

1.4 Financial Applications

Question Paper

Course	DPIB Maths
Section	1. Number & Algebra
Topic	1.4 Financial Applications
Difficulty	Hard

Time allowed: 90
Score: /74
Percentage: /100

Question 1a

Emily decides to buy a new boat at a cost of \$20 000, but cannot afford the full amount. The boat dealership offers two options to finance a loan.

First option:

A 5 year loan at a nominal annual interest rate of 15%, **compounded monthly**. No deposit required and repayments are made each month.

(a) Find the repayment made each month.

[3 marks]

Question 1b**Second option:**

A 5 year loan at a nominal annual interest rate of $r\%$ **compounded quarterly**. Terms of the loan require a 5% deposit and a monthly repayment of \$400.

(b) Find the annual interest rate, r .

[3 marks]

Question 1c

(c) State which option Emily should choose. Justify your answer.

[2 marks]

Question 2a

On the first of January 2018 Phillip invests 25 000 US dollars (USD) in the S&P 500. The S&P 500 is a stock market index that tracks the performance of 500 large companies listed on the stock exchange in the United States. Over the past 30 years the S&P 500 has delivered an average annual growth rate of 10.7%, **compounded annually**.

(a) Calculate the expected value of Phillip's investment on the first of January 2024. Give your answer to 2 decimal places.

[1 mark]

Question 2b

On the first of January 2024 the value of Phillip's investment 32 762.41 USD.

(b) Calculate the percentage error between the expected value of the investment on the first of January 2024, found in part (a), and the actual value of the investment on the first of January 2024.

[2 marks]

Question 2c

(c) Calculate the average annual growth rate of the S&P 500 between 2018 and 2024.

[2 marks]

Question 2d

In 2018 the S&P 500 delivered an annual return of -4.38% .

(d) Calculate the average annual growth rate of the S&P 500 between 2019 and 2024.

[3 marks]

Question 3a

Ben and Evan are twins and both receive €44 000 on their 21st birthday. Ben deposits his €44 000 into a savings account that pays a nominal annual interest rate of 3.27%, **compounded annually**. Evan invests his €44 000 into a fixed income fund that returns €1750 per year.

(a) Calculate:

- (i) the amount of interest that Ben earns if his sum is invested for 20 years, giving your answer to 2 decimal places
- (ii) the amount of interest that Evan earns for his investment if invested for 20 years.

[4 marks]

Question 3b

(b) Find the year in which the amount in Ben's account surpasses the amount in Evan's account.

[4 marks]

Question 4a

Georgia buys a new computer for herself that costs \$1099. At the same time, she buys her son, Duncan, a new gaming computer that costs \$2749.

It is anticipated that Georgia's computer will depreciate at a rate of 11% per year, whereas Duncan's gaming computer will depreciate at 18% per year.

(a) Estimate the value of Georgia's computer after 6 years.

[2 marks]

Question 4b

Georgia and Duncan's computers will have the same estimated value k years after they were purchased.

(b) Find:

- (i) the value of k
- (ii) the estimated value after k years.

[4 marks]

Question 5a

Sasha takes out a loan of \$19 800 to purchase new tools for his gardening business. He agrees to pay the bank \$840 at the end of every quarter to amortise the loan. The loan has a nominal annual interest rate of 4.34% per year, **compounding semi-annually**.

(a) Find out how long it takes to pay back the loan.

[2 marks]

Question 5b

(b) Calculate the total amount that Sasha will pay in amortising the loan.

[1 mark]

Question 5c

The bank offers Sasha an option to make a higher quarterly repayment of \$980.

(c) Calculate how much Sasha will save if he decides to make the higher payment.

[4 marks]

Question 6a

John has just retired at the age of 65. He has \$445 000 in his savings fund. He “rolls over” the money into an annuity fund which returns a nominal annual interest rate of 4.17%, **compounded quarterly**.

John decides to withdraw \$2200 every month to live on.

(a) Find the number of years and months for the money in the fund to run out.

[3 marks]

Question 6b

After 5 years, John wants to make a withdrawal from his annuity to invest in a new online educational platform. Terms of the annuity state John is allowed to withdraw up to 9.5% of his account value without paying a surrender charge.

(b) Calculate the maximum amount John can withdraw so that he does not have to pay the surrender charge.

[4 marks]

Question 7a

Joshua wants to invest in Blueprint Construction LTD and so decides to buy 55% of the business for \$660 000.

(a) Calculate the total value of Blueprint Construction LTD.

[1 mark]

Question 7b

Stephanie, Joshua's financial advisor, has prepared the following table detailing the 5-year expected annual returns from Joshua's investment in different economic states (boom or bust) and the probability of each state occurring.

	Probability of state	Expected return (%)
Boom	0.6	12.2
Bust	0.4	-10.4

(b) Calculate the 5-year expected return of Joshua's investment in Blueprint Construction LTD.

[3 marks]

Question 7c

In 5 years, Blueprint Construction is valued at \$1 800 000.

(c) Find:

- (i) the value of Joshua's stake in Blueprint Construction LTD
- (ii) the average annual return over the 5-year period.

[3 marks]

Question 8a

Malcolm and Julie have decided to move out of their home to a new apartment in town, so that they can rent it to a young family who agree to pay \$3200 a month.

They decide to take out a mortgage to buy the new apartment costing \$545 000. Terms of the mortgage are:

- a minimum deposit of 12%
- a nominal annual interest rate of $r\%$, **compounded annually**
- monthly repayments of \$2695

(a) Calculate:

- (i) the minimum deposit
- (ii) the loan amount.

[2 marks]

Question 8b

Malcolm and Julie pay off the loan in exactly 30 years.

(b) Find the value of the interest rate, r .

[3 marks]

Question 8c

Malcolm and Julie decide to invest the difference between their rental income and their monthly repayments into a savings account paying a nominal annual interest rate of 2.91%, **compounded annually**.

(c) Calculate the total amount in the savings account once they have finished paying off their loan.

[3 marks]

Question 9a

Aaron has been renting out his apartment for \$1200 per month, however he wants to renovate the apartment to increase the monthly rent to \$1750. He notifies his current tenants and they agree to move out. The renovations are expected to take 5 months.

(a) Calculate the amount of rental income Aaron is foregoing by deciding to renovate the apartment.

[2 marks]

Question 9b

Aaron takes out a loan of \$25 000 to renovate the apartment. Terms of the loan are

- A nominal annual interest rate of 6.2%, **compounded monthly**
- Monthly repayments of \$850.

(b) Calculate the total amount paid for the renovations.

[3 marks]

Question 9c

Aaron has some new tenants ready to move in as soon as the renovations are finished.

(c) Find the number of months it takes for the increase in Aaron's rental income to match the amount paid for the renovations, found in part (b).

[3 marks]

Question 10a

Phillip takes out a loan of \$47 000. The unpaid balance on the loan has an interest rate of 4.97%, **compounded semi-annually**. The loan is to be repaid in payments of \$2120 at the end of every quarter.

(a) Calculate the number of years it will take to repay the loan.

[2 marks]

Question 10b

After 1.5 years, Phillip misses a payment. The penalty for missing a payment is 5.5% of the remaining balance.

(b) Calculate the total amount paid for the loan.

[5 marks]