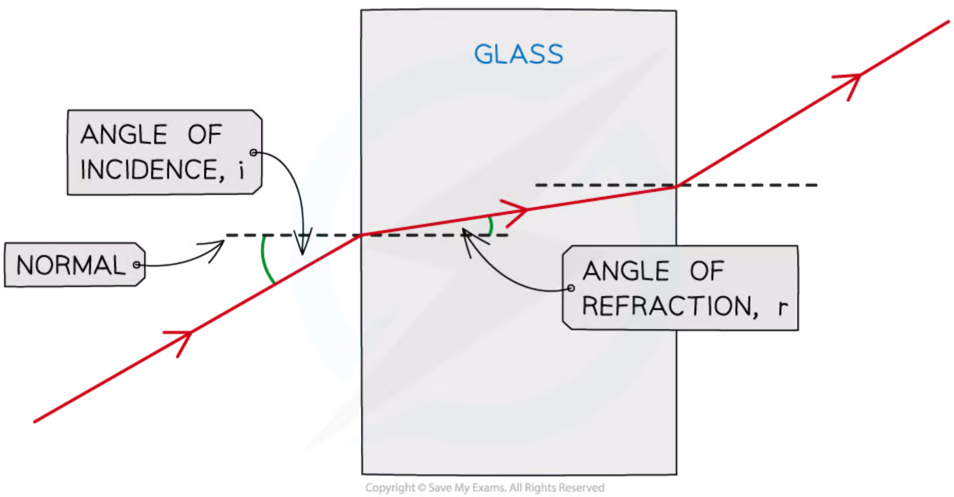

How to measure the angle of incidence and angle of refraction

Evaluating the Experiment

Systematic Errors:

- An error could occur if the 90° lines are drawn incorrectly
  - Use a set square to draw perpendicular lines
- If the mirror is distorted, this could affect the reflection angle, so make sure there are little to no blemishes on it

Random Errors:

- The points for the incoming and reflected beam may be inaccurately marked
  - Use a sharpened pencil and mark in the middle of the beam
- The protractor resolution may make it difficult to read the angles accurately
  - Use a protractor with a higher resolution

Safety Considerations

- The ray box light could cause burns if touched
  - Run burns under cold running water for at least five minute
- Looking directly into the light may damage the eyes
  - Avoid looking directly at the light
  - Stand behind the ray box during the experiment
- Keep all liquids away from the electrical equipment and paper

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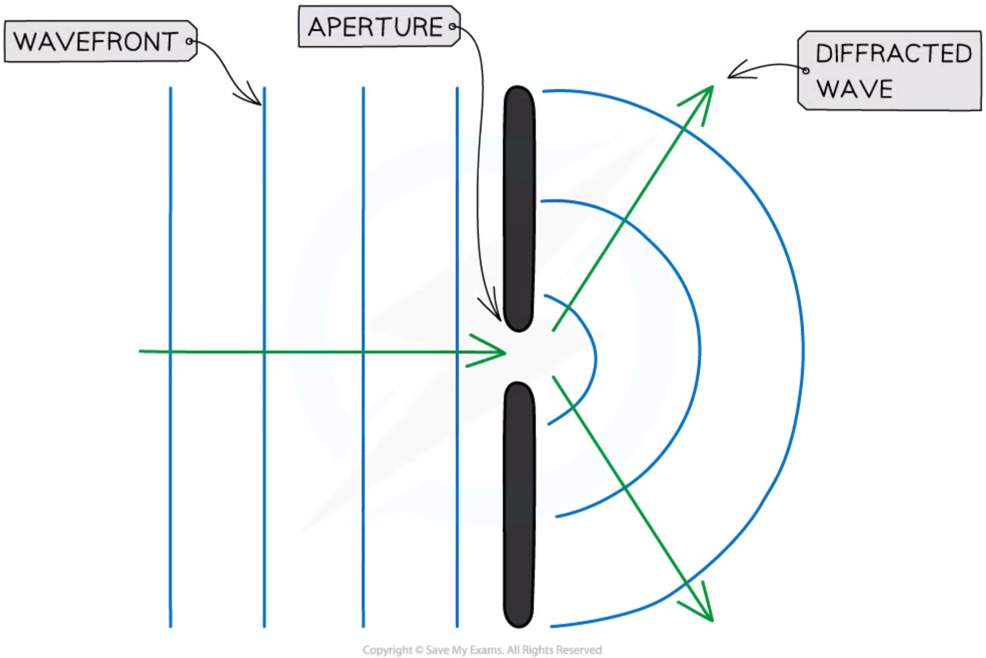
Single-Slit Diffraction

- Diffraction is the **spreading out** of waves when they pass an obstruction
    - This obstruction is typically a narrow slit known as an aperture



As waves pass through the aperture, they spread out

- The frequency of the diffracted waves is **less** than that of the incident waves, since **energy is distributed over a larger area**
- The extent of diffraction depends on the width of the gap compared to the wavelength of the wave
    - Diffraction is the most prominent when the width of the slit is approximately **equal to or smaller** than the wavelength
    - As the gap size increases, the effect gradually gets less pronounced until, in the case that the gap is much larger than the wavelength, the waves are no longer spread out

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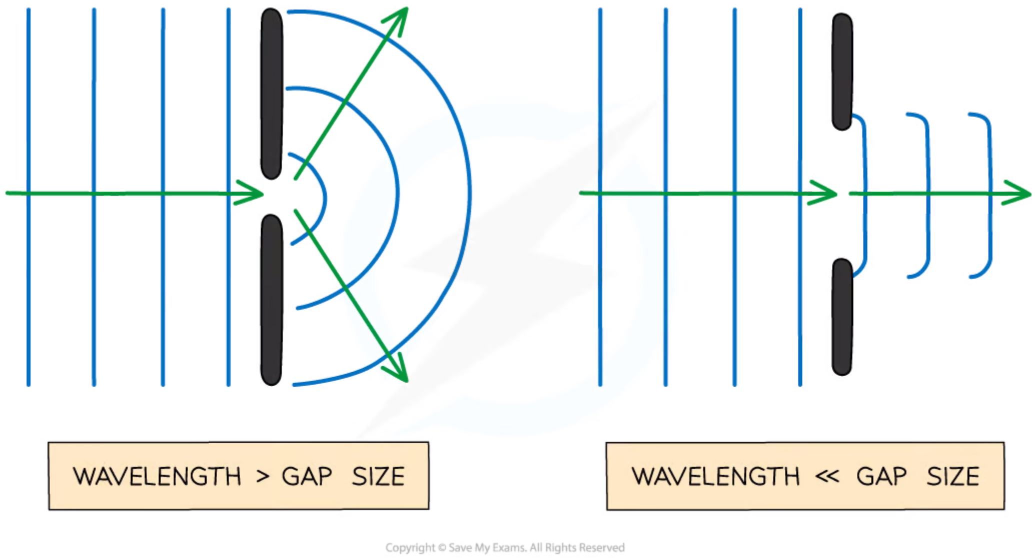
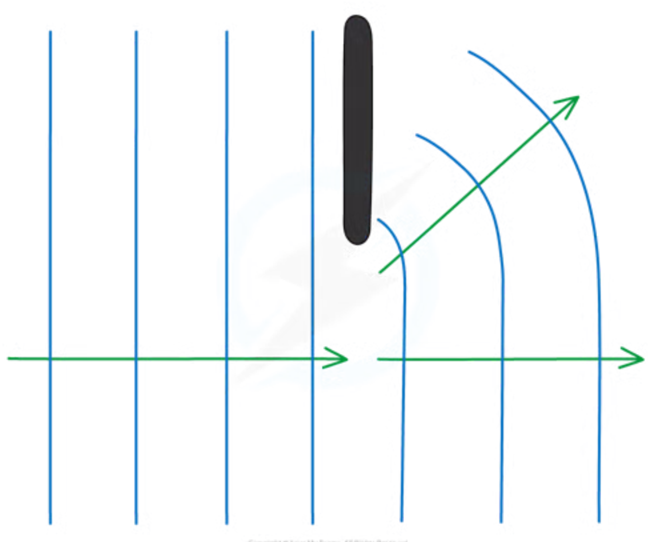


The size of the gap (compared to the wavelength) affects how much the waves spread out

- Any type of wave can be diffracted i.e. sound, light, water
- The only property of a wave that changes when its diffracted is its **amplitude**
  - This is because some energy is dissipated when a wave is diffracted through a gap
- Diffraction can also occur when waves curve around an edge:



When a wave goes past the edge of a barrier, the waves can curve around it

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Worked Example

An electric guitar student is practising in his room. He has not completely shut the door of his room, and there is a gap of about 10 cm between the door and the door frame.

Determine the frequencies of sound that are best diffracted through the gap.

The speed of sound can be taken to be 340 m s^{-1}

Step 1: Optimal diffraction happens when the wavelength of the waves is comparable to (or larger than) the size of the gap

$$\lambda = 10 \text{ cm} = 0.1 \text{ m}$$

Step 2: Write down the wave equation

$$c = f\lambda$$

- where c = 340 m s^{-1}

Step 3: Rearrange the above equation for the frequency *f*

$$f = \frac{c}{\lambda}$$

Step 4: Substitute the numbers into the above equation

$$f = \frac{340}{0.1}$$

$$f = 3400 \text{ Hz}$$

The frequencies of sound that are best diffracted through the gap are:

$$f \leq 3400 \text{ Hz}$$

Worked Example

When a wave is travelling through the air, which scenario best demonstrates diffraction?

A. UV radiation through a gate post

B. Sound waves passing a diffraction grating

C. Radio waves passing between human hair

D. X-rays passing through atoms in a crystalline solid

ANSWER: D

- Diffraction is most prominent when the wavelength is close to the aperture size

Consider option **A**:

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- UV waves have a wavelength between (4×10^{-7}) and (1×10^{-8}) m so would **not** be diffracted by a gate post
  - Radio waves, microwaves or sound waves would be more likely to be diffracted at this scale

Consider option **B**:

- Sound waves have a wavelength of (1.72×10^{-2}) to 17 m so would **not** be diffracted by the diffraction grating
  - Infrared, light and ultraviolet waves would be more likely to be diffracted at this scale

Consider option **C**:

- Radio waves have a wavelength of 0.1 to 10^6 m so would **not** be diffracted by human hair
  - Infrared, light and ultraviolet waves would be more likely to be diffracted at this scale

Consider option **D**:

- X-rays have a wavelength of (1×10^{-8}) to (4×10^{-13}) m
  - This is a suitable estimate for the size of the gap between atoms in a crystalline solid
  - Hence X-rays could be diffracted by a crystalline solid
- Therefore, the correct answer is **D**

Exam Tip

When drawing diffracted waves, take care to keep the wavelength (the distance between each wavefront) constant.

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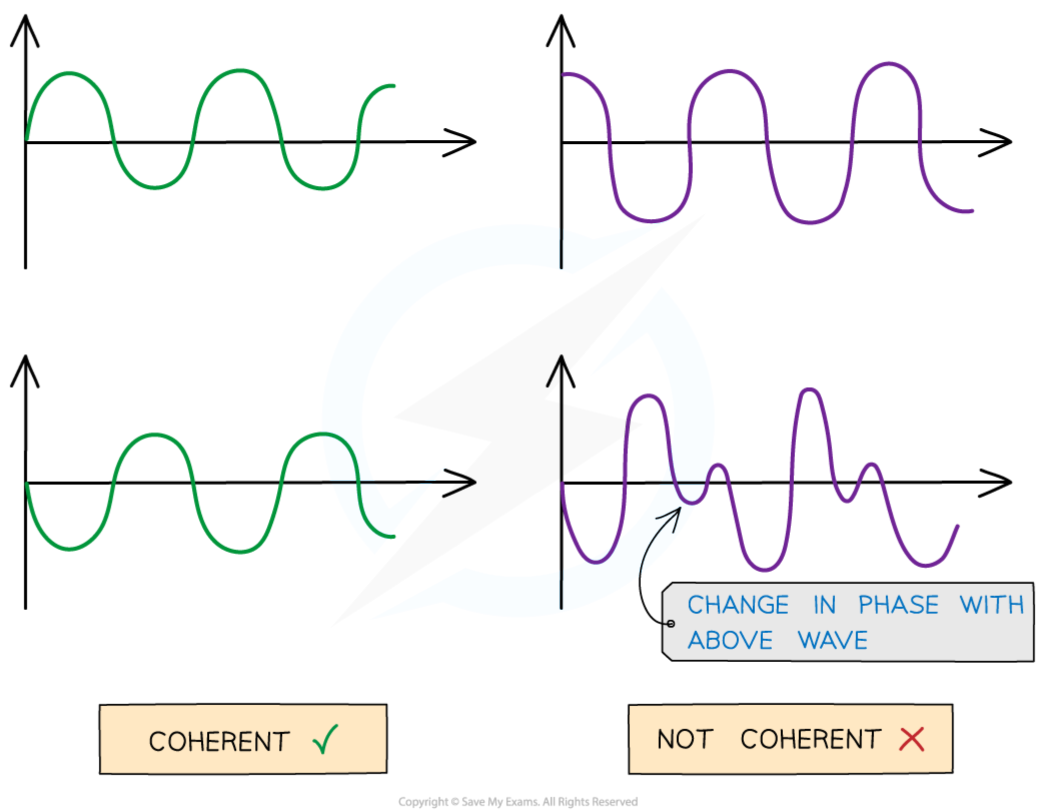
4.4.6 Interference & Path Difference

Double-Slit Interference

- **Interference** occurs whenever two or more waves combine to produce a resultant wave with a new resultant **displacement**
- The waves combine according to the **principle of superposition**
  - **Constructive interference** happens when the resultant wave has a larger displacement than any of the individual displacements
  - **Destructive interference** happens when the positive displacement of one wave and the negative displacement of another wave exactly cancel out giving a resultant displacement of zero

Coherence

- Interference is only observable if produced by a **coherent** source
- Waves are said to be coherent if they have:
  - A **constant phase difference**
  - The **same frequency**



Coherent waves (on the left) and non-coherent waves (on the right). The abrupt change in phase creates an inconsistent phase difference

- A coherent beam of light contains light waves that are **monochromatic** and have a constant phase difference
  - Monochromatic light consists of light waves of a **single frequency**

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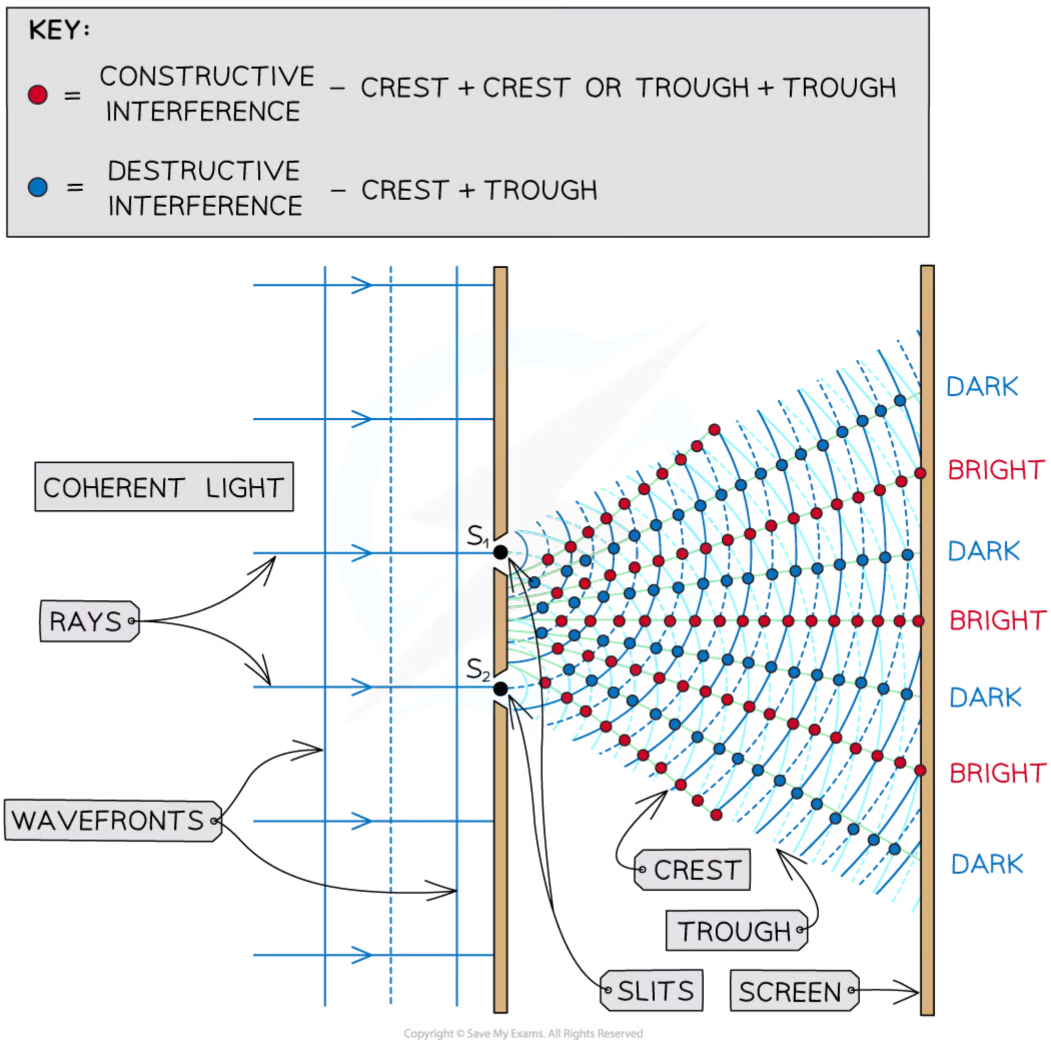
- Laser light is an example of a coherent light source
- Filament lamps produce incoherent light waves

Double-Slit Interference of Light

- When a **coherent beam of light** is incident on two narrow slits very close together, **diffraction** occurs at each slit (i.e. the waves spread out)
- As the diffracted waves cross, they **interfere** with each other
- If a screen is placed some distance away from the slits, a pattern of equally spaced bright and dark **fringes** is observed on the screen
  - The bright fringes form where the waves interfere constructively (i.e. a crest meets a crest or a trough meets a trough)
  - The dark fringes form where the waves interfere destructively (i.e. a crest meets a trough)



Coherent light waves interfere after passing through two narrow slits. Alternating bright and dark fringes are observed on the screen

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Path Difference

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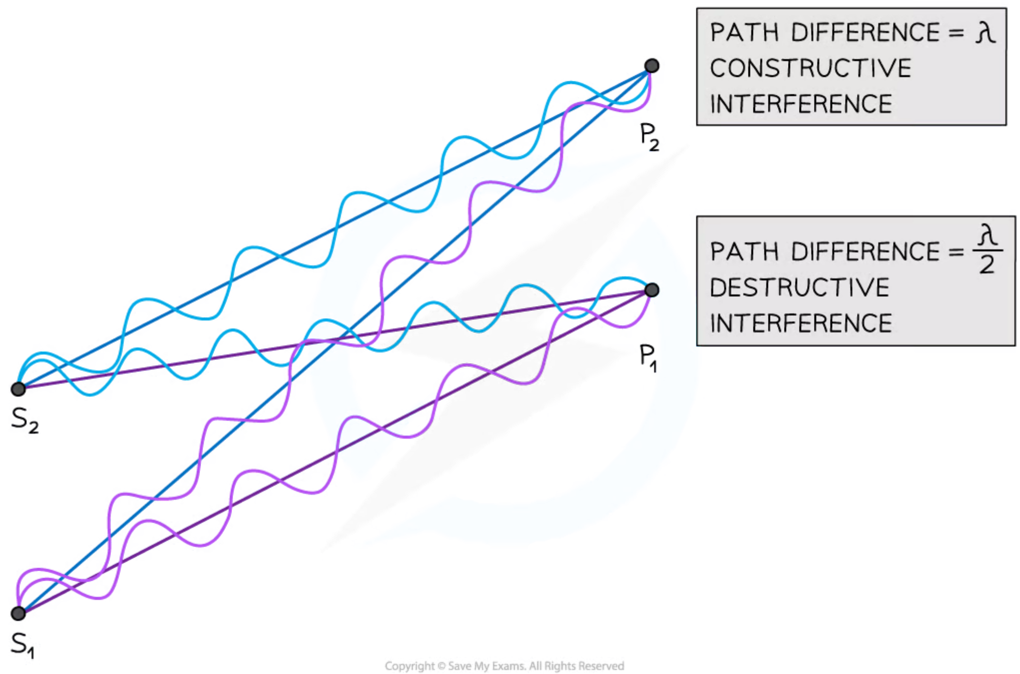
- The type of interference occurring at a given point (i.e. constructive or destructive) depends on the **path difference** of the overlapping waves
- Path difference is defined as:

The difference in distance travelled by two waves from their sources to the point where they meet

- Path difference is generally expressed in multiples of wavelength



At point P_2 the waves have a path difference of a whole number of wavelengths resulting in constructive interference. At point P_1 the waves have a path difference of an odd number of half wavelengths resulting in destructive interference

- In the diagram above, the number of wavelengths between:
  - $S_1 \rightarrow P_1 = 6\lambda$
  - $S_2 \rightarrow P_1 = 6.5\lambda$
  - $S_1 \rightarrow P_2 = 7\lambda$
  - $S_2 \rightarrow P_2 = 6\lambda$

- The path difference is:
  - $(6.5\lambda - 6\lambda) = \boldsymbol{\lambda / 2}$ at point P_1
  - $(7\lambda - 6\lambda) = \boldsymbol{\lambda}$ at point P_2

- Hence:
  - **Destructive interference** occurs **at** point $\mathbf{P_1}$
  - **Constructive interference** occurs **at** point $\mathbf{P_2}$

Conditions for Constructive and Destructive Interference

- In general, for waves emitted by two coherent sources very close together:

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- The condition for **constructive interference** is:

$$path\ difference = n\lambda$$

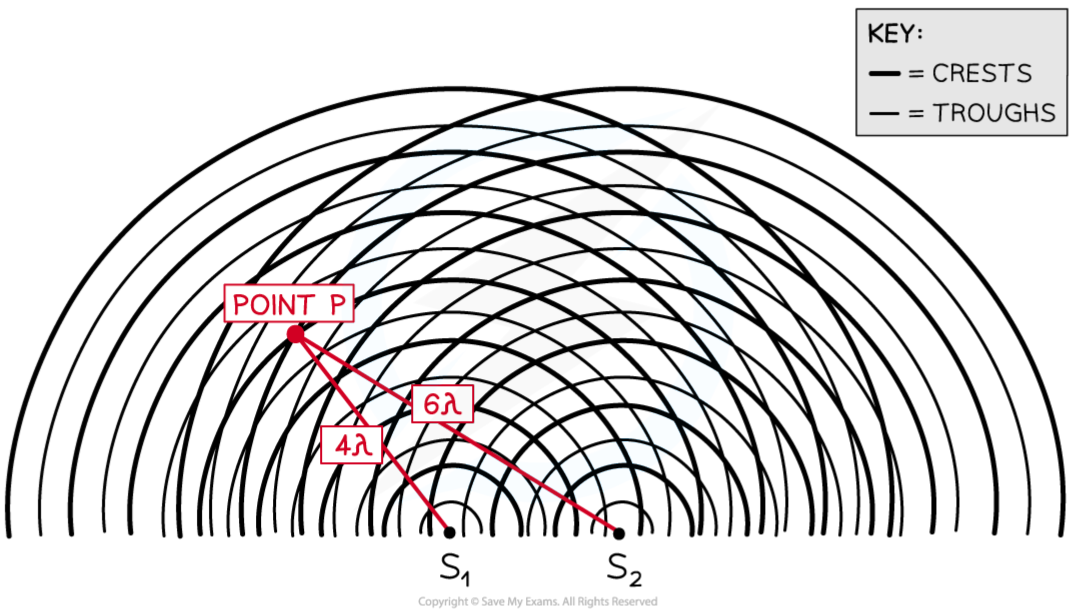
- The condition for **destructive interference** is:

$$path\ difference = (n + ½)\lambda$$

- Where:
  - λ = wavelength of the waves in metres (m)
  - n = 0, 1, 2, 3... (any other integer)

- The same conditions apply to waves emitted by a single coherent source and diffracted by two narrow slits very close together

Path Difference and Wavefronts



At point P the waves have a path difference of a whole number of wavelengths resulting in constructive interference

- Another way to represent waves spreading out from two sources is shown in the diagram above
- At point **P**, the number of **crests** from:
  - Source $S_1 = 4\lambda$
  - Source $S_2 = 6\lambda$
- The path difference at **P** is $(6\lambda - 4\lambda) = \mathbf{2\lambda}$
- This is a whole number of wavelengths ($n = 2$), hence **constructive interference** occurs **at** point **P**

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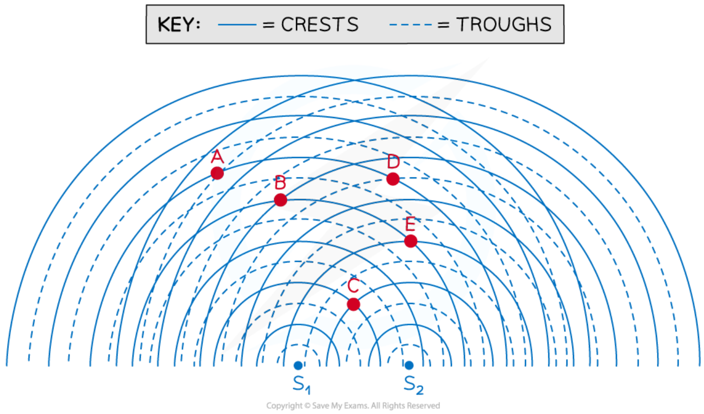
Worked Example

The diagram below is a snapshot of overlapping wavefronts resulting from the interference of coherent waves diffracted by two narrow slits S_1 and S_2.



For each of the points shown, determine:

- The path difference from the sources
- The value of *n* in the path difference formula
- Whether they are locations of constructive or destructive interference

Step 1: Count the number of wavelengths between each source and the desired point

- E.g. Number of wavelengths between:
  - S_1 ➜ A = 5λ
  - S_2 ➜ A = 6.5λ

Step 2: Determine the path difference by subtracting the distances of the point from the two sources

- E.g. Path difference at A = (6.5λ – 5λ) = 1.5λ

Step 3: Compare the path difference calculated in Step 2 with the condition for constructive or destructive interference and give the value of *n*

- E.g. Path difference at A = 1.5λ = $(n + ½)λ$ ➜ n = 1

Step 4: Decide whether the point is a location of constructive or destructive interference

- E.g. A is a location of destructive interference

- Point **A**:
  - Path difference = (6.5λ – 5λ) = 1.5λ
  - n = 1
  - Destructive interference
- Point **B**:
  - Path difference = (5λ – 4λ) = λ

- $n = 1$
- Constructive interference

- Point **C**:
  - Path difference = (2λ – 2λ) = 0
  - $n = 0$
  - Constructive interference
- Point **D**:
  - Path difference = (5λ – 4.5λ) = 0.5λ
  - $n = 0$
  - Destructive interference
- Point **E**:
  - Path difference = (4λ – 3λ) = λ
  - $n = 1$
  - Constructive interference

Exam Tip

You are not required to memorise the conditions for constructive and destructive interference, as these are given in the data booklet. You must be able to determine the path difference of waves from two sources (or two narrow slits) at a given point. You can then compare this with the given conditions for constructive and destructive interference, in order to decide which type of interference occurs at the point you are considering.

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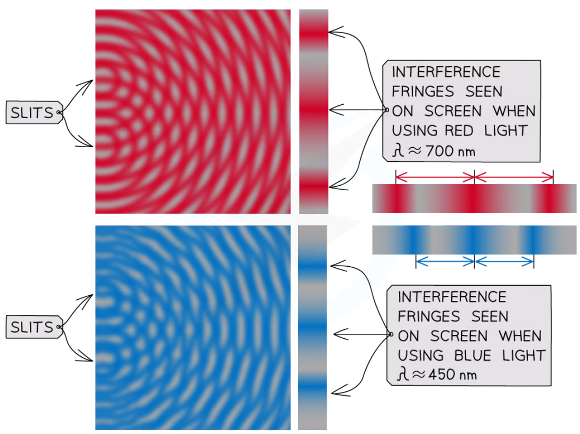
Double-Slit Equation

Interference Patterns

- Interference patterns depend on:
  - The coherent light incident on the slits
  - The **distance between the slits and the screen** where the interference pattern is observed
  - The **separation between the slits**
- In particular:
  - Successive bright fringes are further apart from each other if the wavelength λ of the incident light is longer



Difference in the interference pattern of red and blue light. Red light has a longer wavelength than blue light, so the fringes are more spaced out

  - Similarly, bright fringes are more spaced out if the screen is placed further away from the slits
  - If the separation between the slits is increased, instead successive bright fringes are closer to each other

Explaining the Interference Pattern

- Two slits are separated by a length *d*, and a screen is placed a distance *D* away

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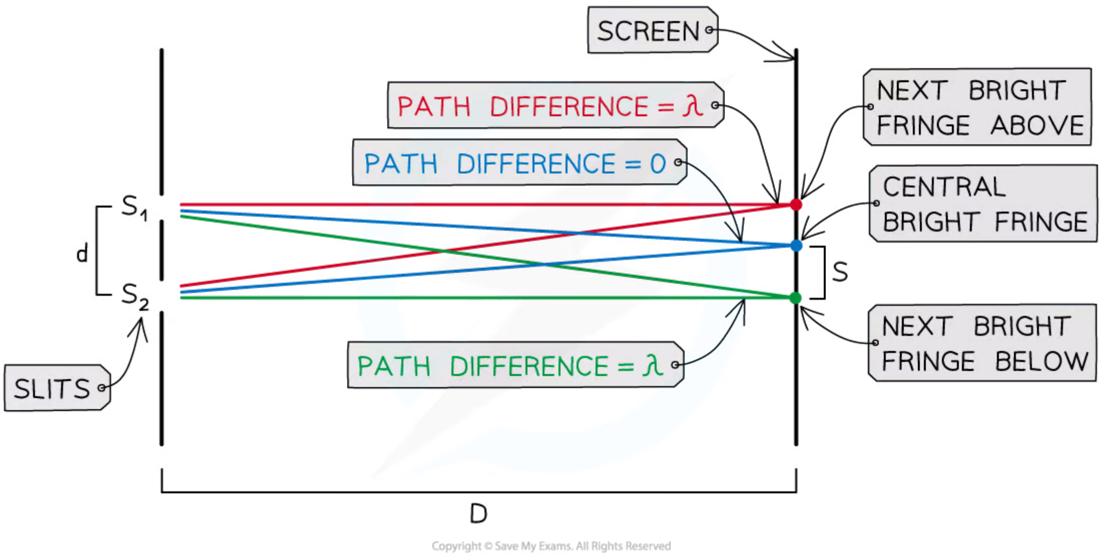

Light rays of wavelength λ incident on two slits a distance s apart interferes and form bright fringes on a screen placed a distance D away from the slits

- When light rays of wavelength λ are incident on the two slits:
  - A **central bright fringe** where the two waves have travelled the **same** distance to the screen and their path difference is **zero**
  - Another **bright fringe** is formed either side of the central one where the path difference between the waves is **exactly one wavelength**
    - This is constructive interference
  - **Dark fringes** (an absence of light) are formed where the path difference between the waves is some number of wavelengths **plus half a wavelength**
    - This is destructive interference

- Further bright fringes will be located at each position on the screen where the path difference is exactly equal to *n* number of wavelengths $\boldsymbol{n\lambda}$
- For example: 2λ, 3λ, 4λ...
- Dark fringes are located in between the bright ones, where the path difference is exactly $\boldsymbol{(n + \frac{1}{2})\lambda}$
- For example: $1\frac{1}{2}\lambda,\ 2\frac{1}{2}\lambda,\ 3\frac{1}{2}\lambda$...

Double-Slit Equation

- The separation *s* of successive fringes is given by:

$$s = \frac{\lambda D}{d}$$

- Where:
  - s = separation between successive fringes on the screen (m)
  - λ = wavelength of the waves incident on the slits (m)

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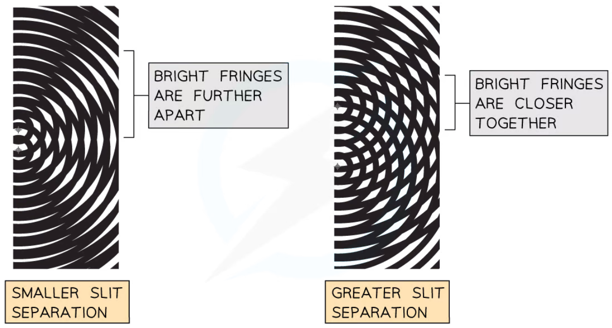
- D = distance between the screen and the slits (m)
- d = separation between the slits (m)

- Note that s is the separation between two successive bright fringes or two successive dark ones
- The above equation shows that the separation between the fringes will increase if:
  - The wavelength of the incident light increases
  - The distance between the screen and the slits increases
  - The separation between the slits decreases



Dependence of the interference pattern on the separation between the slits. The further apart the slits, the closer together the bright fringes

Worked Example

In a double-slit experiment, two slits are placed a distance of 0.40 mm apart and a screen is located 0.50 m away from the slits.

Coherent electromagnetic waves incident on the slits produce an interference pattern on the screen. The separation between dark fringes is 0.50 cm.

Determine the wavelength and state the type of electromagnetic waves used in the experiment.

Step 1: Write down the known quantities

- d = 0.40 mm = 4.0×10^{-4} m
- D = 0.50 m
- s = 0.50 cm = 5.0×10^{-3} m

Note that you must convert all lengths into metres (m)

Step 2: Write down the double-slit equation

$$s = \frac{\lambda D}{d}$$

Step 3: Rearrange the above equation to calculate the wavelength λ

$$\lambda = \frac{sd}{D}$$

Step 4: Substitute the numbers into the above equation

$$\lambda = \frac{(5.0 \times 10^{-3}) \times (4.0 \times 10^{-4})}{0.50}$$

$$\lambda = 4.0 \times 10^{-6} \text{ m} = 4.0 \text{ } \mu\text{m}$$

This corresponds to the infrared area of the electromagnetic spectrum

YOUR NOTES

Exam Tip

Remember that the separation between dark fringes is exactly the same as the separation between bright fringes. Whether a question gives you or asks about the separation between dark fringes instead of bright ones it makes no difference.

Some tricky questions might give you the separation between a bright and a dark fringe. This is equal to half the value of s!

YOUR NOTES
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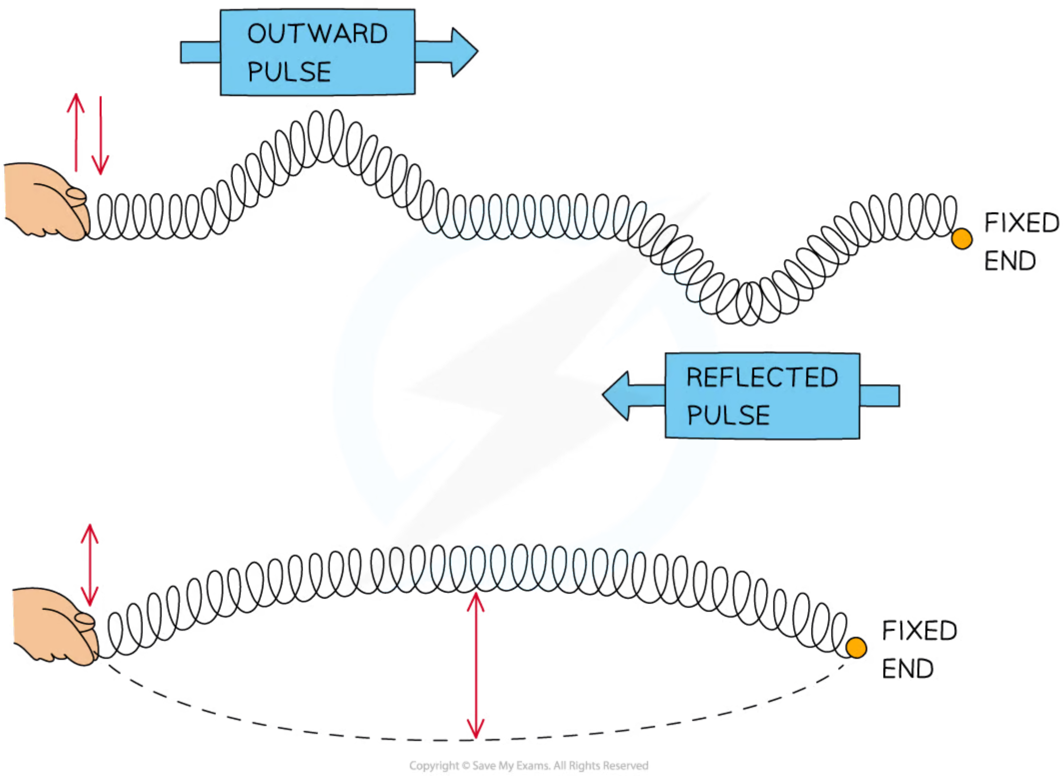
4.5 Standing Waves

4.5.1 The Nature of Standing Waves

The Nature of Standing Waves

- **Standing waves** are produced by the amplitude travelling in **opposite directions**
- This is usually achieved when a travelling wave superimposes its reflection
  - The superposition produces a wave pattern where the crests and troughs do not move



Formation of a stationary wave on a stretched spring fixed at one end

Formation of Standing Waves

- Standing waves are formed from the principle of superposition. This is when:

  Two waves travelling in opposite directions along the same line with the same frequency superpose

- The principle of superposition applies to all types of waves i.e. transverse and longitudinal, progressive and stationary
- The waves must have:
  - The same wavelength
  - A similar amplitude
- As a result of superposition, a resultant wave is produced

YOUR NOTES
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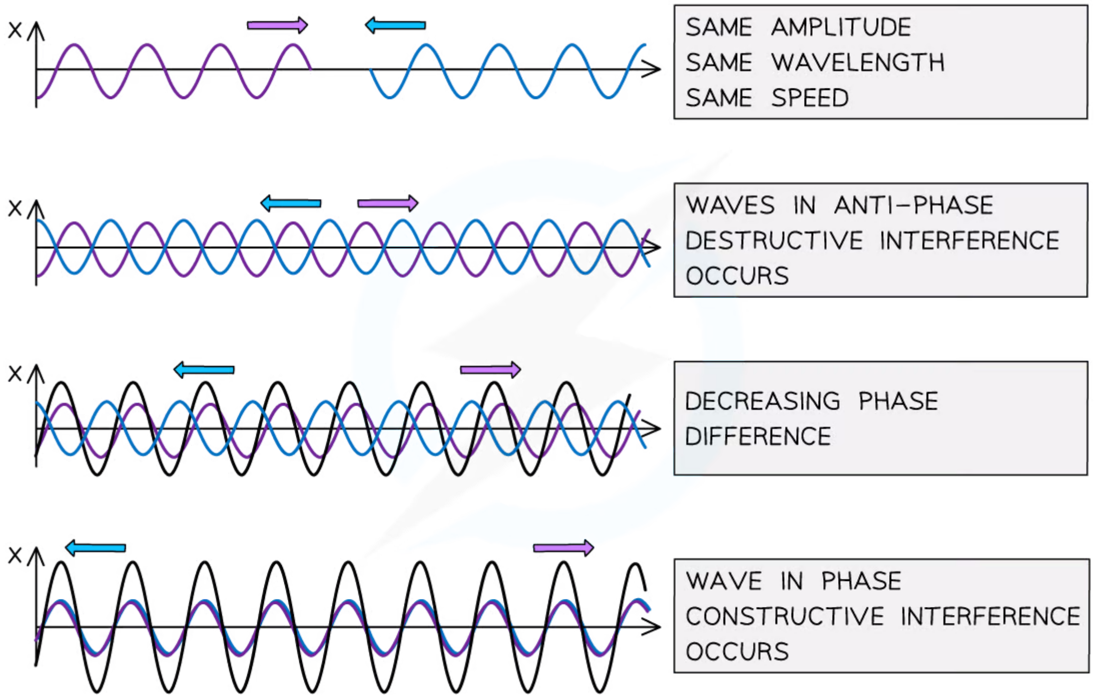

A graphical representation of how stationary waves are formed – the black line represents the resulting wave

Comparing Progressive and Standing Waves

- Standing waves (or stationary waves) **store** energy
- **Progressive waves** (or travelling waves) **transfer** energy
- The table below outlines the main differences between progressive and stationary waves

Table of Differences Between Progressive and Stationary Waves

Progressive Waves	Stationary Waves
All points have the same amplitude (in turn)	Each point has a different amplitude depending on the amount of superposition
Points exactly a wavelength apart are in phase. The phase of points within one wavelength can be between 0 to 360°	Points between nodes are in phase. Points on either side of a node are out of phase
Energy is transferred along the wave	Energy is stored, not transferred
Does not have nodes or antinodes	Has nodes and antinodes
The wave speed is the speed at which the wave moves through a medium	Each point on the wave oscillates at a different speed. The overall wave does not move

YOUR NOTES

Worked Example

A travelling wave is incident on a barrier. The wave profile is shown below.

The travelling wave reflects off the barrier. The reflected and incident waves superimpose.

State whether or not a standing wave is formed.

- For standing waves to be formed, the half-cycles of the wave profile must be symmetrical (i.e. the same but inverted)
- For this wave, the half-cycles are not symmetrical
  - The leading edge is straight
  - The trailing edge is sinusoidal
- When the incident and reflected waves superimpose, they will not cancel out at any point
- Therefore a standing wave is not formed

4.5.2 Nodes & Antinodes

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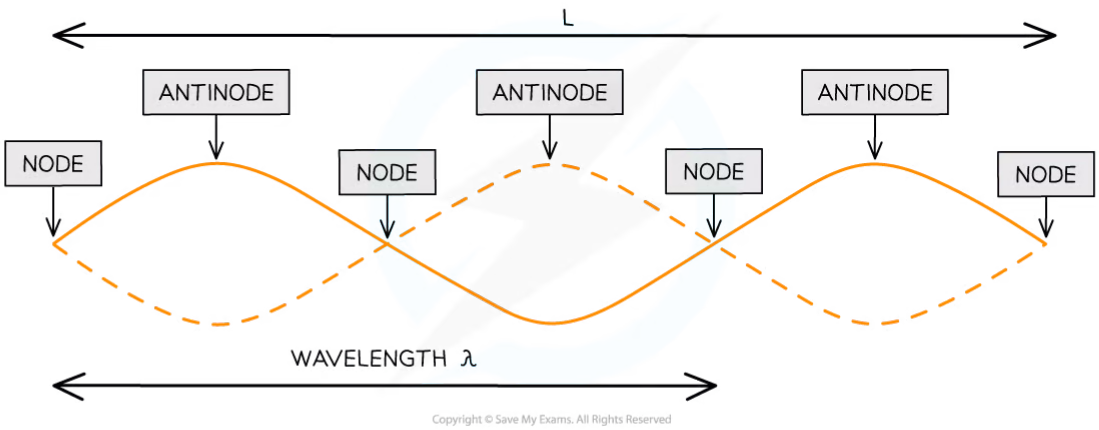
Nodes & Antinodes

- A standing wave is made up **nodes** and **antinodes**
  - **Nodes** are locations of **zero amplitude** and they are **separated by half a wavelength** ($\lambda/2$)
  - **Antinodes** are locations of **maximum amplitude**

- The nodes and antinodes **do not** move along the wave
  - Nodes are fixed and antinodes only oscillate in the vertical direction



Nodes and antinodes of a stationary wave of wavelength λ on a string of length L at a point in time

The Formation of Nodes and Antinodes

- Nodes are formed as a result of destructive interference
  - The amplitude of both waves cancel out
- Antinodes are formed as a result of constructive interference
  - The amplitude of both waves add together

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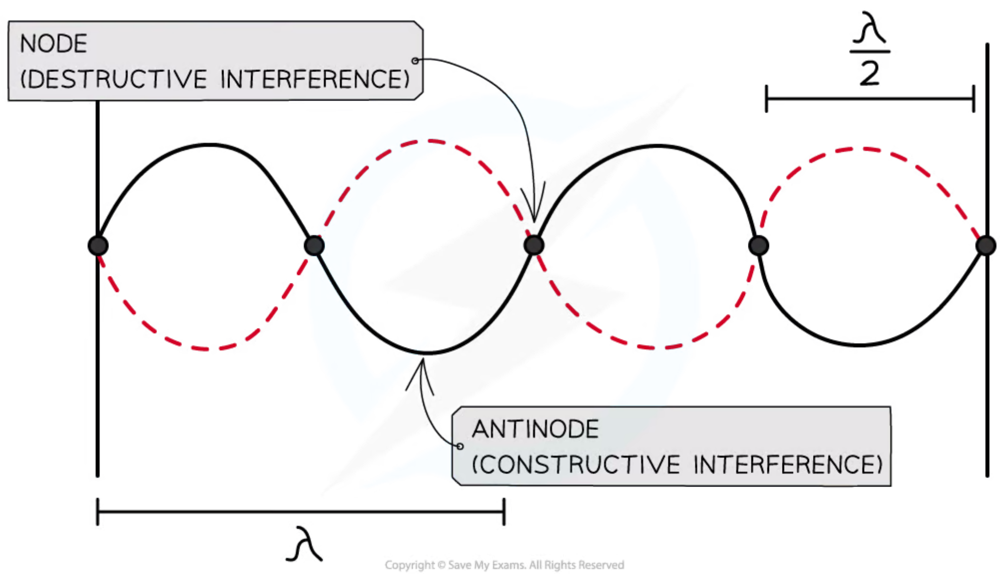
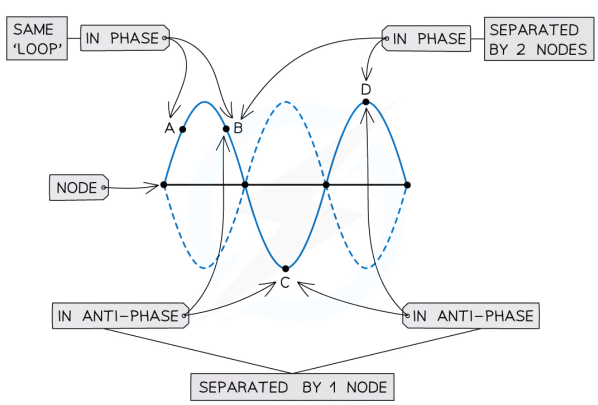


Nodes and antinodes are a result of destructive and constructive interference respectively

- At the **nodes**:
  - The waves are in anti-phase meaning destructive interference occurs
  - This causes the two waves to cancel each other out
- At the **antinodes**:
  - The waves are in phase meaning constructive interference occurs
  - This causes the waves to add together

Phase on a Standing Wave

- Two points on a standing wave are either In Phase
  - Points that have an **odd** number of nodes between them are in **anti-phase**
  - Points that have an **even** number of nodes between them are **in phase**
  - All points within a **"loop"** are **in phase**



Points A, B and D are all in phase. While points A and D are in antiphase with point C

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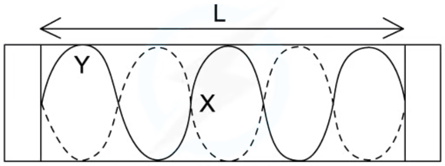
Worked Example

Which row in the table correctly describes the length of L and the name of **X** and **Y**?

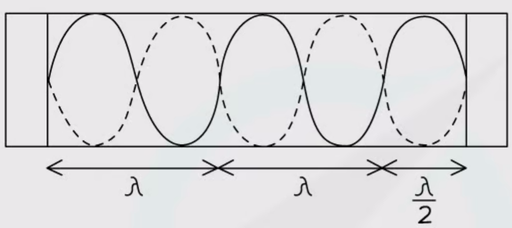


	Length L	Point X	Point Y
A	5 wavelengths	Node	Antinode
B	$2\frac{1}{2}$ wavelengths	Antinode	Node
C	$2\frac{1}{2}$ wavelengths	Node	Antinode
D	5 wavelengths	Antinode	Node

ANSWER: C

STEP 1

CALCULATE HOW MANY WAVELENGTHS IN THE LENGTH OF THE STRING



$= 2\frac{1}{2}$ WAVELENGTHS

THIS RULES OUT A AND D

STEP 2

X IS A POINT OF 0 DISPLACEMENT – A NODE

STEP 3

Y IS A POINT OF MAXIMUM DISPLACEMENT – AN ANTINODE

STEP 4

THE CORRECT ROW IS C

4.5.3 Boundary Conditions for Standing Waves

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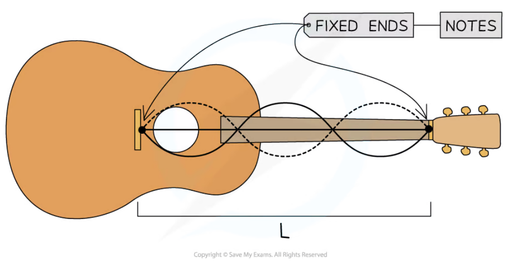
Boundary Conditions

- Stationary waves can form on strings or in pipes
- In both cases, progressive waves travel in a medium (i.e. the string or air) and superimpose with their reflections
- The number of nodes and antinodes that fit within the available length of medium depends on:
  - The frequency of the progressive waves
  - The **boundary conditions** (i.e. whether the progressive waves travel between two fixed ends, two free ends or a fixed and a free end)

Standing Waves on Stretched Strings

- When guitar strings are plucked, they can vibrate with different frequencies
- The frequency with which a string vibrates depends on:
  - The tension, which is adjusted using rotating 'tuning pegs'
  - The mass per unit length, which is the reason why a guitar has strings of different thicknesses



Standing wave on a guitar string

- For a string, the boundary condition can be
  - Fixed at both ends
  - Free at both ends
  - One end fixed, the other free

- At specific frequencies, known as **natural frequencies**, an **integer number of half wavelengths** will fit on the length of the string
  - As progressive waves of different natural frequencies are sent along the string, standing waves with different numbers of nodes and antinodes form

Standing Waves in Pipes

- When the air within a pipe vibrates, longitudinal waves travel along the pipe
- Simply blowing across the open end of a pipe can produce a standing wave in the pipe
- For a pipe, there is more than one possible boundary condition, theses are pipes that are:
  - Closed at both ends
  - Open at both ends
  - Closed at one end and open on the other

Nodes & Antinodes

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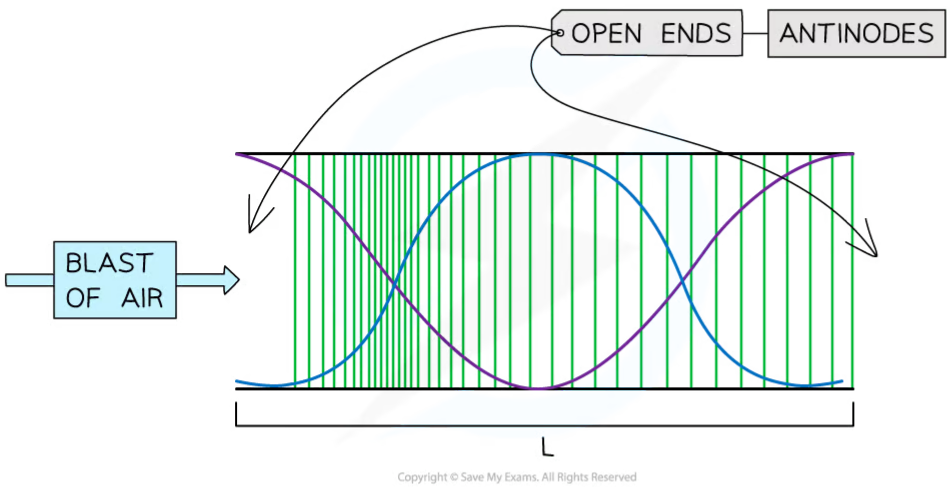
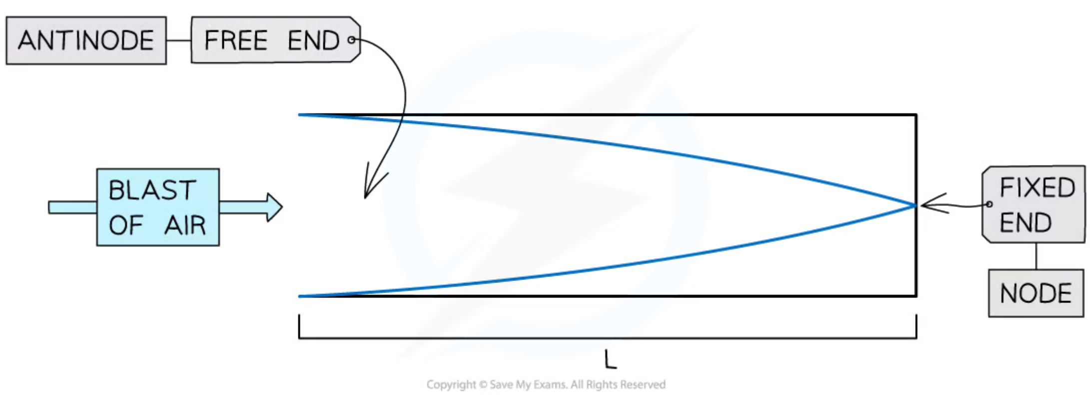
- When a progressive wave travels towards a free end for a string, or open end for a pipe:
  - The **reflected wave** is **in phase** with the incident wave
  - The amplitudes of the incident and reflected waves add up
  - A free end is a location of maximum displacement - i.e. an **antinode**



Standing wave inside a pipe open at both ends

- When a progressive wave travels towards a fixed end for a string, or closed end for a pipe:
  - The **reflected wave** is in **anti-phase** with the incident wave
  - The two waves cancel out
  - A fixed end is a location of zero displacement - i.e. a **node**
  - The open end is therefore a location of maximum displacement - i.e. an **antinode**



Standing wave inside a pipe open at both ends

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4.5.4 Harmonics

Harmonics

- Stationary waves can have different wave patterns, known as **harmonics**
  - These depend on the **frequency** of the vibration and the **boundary conditions** (i.e. fixed and/or free ends)
- The harmonics are the only frequencies and wavelengths that will form standing waves on strings or in pipes

Harmonics on Strings

- The **boundary condition** is that **both ends are fixed**
- The simplest wave pattern is a single loop made up of two nodes (i.e. the two fixed ends) and an antinode
  - This is called the **first harmonic**
  - The wavelength of this harmonic is $\lambda_1 = 2L$
  - Using the wave equation, the frequency is $f_1 = v/2L$, where v is the wave speed of the travelling waves on the string (i.e. the incident wave and the reflected wave)
- As the vibrating frequency increases, more complex patterns arise
  - The **second harmonic** has three nodes and two antinodes
  - The **third harmonic** has four nodes and three antinodes

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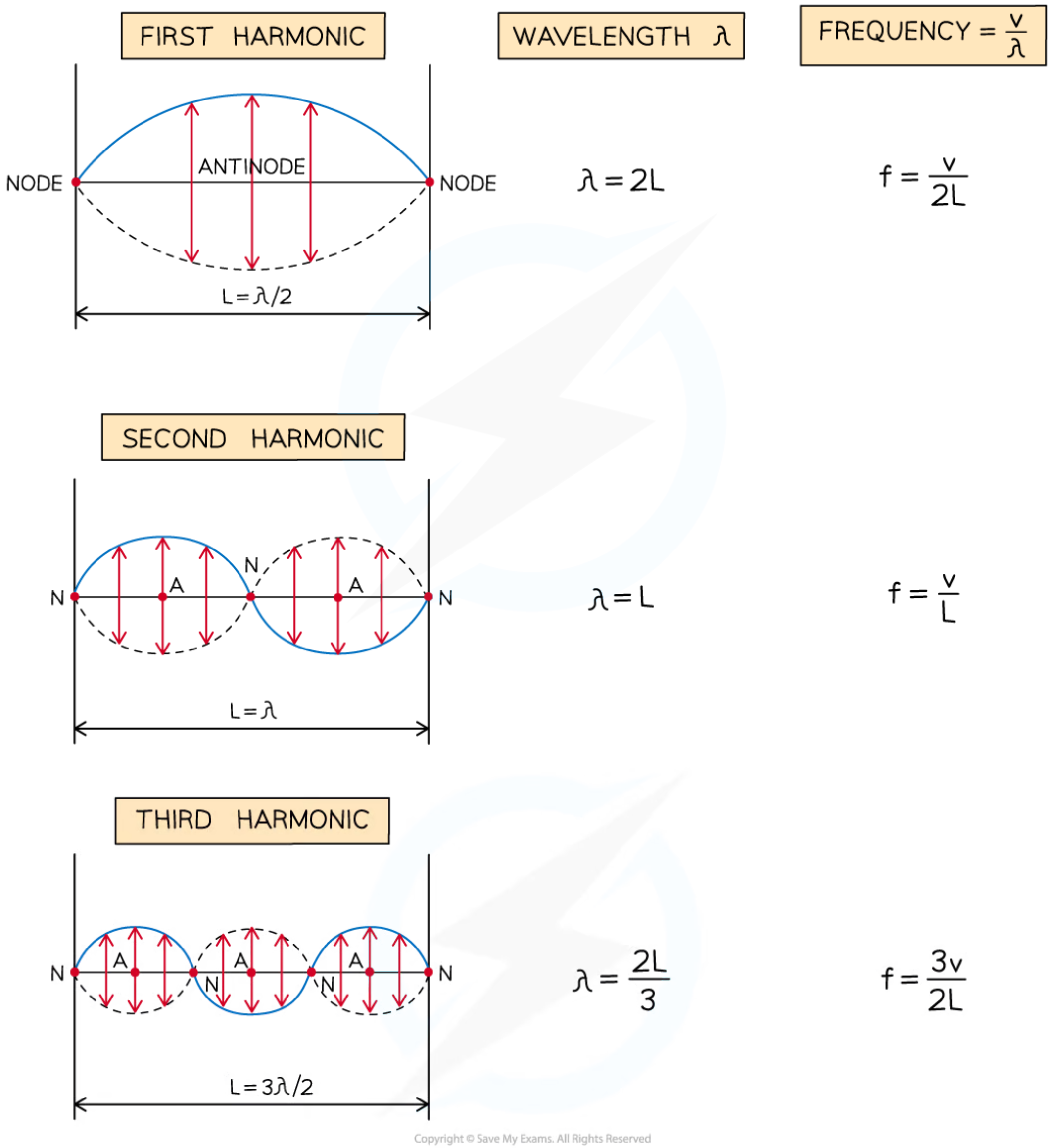

Diagram showing the first three harmonics on a stretched string fixed at both ends

- The nth harmonic will have $(n + 1)$ nodes and n antinodes
- The general expression for the **wavelength of the nth harmonic** on a string that is fixed at both ends is:

$$\lambda_n = \frac{2L}{n}$$

- Where:
  - λ_n = wavelength in metres (m)
  - L = length of the string in metres (m)
  - n = integer number greater than zero - i.e. 1, 2, 3...
- Knowing the wavelength λ_n of the standing wave and the speed v of the travelling waves (i.e. incident and reflected), the **natural frequency** f_n of any harmonic can be calculated

using the wave equation $v = f\lambda_n$, so that:

$$f_n = \frac{nv}{2L}$$

Harmonics in Pipes

- The **boundary conditions** vary, since pipes can have:
  - **two open ends**
  - only **one open end**

- For a pipe that is **open at both ends**:
  - The simplest wave pattern is one central node and two antinodes
  - The second harmonic consists of two nodes and three antinodes
  - The nth harmonic will have $(n + 1)$ antinodes and n nodes
  - The expression for the wavelength of the nth harmonic in a pipe of length L is the same as that given above for nth harmonic on a string

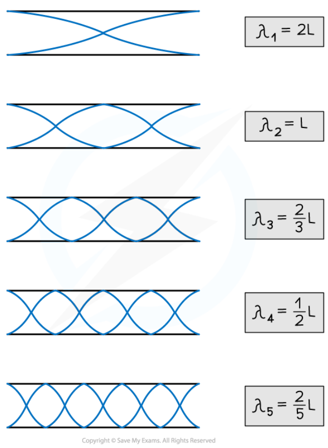

Diagram showing the first five harmonics in a pipe open at both ends

- For a pipe that is **open at one end**:
  - The lowest harmonic is a "half-loop" with one node and one antinode
  - The next possible harmonic will have two nodes and two antinodes
    - This is the third harmonic, not the second one
    - Since **only odd harmonics can exist** under this boundary condition

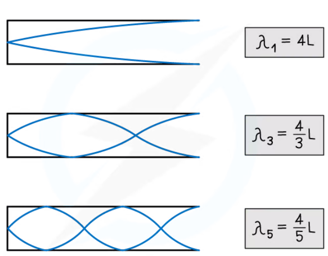

Diagram showing the first three possible harmonics in a pipe open at one end. Only the odd harmonics can form in this case

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- The expression for the wavelength of the *n*th harmonic in a pipe of length *L* is:

$$\lambda_n = \frac{4L}{n}$$

- Where this time, *n* is an odd number - i.e. 1, 3, 5...

- Under both boundary conditions, the natural frequencies are once again calculated from the wavelength of the standing wave and the speed *v* of the travelling waves using the wave equation

Worked Example

Transverse waves travel along a stretched wire 100 cm long. The speed of the waves is 250 m s^{-1}.

Determine the maximum harmonic detectable by a person who can hear up to 15 kHz.

Step 1: Write down the known quantities

- L = 100 cm = 1.00 m
- v = 250 m s^{-1}
- f_n = 15 kHz = 15000 Hz

Note the conversions:

- The length must be converted from centimetres (cm) into metres (m)
- The frequency must be converted from kilohertz (kHz) into hertz (Hz)

Step 2: Write down the equation for the frequency of the *n*th harmonic

$$f_n = \frac{nv}{2L}$$

Step 3: Rearrange the above equation to calculate the number *n* of the maximum harmonic detectable by the person

$$n = \frac{2Lf_n}{v}$$

Step 4: Substitute the numbers into the above equation

$$n = \frac{2 \times 1.00 \times 15000}{250}$$

$$n = 120$$

The person can hear up to the 120th harmonic

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Exam Tip

Before carrying out any calculation on standing waves, you should look carefully at the boundary conditions, since these will determine the wavelengths and natural frequencies of the harmonics.

The expressions for the wavelength of the *n*th harmonic on strings fixed at both ends (or in pipes open at both ends) and in pipes open at one end are not given in the data booklet and you must be able to recall them.

Remember that *n* can take any integer value greater than zero in the case of standing waves on strings and in pipes open at both ends. For pipes open at one end, instead, *n* can only be an odd integer.