

4.6 Normal Distributions

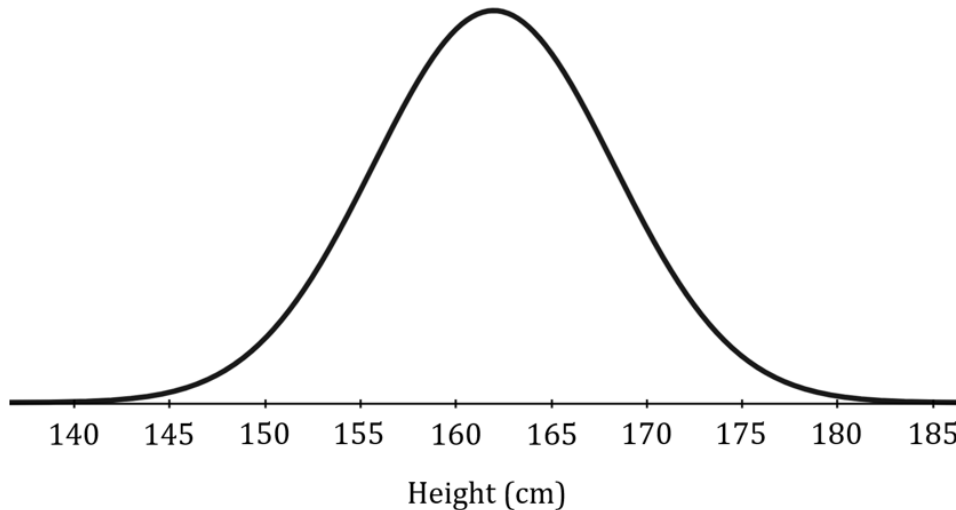
Question Paper

Course	DPIB Maths
Section	4. Statistics & Probability
Topic	4.6 Normal Distributions
Difficulty	Medium

Time allowed: 80
Score: /61
Percentage: /100

Question 1a

The random variable, X , is seen on the following diagram which shows the distribution of heights, in cm, of adult women in the UK:



The distribution of heights follows a normal distribution, with a mean of 162 cm and a standard deviation of 6.3 cm.

(a) On the diagram above, shade in the region representing $P(X > 155)$.

[2 marks]

Question 1b

- (b) (i) Find the probability that a randomly selected woman has a height of more than 155cm.
- (ii) Use your answer from part (b)(i) to find the probability that a randomly selected woman has a height of more than 169cm.

[4 marks]

Question 1c

(c) Suggest a range of heights within which the height of approximately

(i) 68%

(ii) 95%

(iii) 99.7%

of adult women in the UK will fall.

[3 marks]

Question 2a

(a) For the random variable $X \sim N(23, 4^2)$ find the following probabilities:

(i) $P(X < 20)$

(ii) $P(X \geq 29)$

(iii) $P(20 \leq X < 29)$

[3 marks]

Question 2b

(b) For the random variable $Y \sim N(100, 225)$ find the following probabilities:

(i) $P(Y \leq 90)$

(ii) $P(Y > 140)$

(iii) $P(85 \leq Y \leq 115)$

[3 marks]

Question 3a

The weight, W g, of a chocolate bar produced by a certain manufacturer is modelled as $W \sim N(200, 1.75^2)$.

(a) Find:

(i) $P(W < 195)$

(ii) $P(W > 203)$

[2 marks]

Question 3b

Heledd buys a pack containing 12 of the chocolate bars. It may be assumed that the 12 bars in the pack represent a random sample.

(b) Find the probability that all of the bars in the pack have a weight of at least 195 g.

[2 marks]

Question 4a

The random variable $X \sim N(330, 10^2)$.

(a) Find the value of a , to 2 decimal places, such that:

(i) $P(X < a) = 0.25$

(ii) $P(X > a) = 0.25$

(iii) $P(315 \leq X \leq a) = 0.5$

[4 marks]

Question 4b

The random variable $Y \sim N(10, 10)$.

(b) Find the value of b and the value of c , each to 2 decimal places, such that:

(i) $P(Y < b) = 0.4$

(ii) $P(Y > c) = 0.25$

[2 marks]

Question 4c

(c) Use a sketch of the distribution of Y to explain why $P(b \leq Y \leq c) = 0.35$.

[2 marks]

Question 5a

The test scores, X , of a group of RAF recruits in an aptitude test are modelled as a normal distribution with $X \sim N(210, 27.8^2)$.

- (a) (i) Find the values of a and b such that $P(X < a) = 0.25$ and $P(X > b) = 0.25$.
- (ii) Hence find the interquartile range of the scores.

[3 marks]

Question 5b

Those who score in the top 30% on the test move on to the next stage of training.

- (b) One of the recruits, Amelia, achieves a score of 231. Determine whether Amelia will move on to the next stage of training.

[2 marks]

Question 6a

(a) For the standard normal distribution $Z \sim N(0, 1^2)$, find:

- (i) $P(Z < 1.5)$
- (ii) $P(Z > -0.8)$
- (iii) $P(-2.1 < Z < -0.3)$

[4 marks]

Question 6b

The random variable $X \sim N(2, 0.1^2)$.

(b) By using the coding relationship between X and Z , re-express the probabilities from parts (a) (i), (ii) and (iii) in the forms $P(X < a)$, $P(X > b)$ and $P(c < X < d)$ respectively, where a , b , c and d are constants to be found.

[3 marks]

Question 7a

The table below shows the percentage points of the normal distribution. The values z in the table are those which a random variable $Z \sim N(0, 1)$ exceeds with probability p .

p	z	p	z
0.5000	0.0000	0.0500	1.6449
0.4000	0.2533	0.0250	1.9600
0.3000	0.5244	0.0100	2.3263
0.2000	0.8416	0.0050	2.5758
0.1500	1.0365	0.0010	3.0902
0.1000	1.2816	0.0005	3.2905

- (a) (i) Use the percentage points table for the standard normal distribution to find the value of z for which $P(Z > z) = 0.2$.
- (ii) Use your answer to part (a)(i) along with the properties of the normal distribution to work out the values of a and b for which $P(Z < a) = 0.2$ and $P(Z < b) = 0.8$.

[3 marks]

Question 7b

The weights, W kg, of coconuts grown on the Coconutty As They Come coconut plantation are modelled as a normal distribution with mean 1.25 kg and standard deviation 0.38 kg. The plantation only considers coconuts to be exportable if their weight falls into the 20% to 80% interpercentile range.

(b) Use your answer to part (a)(ii) to find the range of possible weights, to the nearest 0.01 kg, for an exportable coconut.

[2 marks]

Question 8a

A machine is used to fill cans of a particular brand of soft drink. The volume, V ml, of soft drink in the cans is normally distributed with mean 330 ml and standard deviation σ ml. Given that 15% of the cans contain more than 333.4 ml of soft drink, find:

(a) the value of σ

[2 marks]

Question 8b

(b) $P(320 \leq V \leq 340)$.

[1 mark]

Question 8c

Six cans of the soft drink are chosen at random.

(c) Find the probability that all of the cans contain less than 329 ml of soft drink.

[3 marks]

Question 9a

The random variable $X \sim N(\mu, \sigma^2)$. It is known that $P(X > 36.88) = 0.025$ and $P(X < 27.16) = 0.1$

(a) Find the values of a and b for which $P(Z > a) = 0.025$ and $P(Z < b) = 0.1$, where Z is the standard normal variable. Give your answers correct to 4 decimal places.

[2 marks]

Question 9b

(b) Use your answers to part (a), along with the relationship between Z and X , to show that the following simultaneous equations must be true:

$$\begin{aligned}\mu + 1.96\sigma &= 36.88 \\ \mu - 1.2816\sigma &= 27.16\end{aligned}$$

[2 marks]

Question 9c

(c) By solving the simultaneous equations in (b), determine the values of μ and σ . Give your answers correct to 2 decimal places.

[2 marks]

Question 10

The ages, A , in years, that Liverpool players have made their debuts over the past 20 years are normally distributed with a mean of 22.5 years and a standard deviation of σ years.

Given that 10% of Liverpool players make their debuts before turning 20 years old, find:

- (i) the value of σ ,
- (ii) the probability that a randomly selected player made his debut before his 18th birthday.

[5 marks]

