5.2 Further Differentiation

Question Paper

Course	DP IB Maths
Section	5. Calculus
Topic	5.2 Further Differentiation
Difficulty	Hard

Time allowed: 90

Score: /74

Percentage: /100

Question 1a

(a) Use the product rule to find the derivative of $f(x) = (3x - 7)(4 - 2x^2)$

[2 marks]

Question 1b

(b) Use the quotient rule to find the derivative of $g(x) = \frac{-7x}{x^3 - 1}$

[3 marks]

Question 1c

(c) Use the chain rule to find the derivative of $h(x) = (5 - 3x)^5$

[2 marks]

Question 2a

Find an expression for the derivative of each of the following functions:

(a) $f(x) = e^{3x} \cos x$

[2 marks]

Question 2b

(b) $g(x) = \sin(3x^2 + 5)$

[2 marks]

Question 2c

(c) $h(x) = \dfrac{-\cos^2 x}{\ln x}$

[3 marks]

Question 3

Consider the function f defined by $f(x) = 2x + \cos^3 x,\ x \in \mathbb{R}$.

By considering the derivative of the function, show that f is increasing everywhere on its domain.

[5 marks]

Question 4a

Consider the function g defined by $g(x) = e^x - 7x,\ x \in \mathbb{R}$.

(a) Show that the equation of the tangent to the graph of g at $x = \ln 3$ may be written in the form $y = -4x - 3(\ln 3 - 1)$.

[5 marks]

Question 4b

(b) Show that there is a point on the graph of g at which the normal to the graph is vertical, and determine the coordinates of that point.

[3 marks]

Question 5a

Consider the function h defined by $h(x) = \cos x - e^{2x} \sin x,\ x \in \mathbb{R}$.

(a) Find an expression for $h'(x)$.

[3 marks]

Question 5b

(b) Hence determine an equation for the tangent to the graph of h at $x = \pi$.

[4 marks]

Question 6

Let $f(x) = g(x)h(x)$, where g and h are functions such that $g(x) = 3x^2h(x)$ for all $x \in \mathbb{R}$.

Given that $h(-1) = 2$ and $h'(-1) = -2$, find the equation of the tangent to the graph of f at $x = -1$.

[7 marks]

Question 7a

Let f be a function defined by $f(x) = e^{x^3}$, $x \in \mathbb{R}$.

(a) Find an expression for $f''(x)$.

[5 marks]

Question 7b

(b) Determine the values of x for which the graph of f is

(i) concave up

(ii) concave down.

Your answers should be given as exact values.

[4 marks]

Question 7c

(c) Hence show that the graph of f has two points of inflection, and determine the exact values of their coordinates.

[4 marks]

Question 8a

Consider the function f defined by $f(x) = xe^{3\cos x}$, for $-\pi \leq x \leq \pi$.

(a) Find the number of points at which the graph of f has a horizontal tangent.

[1 mark]

Question 8b

The point A is the point on the graph of f for which the x-coordinate is $\frac{\pi}{2}$.

(b) Show algebraically that the gradient of the tangent to the graph of f at point A is $\frac{2-3\pi}{2}$.

[4 marks]

Question 8c

(c) Hence find the equation of the normal line to the graph of f at point A, and determine where that line intersects the x-axis.

[5 marks]

Question 8d

(d) Show algebraically that the graph of f intersects the line $y = x$ in exactly three places, and determine the coordinates of the points of intersection.

[4 marks]

Question 9

Let $f(x) = \frac{\sqrt{3}}{2}\cos 2x$ and $g(x) = \sin x \cos x$, for $0 \leq x \leq \pi$.

Solve the equation $f'(x) = g'(x)$.

[6 marks]