1.2 Exponentials & Logs

Question Paper

Course	DP IB Maths
Section	1. Number & Algebra
Topic	1.2 Exponentials & Logs
Difficulty	Medium

Time allowed: 90

Score: /69

Percentage: /100

Question 1a

Let $\log_2 16 = \log_2 a^b$, where a and b are integers and $a < b$.

(a) (i) Find the values of a and b.

     (ii) Hence, or otherwise, find the value of $\log_2 16$.

[3 marks]

Question 1b

Let $\log 25 + \log 4 = \log c$.

(b) (i) Find the value of c.

     (ii) Hence, or otherwise, find the value of $\log 25 + \log 4$.

[2 marks]

Question 1c

Let $\log_5 500 - \log_5 4 = \log_5 d$.

(c) (i) Find the value of d.

     (ii) Hence, or otherwise, find the value of $\log_5 500 - \log_5 4$.

[2 marks]

Question 2a

Let $x = \ln 15$ and $y = \ln 3$. Write down the following expressions in terms of x and y.

(a) $\ln 5$.

[2 marks]

Question 2b

(b) $\ln 45$.

[2 marks]

Question 2c

(c) $\ln 135$.

[3 marks]

Question 3a

Let $r = \log 2$ and $s = \log 12$. Write down the following expressions in terms of r and s.

(a) $\log 24$.

[2 marks]

Question 3b

(b) $\log 3$.

[3 marks]

Question 3c

(c) $\log 72$.

[3 marks]

Question 4a

Simplify the following:

(a) $\dfrac{(4xy^{-2})(-12x^{-4}y^{12})}{6x^2y}$.

[2 marks]

Question 4b

(b) $(2x^{-1}y^{-2})^{-3}(4x^2y^3)^4$.

[2 marks]

Question 4c

(c) $\sqrt[2]{(9x^6y^{-2}z^4)}^3(3xyz)^{-2}$.

[2 marks]

Question 5

Solve the equation $2 - x\sqrt{3} = \dfrac{7x}{\sqrt{3}}$, giving your answer in the form $\dfrac{\sqrt{a}}{b}$ where a and b are integers. State the values of a and b.

[5 marks]

Question 6a

Given that $\log_a 8 = 3$.

(a) Find the value of $\log_a 64$.

[2 marks]

Question 6b

(b) Find the value of a.

[2 marks]

Question 6c

(c) Find the value of $\log_{a^2} 8$.

[3 marks]

Question 7a

Let $\log_b 3 = x$ and $\log_b 16 = y$

(a) Find an expression for $\log_b 9$ in terms of x.

[2 marks]

Question 7b

(b) Find an expression for $\log_b 4$ in terms of y.

[2 marks]

Question 7c

(c) Find an expression for $\log_b 48$ in terms of x and y.

[3 marks]

Question 8a

(a) Show that $\dfrac{\left(4-2\sqrt{x}\right)^2}{8x}$ can be written as $2x^{-1} - 2x^{-\frac{1}{2}} + \dfrac{1}{2}$.

[2 marks]

Question 8b

(b) Given that $8\sqrt{2} = 2^a$, find the value of a.

[2 marks]

Question 8c

(c) Show that $\dfrac{x\left(2x^4-\sqrt{x}\right)}{x^2}$ can be written as $2x^a - x^b$, where a and b are rational numbers. State the value of a and b.

[2 marks]

Question 9

Solve the equation $16^x - 3(4^{x+1}) = 28$. Write your answer in the form $\dfrac{\ln a}{\ln b}$, where a and b are integers.

[5 marks]

Question 10

$\sqrt{425}$ can be written in the form $a\sqrt{b}$. Find the values of a and b. Show all of your working.

[5 marks]

Question 11a

The expression $a^{\frac{1}{5}} \times a^{\frac{2}{5}}$ can be expressed in the form a^p.

(a) Find the value of p.

[1 mark]

Question 11b

Let $a^{\frac{1}{5}} \times a^{\frac{2}{5}} = 8$.

(b) Find the value of a.

[2 marks]

Question 11c

The expression $b \times b^{-\frac{3}{2}}$ can be written in the form $\frac{1}{b^q}$.

(c) Find the value of q.

[1 mark]

Question 11d

Let $b \times b^{-\frac{3}{2}} = \sqrt{2}$

(d) Find the value of b.

[2 marks]