

1.2 Exponentials & Logs

Question Paper

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| Course | DPIB Maths |
| Section | 1. Number & Algebra |
| Topic | 1.2 Exponentials & Logs |
| Difficulty | Hard |

Time allowed: 70
Score: /56
Percentage: /100

Question 1a

Let $f(x) = 5\ln(x - 7)$.

(a) Find the values of x for which $f(x)$ is undefined.

[2 marks]

Question 1b

(b) Given that point P has coordinates $(p, 0)$, find the value of p .

[3 marks]

Question 2a

(a) Given that $2^m = 8$ and $2^n = 16$, write down the value of m and of n .

[2 marks]

Question 2b

(b) Hence or otherwise solve $8^{2x+1} = 16^{2x-3}$.

[4 marks]

Question 3a

(a) Write the expression $3 \ln 2 - \ln 4$ in the form $\ln k$, where $k \in \mathbb{Z}$.

[3 marks]

Question 3b

(b) Hence, or otherwise, solve $3 \ln 2 - \ln 4 = -\ln x$.

[3 marks]

Question 4

Solve the equation $49^{p+4} = 35^{2p}$ for p . Express your answer in terms of $\ln 7$ and $\ln 5$.

[5 marks]**Question 5**

Solve the equation $4^x - 3 \times 2^{x+2} = 64$.

[5 marks]

Question 6a

Simplify the following expressions, giving your answers in the form ax^n where a and n are rational numbers and any fractions are in lowest terms.

(a) $8x^{-3} \div 2x^{-\frac{1}{3}}$

[2 marks]

Question 6b

(b) $\frac{2}{3}x^{\frac{7}{6}} \times \frac{5}{2}x^{-\frac{5}{3}}$

[2 marks]

Question 6c

(c) $\left(3x^{-\frac{2}{3}}\right)^2 \div 6x^{-\frac{1}{6}}$

[3 marks]

Question 7a

Given that $y = \frac{8}{27}x^6$, express each of the following in the form ax^n , where a and n are constants.

(a) $y^{\frac{1}{3}}$

[2 marks]

Question 7b

(b) y^{-1}

[2 marks]

Question 7c

(c) $y^{-\frac{4}{3}}$

[3 marks]

Question 8a

A block of ice cream in the shape of a cuboid that has been taken out of the freezer and left at room temperature. The rate of decrease of the volume of the block of ice cream can be modelled by the formula

$$V = Ae^{xt}$$

where A and x are constants and t is the time in minutes after the ice cream was removed from the freezer.

Before it was taken out of the freezer the block of ice cream measured 12 cm by 9 cm by 7 cm. Eight minutes later the block of ice cream measured 8.4 cm by 7.1 cm by 5.8 cm.

(a) By taking logarithms of both sides, show that the rate of decrease of the volume of the block of ice cream can be rewritten as

$$xt = \ln V - \ln A$$

[3 marks]

Question 8b

(b) Find the value of x and A .

[4 marks]

Question 9a

A zoologist is researching the connection between the mass of a particular breed of mouse, M g, and its heartrate, H bpm. It is suggested that the relationship can be modelled by the equation

$$H = kM^x, \text{ where } x, k \in \mathbb{R}.$$

The zoologist takes measurements from two different mice and records the data as follows.

| Measurement | M (g) | H (bpm) |
|-------------|---------|-----------|
| 1 | 17.12 | 513 |
| 2 | 24.51 | 401 |

(a) Find the values of x and k .

[5 marks]

Question 9b

(b) Use the model to predict

- (i) the heartrate for a mouse of 20.2 grams,
- (ii) the mass of a mouse that has a heartrate of 420 bpm.

[3 marks]