

4.6 Normal Distributions

Question Paper

Course	DPIB Maths
Section	4. Statistics & Probability
Topic	4.6 Normal Distributions
Difficulty	Hard

Time allowed: 110
Score: /85
Percentage: /100

Question 1a

A scientist is studying the movement of snails and has observed that the distribution of their speeds, S , follows a normal distribution with a mean of 48 m/h and a standard deviation of 1.5 m/h.

(a) Sketch a diagram to represent this information.

[2 marks]

Question 1b

(b) Find the probability that a randomly selected snail has a speed of less than 46.5 m/h.

[2 marks]

Question 1c

(c) From a sample of 80 snails, calculate the expected number of snails that would have a speed of less than 46.5 m/h. Give your answer to the nearest integer.

[2 marks]

Question 2a

The height of the average female giraffe, from a population where the heights follow a normal distribution, is 4.57 m and has a standard deviation of 1.28 m.

(a) Find:

(i) Q_1

(ii) Q_3

(iii) IQR

[4 marks]

Question 2b

The heights, H , of male giraffes are also normally distributed, where $H \sim N(4.96, \sigma^2)$.

(b) If 68% of male giraffes are shorter than the upper quartile of the female giraffe population, find the standard deviation of the male giraffes.

[3 marks]

Question 3a

If $X \sim N(24, 16)$

(a) Find x given that:

(i) $P(X < x) = 0.7$

(ii) $P(X > x) = 0.15$

[3 marks]

Question 3b

(b) Find a and b given that $P(a < X < b) = 0.95$ and a and b are equal distances from the mean.

[3 marks]

Question 4a

The weight, W , of pumpkins purchased are normally distributed with a mean of 11.3 kg and a standard deviation of 2.1 kg.

In one year, a farmer grows 350 pumpkins on his farm.

(a) Predict the number of pumpkins that weigh between 7.2 kg and 12.5 kg from the farm.

[3 marks]

Question 4b

The heaviest 7% of pumpkins are classified as large and are sold at a higher price.

(b) Find the range of weights of pumpkins that can be sold for a higher price.

[3 marks]

Question 5a

The distribution of birth weight of a newborn can be assumed to follow a normal distribution with a mean of 3369 g and a standard deviation of 567 g.

A baby is classified as being of low birth weight if its weight is less than 2500 g.

- (a) Draw a diagram to represent the situation, labelling clearly the mean and the boundary for low birth weight.

[2 marks]

Question 5b

- (b) Find the expected number of babies from a sample of 1000 that are born with a low birth weight. Give your answer to the nearest integer.

[2 marks]

Question 5c

14% of babies born at a particular hospital weigh more than 4 kg.

- (c) Show that the babies born at this hospital are not representative of the population.

[2 marks]

Question 5d

(d) Given that the standard deviation is the same as the population, find the mean birth weight of a newborn at this particular hospital. Give your answer to the nearest gram.

[3 marks]

Question 6a

A reaction time test has been used to collate data about how quickly a person can react to a given signal by pressing the space bar on a computer. It is found that the results of the test, X , are distributed normally with a mean of 273 milliseconds and a variance of 121 milliseconds.

(a) Use a sketch of the distribution to show that $P(284 < X < 295) = 0.135$.

[3 marks]

Question 6b

A result that lies outside of two and a half standard deviations from the mean is considered to be extreme. A group of 145 students decide to measure their reaction times using the test.

(b) Estimate the number of students that will receive a result that would be considered extreme. Give your answer to the nearest integer.

[4 marks]

Question 7a

A software firm has recorded the distance, D , that their users have been scrolled on a computer by a mouse over the course of one year. The data is distributed normally with a mean of 18.4 miles and a standard deviation of 5.8 miles.

(a) Calculate the interquartile range of the data.

[4 marks]

Question 7b

It is known that 8% of the users scroll more than d miles but less than 22 miles.

(b) Find the value of d .

[4 marks]

Question 8a

A company states that the lifespan in hours, H , of the bulbs that they manufacture follows a normal distribution with the parameters $H \sim N(3000, 720^2)$.

The company advertises that their lightbulbs exceed a lifespan of h hours.

(a) Given that $P(H < h) = 0.4$, find h .

[2 marks]

Question 8b

(b) From a batch of 4000 lightbulbs, calculate the expected number of lightbulbs that will have a lifespan greater than h found in part (a) but less than 3150 hours.

[3 marks]

Question 8c

The 5% of light bulbs with the shortest life span are considered to be defective.

(c) One of the lightbulbs that is tested has a lifespan of 2213 hours. Determine whether the lightbulb is considered to be defective. Give a reason for your answer.

[3 marks]

Question 8d

A second company also manufactures lightbulbs, whose lifespan again follows a normal distribution, $H \sim (\mu, \sigma^2)$.

(d) Given that $P(H < 2600) = 0.404$ and $P(H > 3050) = 0.358$, find the values of μ and σ .

[4 marks]

Question 9a

From a given population it is found that the average person spends on average 122 minutes exercising each week. The data follows a normal distribution and has a standard deviation of σ minutes.

It is known that approximately 81.5% of people spend between 50 and 158 minutes exercising each week.

(a) Using a sketch of the distribution or otherwise, explain why 36 minutes would provide a good approximation for the value of σ .

[4 marks]

Question 9b

From a sample of people within the population, it is known that 15 of them spent less than 65 minutes exercising each week.

- (b) Using the value above, of 36 minutes for the standard deviation, find the total number of people within the sample.

[3 marks]

Question 10a

The running time of films is normally distributed with a mean time of 102 minutes and a standard deviation of 13 minutes.

- (a) Find the probability that, on a randomly selected day, the feature film playing at the cinema has a running time of between 97 and 108 minutes.

[2 marks]

Question 10b

Jonah watches a film on 18 different occasions.

- (b) Find the expected number of occasions on which the film he watches will last less than 95 minutes.

[3 marks]

Question 10c

(c) Find the probability that on at least 6 out of the 18 occasions, the film will last for longer than 99 minutes.

[3 marks]

Question 11a

The length L of carrots in centimetres is normally distributed with mean μ . The following table shows the probabilities for values of L .

Values of L	$L < 13.9$	$13.9 \leq L \leq 21.7$	$21.7 < L$
$P(L)$	k	0.94	0.03

Any carrots that have a length longer than 20.3 cm are classed as oversized carrots.

(a) Write down the value of k .

[2 marks]

Question 11b

(b) Show that the value of μ is 17.8 cm.

[2 marks]

Question 11c

(c) Find the probability that when picking a carrot at random, an oversized carrot is chosen.

[5 marks]