2.5 Reciprocal & Rational Functions

Question Paper

Course	DP IB Maths
Section	2. Functions
Topic	2.5 Reciprocal & Rational Functions
Difficulty	Hard

Time allowed: 100

Score: /77

Percentage: /100

Question 1a

Let $f(x) = \dfrac{b}{x-a} + 2$, for $x \neq a$.

The line $x = -3$ is a vertical asymptote of the graph of f.

(a)
Write down the value of a.

[1 mark]

Question 1b

The graph of f passes through the point $A(4, -2)$.

(b)
Find the value of b.

[3 marks]

Question 1c

(c)
Find $\displaystyle\lim_{x \to \infty} f(x)$.

[2 marks]

Question 2a

Consider $f(x) = \dfrac{ax + b}{cx + 1}$, where $a,\, b,\, c \in \mathbb{Z}$.

$y = 4$ and $x = -1$ are the equations of the asymptotes of the graph of f. Point $P\left(5, \dfrac{9}{2}\right)$ lies on the graph.

(a)
Find the values of $a,\, b$ and c.

[6 marks]

Question 2b

(b)
Sketch the graph of $y = f(x)$.

[4 marks]

Question 3a

Let $f(x) = \dfrac{3}{x-a} + b$, for $x \neq a$.

The line $x = 2$ is a vertical asymptote of the graph of f.

(a)
Write down the value of a.

[1 mark]

Question 3b

The graph of f passes through the point $A(1,5)$.

(b)
Find the value of b.

[3 marks]

Question 3c

(c)
Find $\lim\limits_{x \to \infty} f(x)$.

[2 marks]

Question 4a

Consider the function f defined by $f(x) = \dfrac{3x - 7}{x^2 - 6x + 5}$, for $x \neq 1.5$.

(a)
Find the coordinates where the graph of f intersects the coordinate axes.

[2 marks]

Question 4b

(b)
(i)
Express $f(x)$ as partial fractions.
(ii)
Hence, write down the equation of the horizontal asymptote.
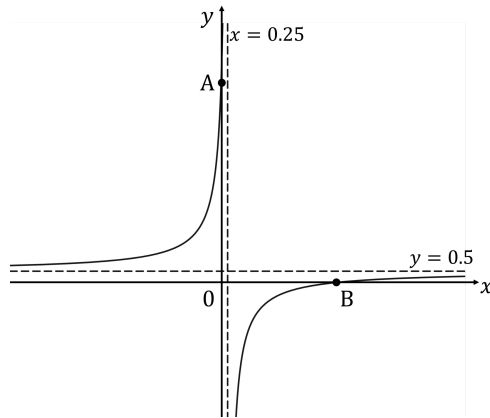
[3 marks]

Question 4c

(c)
Sketch the graph of $y = f(x)$.

[4 marks]

Question 5a

The graph of a function f is shown below. The equations of the asymptotes are $x = 0.25$ and $y = 0.5$. The graph crosses the coordinate axes at the points $A(0, m)$ and $B(5,0)$.

(a)

Find an equation for $f(x)$ in the form $\dfrac{ax + b}{cx + d}$, where $a, b, c, d \in \mathbb{Z}$.

[6 marks]

Question 5b

(b)
Write down the value of m.

[1 mark]

Question 6a

Consider the function $f(x) = \dfrac{2x^2 - x - 15}{x - 2}$, $x \in \mathbb{R}$, $x \neq 2$.

(a)
Find the coordinates where the graph of f crosses the
(i)
x-axis,
(ii)
y-axis.

[3 marks]

Question 6b

(b)
Find the equation of the oblique asymptote of the graph of f, giving the answer in the form $y = ax + b$ where $a, b \in \mathbb{Z}$..
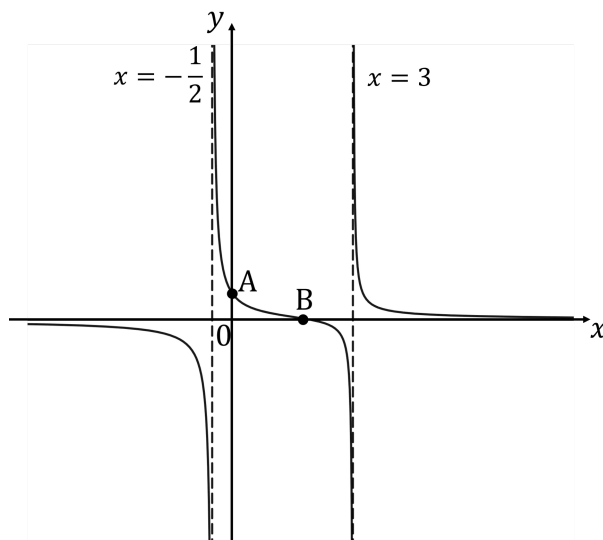
[4 marks]

Question 6c

(c)
Sketch the graph of $y = f(x)$. Clearly indicate the asymptotes and give coordinates of the points where the graph intersects the axes.

[4 marks]

Question 7

Consider the function f defined by $f(x) = \dfrac{ax + b}{2x^2 + cx + d}$, where $a,\ b,\ c,\ d \in \mathbb{Z}$. The lines $x = -\dfrac{1}{2}$ and $x = 3$ are vertical asymptotes of the graph of f as shown below. The graph crosses through the points $A(0,4)$ and $B(2,0)$.
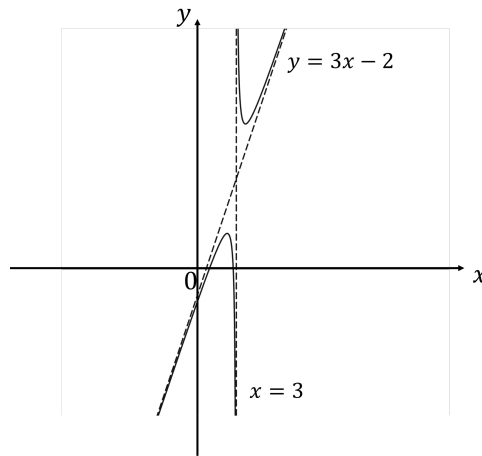

Find the values of $a,\ b,\ c$ and d.

[6 marks]

Question 8a

The graph of a function f is shown below. The equations of the asymptotes are $x = 3$ and $y = 3x - 2$. The graph crosses the y-axis at $(0, -2.5)$.

The function can be written as $f(x) = ax + b + \dfrac{3}{cx + d}$, where $a, b, c, d \in \mathbb{Z}$.

(a)

Find the values of a, b, c and d.

[2 marks]

Question 8b

(b)
Find the exact coordinates of the points where $y = f(x)$ crosses the x-axis.

[3 marks]

Question 8c

(c)
Given that $f(x) = k$ has no real solutions where $k \in \mathbb{R}$, find the set of possible values of k. Give the bounds correct to 2 decimal places.

[2 marks]

Question 9a

Consider the function $f(x) = \dfrac{2x^2 - 18}{x^2 - 25}$, for $x \neq \pm 5$.

(a)
(i)
Show that $\dfrac{2x^2 - 18}{x^2 - 25} = 2 + \dfrac{16}{5(x - 5)} - \dfrac{16}{5(x + 5)}$.

(ii)
Hence, write down the value of $\lim\limits_{x \to \infty} f(x)$.

[4 marks]

Question 9b

(b)

(i)

Sketch the graph of $y = f(x)$.

(ii)

Write down the range of f.

[4 marks]

Question 10a

Consider the function $f(x) = \dfrac{2x^2 - 72}{x^2 - 9}$, for $x \neq \pm 3$.

(a)

Prove that f is an even function.

[2 marks]

Question 10b

(b)

(i)

Sketch the graph of $y = f(x)$.

(ii)

Write down the range of f.

[5 marks]