

2.2 Further Functions & Graphs

Question Paper

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| Course | DPIB Maths |
| Section | 2. Functions |
| Topic | 2.2 Further Functions & Graphs |
| Difficulty | Very Hard |

Time allowed: 100
Score: /77
Percentage: /100

Question 1a

A function is defined by $f(x) = e^{x^2+bx+4}$. The graph of f has an axis of symmetry of $x = 2$.

(a) Find the value of b .

[2 marks]

Question 1b

(b) Find the range of f .

[1 mark]

Question 1c

Another function is defined by $g(x) = -\frac{(x^2-25)}{5}$. The graph of f and g intersect at points A and B.

(c) Find the equation of the line passing through points A and B. Give your answer in the form $y = mx + c$.

[3 marks]

Question 1d

(d) Find the distance of the line AB.

[2 marks]

Question 2a

Consider the function $f(x) = 5 - \log(6 - 4x)$. The line l_1 intersects the graph of f at point $A(-1, y)$ and $B(x, 5)$.

(a) Find the value of x and y .

[2 marks]

Question 2b

(b) Find the equation of l_1 . Give your answer in the form $y = mx + c$, where m and c are fractions.

[2 marks]

Question 2c

The line l_2 is the perpendicular bisector to $[AB]$.

(c) Find the equation of l_2 . Give your answer in the form $ax + by + d = 0$, where a, b and d are integers.

[4 marks]

Question 3a

A rectangle has length $l = 4x$ and width $w = x$.

(a) Find an expression for

- (i) the perimeter of the rectangle, P , in terms of x .
- (ii) the area of the rectangle, A , in terms of x .

[2 marks]

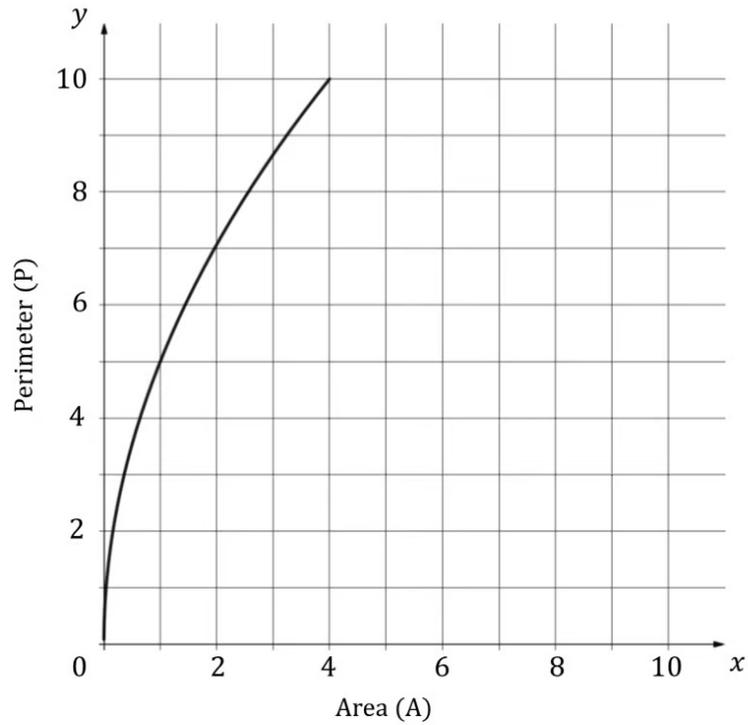
Question 3b

(b) Show that $P(A) = 5\sqrt{A}$.

[2 marks]

Question 3c

The graph of the function P , for $0 \leq A \leq 4$, is shown below.



(c) On the grid above, draw the graph of the inverse function P^{-1} .

[3 marks]

Question 4a

The function f is a quadratic in the form $f(x) = ax^2 + bx - 2$, for $-10 \leq x \leq 10$.

The graph of f has x -intercepts $\left(\frac{1+\sqrt{5}}{2}, 0\right)$ and $\left(\frac{1-\sqrt{5}}{2}, 0\right)$.

(a) Find the values of a and b .

[4 marks]

Question 4b

Another function can be defined by $g(x) = 6(0.8)^{-x} - 1$, for $-10 \leq x \leq 10$.

The graph of f and g intersect at points A and B.

(b) Find the coordinates of A and B.

[2 marks]

Question 4c

(c) Solve the inequality $f(x) < g(x)$.

[2 marks]

Question 5a

Let $f(x) = e^{2\cos(30x)} + 1$, for $x > 0$.

The n th minimum point on the graph of f has x coordinate x_n , where $n \in \mathbb{Z}^+$.

(a) Given that $x_n = x_1 + (n - 1)d$, find x_1 and d .

[4 marks]

Question 5b

(b) (i) Using sigma notation, write down an expression for $x_1 + x_2 + x_3 + \dots + x_{16}$.

(ii) Find the value of the sum from part (b) (i). Give your answer to the nearest integer.

[4 marks]

Question 6a

Let $f(x) = 2x^4 - 2x^3 - 4x^2 + x + 1$, where $x \in \mathbb{R}$.

(a) Solve the inequality $f(x) < 0$.

[3 marks]

Question 6b

(b) For the graph of f , find the coordinates of the

- (i) local maximum point.
- (ii) local minimum points.

[3 marks]

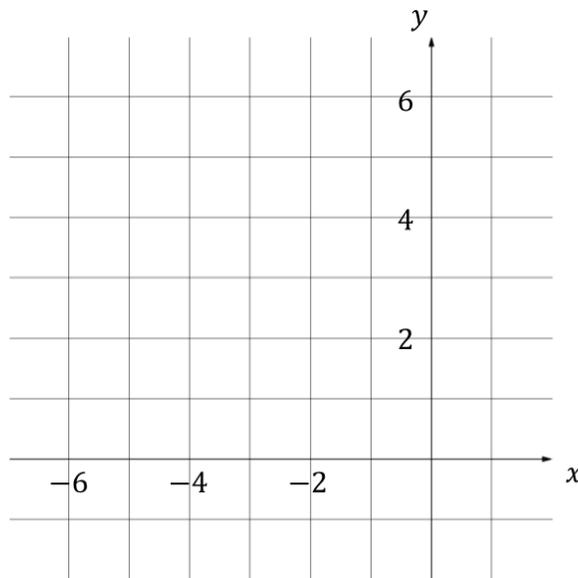
Question 6c

(c) Write down the possible domains of f for which f has an inverse and explain why the domain must be restricted.

[3 marks]

Question 7a

The graph of a quadratic function has equation $y = \frac{1}{4}x^2 + bx + c$, where $b, c \in \mathbb{Z}$, and the axis of symmetry is $x = -4$.



(a) Draw the axis of symmetry on the grid above.

[1 mark]

Question 7b

The graph of the quadratic function intersects the x -axis at points $A(-6, 0)$ and B .

- (b) (i) Write down the coordinates of B .
- (ii) Find the values of b and c .

[3 marks]

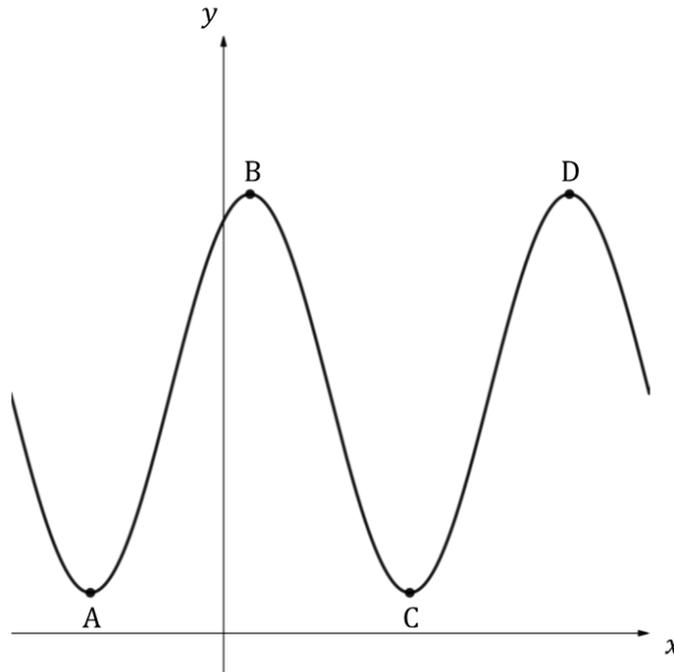
Question 7c

- (c) (i) Mark and label A and B on the grid above.
- (ii) Write down the coordinates of the Vertex, V , and label it on the grid above.
- (iii) Write down the coordinates of the y -intercept, C , and label it on the grid above.
- (iv) Draw the graph of the quadratic function on the grid above.

[4 marks]

Question 8a

The diagram below shows part of the graph of the function $f(x) = 5 \cos(12x - 30) + 6$, where points A, B, C and D represent stationary points. The points A, B, C and D are joined together forming a quadrilateral ABDC.



(a) Write down the coordinates for

- (i) A.
- (ii) B.
- (iii) C.
- (iv) D.

[4 marks]

Question 8b

(b) Find the area of the quadrilateral ABDC.

[2 marks]

Question 9a

The intensity of light, I , is assumed to be 100% at the surface of the ocean and decreases with depth, d , and can be estimated by the function

$$I(d) = k(1.08)^{-d}$$

where I is expressed as a percentage, d is the depth below the surface, in metres, and k is a constant.

(a) Calculate the value of k .

[2 marks]

Question 9b

(b) State the domain and range of I .

[2 marks]

Question 9c

On a particularly bright day the intensity of light 6.2 m below the surface is 75%.

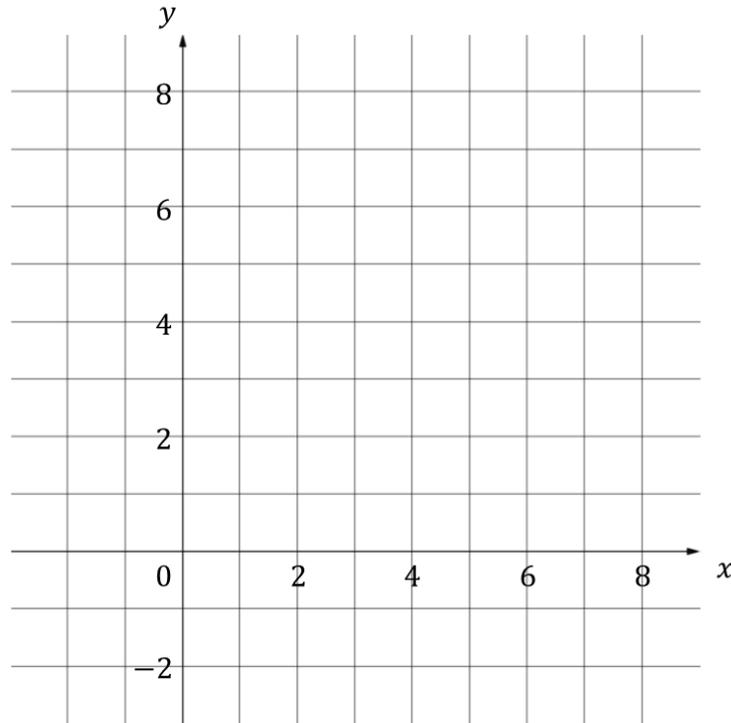
(c) Calculate the percentage error between the estimate made by the function and the actual intensity of light.

[4 marks]

Question 10a

(a) Graph the following piecewise function on the grid below

$$f(x) = \begin{cases} -x + 5, & -2 \leq x < 2 \\ -4x + 11, & 2 \leq x < 3 \\ 2x - 7, & 3 \leq x \leq 6 \end{cases}$$



[4 marks]

Question 10b

(b) Find the values of x when $f(x) = 0$.

[3 marks]