3.5 Trigonometric Functions & Graphs

Question Paper

Course	DP IB Maths
Section	3. Geometry & Trigonometry
Topic	3.5 Trigonometric Functions & Graphs
Difficulty	Hard

Time allowed: 80

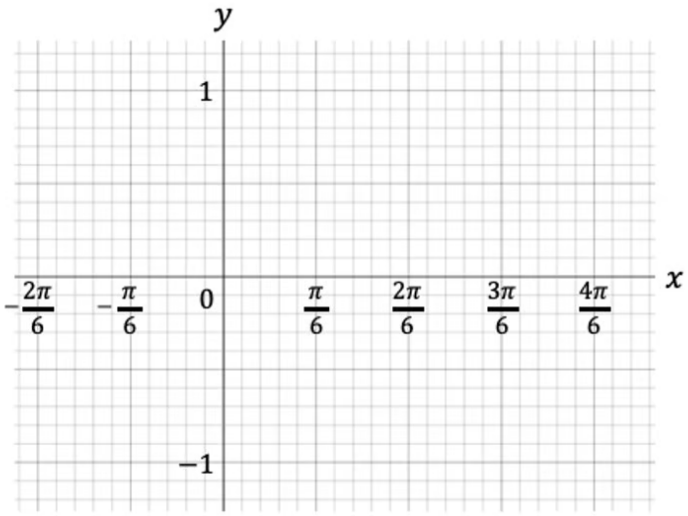
Score: /65

Percentage: /100

Question 1a

A graph has the equation $y = \cos 3x$ for the interval $-\frac{\pi}{3} \leq x \leq \frac{2}{3}\pi$.

(a) Sketch the graph on the axes below.



[3 marks]

Question 1b

A straight line with equation $y = \frac{1}{2}$ intersects the graph of $y = \cos 3x$.

(b) (i) Sketch the line $y = \frac{1}{2}$ on to the same set of axes.

(ii) Find the coordinates of the points of intersection between $y = \cos 3x$ and $y = \frac{1}{2}$.

[5 marks]

Question 2

(i) Sketch the graph of $y = \sin(2\theta - 60)$ in the interval $-180° \leq \theta \leq 180°$.

(ii) Write down all the values for θ, where $y = \sin(2\theta - 60)° = 0$ in the given interval.

[5 marks]

Question 3a

An average heart contains a volume of approximately 140 millilitres and pumps out one half of its volume with each beat. A healthy adult has a heart rate of about 70 beats per minute.

Assuming that the heart starts at full capacity, the volume of blood, $V(t)$, in the heart can be modelled as a function of the time, t, in seconds.

(a) Write down a model for the volume of blood, $V(t)$, giving your answer in the form $V(t) = A\cos(Bt) + D$, where A, B and D are constants to be found.

[4 marks]

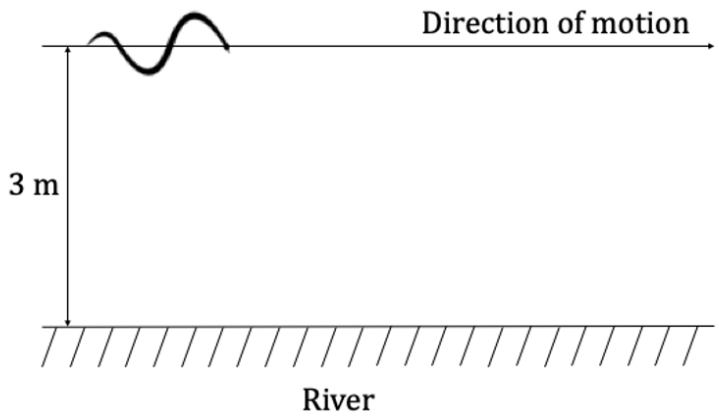
Question 3b

(b) Show that the heart is at its minimum capacity at $t = \frac{3}{7}$ seconds.

[3 marks]

Question 4a

A snake moves along a horizontal surface following a line that is 3 m from, and parallel to, the edge of a straight section of river, as shown in the diagram below.



The perpendicular distance of the tip of the snake's tail from the edge of the river, y cm, can be modelled by the function

$$y = 8\sin(15x)° + 300 \qquad \text{for the interval } 0 \leq x \leq 120$$

where x is the horizontal distance, in cm, moved by the tip of the tail from the start of the movement.

(a) (i) State the maximum perpendicular distance that the tip of the tail will be from the edge of the river.

(ii) State the number of times that the tip of the tail will be at the maximum distance in the given domain.

[3 marks]

Question 4b

A stone is located at a perpendicular distance of 294 cm from the river when the snake has travelled $\frac{1}{6}$ of the total horizontal distance.

(b) Show that the tip of the tail will collide with the stone.

[3 marks]

Question 5a

A hamster runs in its exercise wheel, rotating the wheel at a constant speed. The wheel has a diameter of 14 centimetres and the top of the wheel is positioned at a height of k centimetres above the floor of the cage.

A point at the top of the wheel is marked before the hamster starts to run, turning the wheel clockwise. The hamster takes 4 seconds to turn the wheel one complete revolution.

After t seconds, the height of the mark on the wheel above the floor of the cage is given by

$$h(t) = 10 + a\cos\left(\frac{\pi}{2}t\right) \text{ for } 0 \leq t \leq 150$$

(a) After 26 seconds, the mark is 3 cm above the cage floor. Find k.

[2 marks]

Question 5b

(b) Find the value of a.

[3 marks]

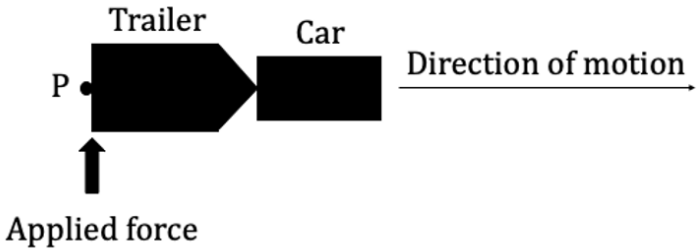
Question 5c

(c) Find the value of t when the mark is 8 cm above the floor of the cage for the 5^{th} time in the given time period and state whether the mark is ascending or descending.

[4 marks]

Question 6a

A student sets up an experiment with a model car moving along a horizontal surface in a straight line. The car has a trailer attached by a pin joint. A momentary force is applied to the end of the trailer, perpendicular to the direction of travel, which causes it to move sideways back and forth as the car continues to move forwards. A diagram can be seen below.



Point P is situated at the midpoint of the end of the trailer. The displacement, d in cm, of point P relative to the centre line of the car in the direction of motion can be modelled by the function

$$d = e^{-\frac{2}{5}t+\frac{8}{15}\pi}\sin(3t-4\pi)$$

where t is the time in seconds since the application of the force.

(a) Find the total distance that point P has moved perpendicular to the line of motion, in the first 3 seconds.

[3 marks]

Question 6b

When the maximum displacement from the centre line does not exceed 2.5 mm, the trailer is considered to be stable.

(b) State the time after which the trailer can be considered stable.

[3 marks]

Question 7a

Let $f(x) = 5\sin\left(\frac{\pi}{4}x\right) - 4$ for $x \in \mathbb{R}$.

Let $g(x) = 3f(2x)$.

The function g can be written in the form $g(x) = 15\sin(bx) + c$.

(a) The range of f is $k \leq f(x) \leq m$. Find k and m.

[3 marks]

Question 7b

(b) Find the range of g.

[2 marks]

Question 7c

(c) (i) Find the value of b and c.

(ii) Find the period of g.

[5 marks]

Question 7d

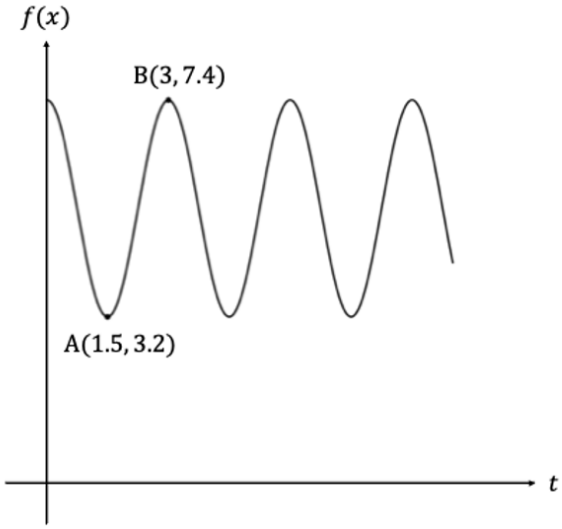
(d) The equation $g(x) = f(x)$ has two solutions where $\pi \leq x \leq 2\pi$. Find both solutions.

[3 marks]

Question 8a

Let $f(x) = p\cos(q \times x) + r$

The diagram below shows the graph of f, for $0 \leq x \leq 10$



The first local minimum is at point A $(1.5, 3.2)$ and the next local maximum is at point B $(3, 7.4)$.

(a) Find the value of

(i) p

(ii) q

(iii) r.

[7 marks]

Question 8b

Let $g(x) = 4\tan(2x)$

(b) Find the two solutions for $f(x) = g\left(\frac{1}{4}x - 3\right) + 4$, for $0 \leq x \leq 10$.

[4 marks]