IB Physics DP

10. Fields (HL only)

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10.1 Describing Fields

10.1.1 Gravitational & Electrostatic Fields

Gravitational & Electrostatic Fields

- A field can be defined as

  A region in which an object will experience a force, such as gravitational or electrostatic, at a distance

- A gravitational field can be defined as:

  The gravitational force per unit mass exerted on a point mass

- An electrostatic field can be defined as:

  The electric force per unit charge exerted on a small positive test charge

- Fields can be described in terms of field strength, which is defined as:

$$\textit{Field strength} = \frac{\textit{force acting on a test object}}{\textit{size of test object}}$$

- Electric field strength, E, and gravitational field strength, g, therefore, have very similar equations
  - Despite a few differences, they are analogous to one another in many ways
- In both cases, the nature of the test object is as follows:
  - **Gravitational fields:** small mass, m
  - **Electrostatic fields:** small positive charge, q

Uniform Fields

- A gravitational field is a region of space in which objects with mass will experience a force
- The gravitational field strength can be calculated using the equation:

$$g = \frac{F}{m}$$

- Where:
  - g = gravitational field strength (N kg^{-1})
  - F = gravitational force on the mass (N)
  - m = mass (kg)

- The direction of the gravitational field is always directed **towards** the centre of the mass
  - Gravitational forces are **always attractive** and cannot be repulsive

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- An electric field is a region of space in which an electric charge will experience a force
- The electric field strength can be calculated using the equation:

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$$E = \frac{F}{Q}$$

- Where:
  - E = electric field strength (N C^{-1})
  - F = electrostatic force on the charge (N)
  - Q = Charge (C)
- It is important to use a **positive** test charge in this definition, as this determines the **direction** of the electric field
- The electric field strength is a **vector** quantity, it is always directed:
  - **Away** from a positive charge
  - **Towards** a negative charge
- **Opposite charges** (positive and negative) **attract** each other
- Conversely, **like charges** (positive-positive or negative-negative) **repel** each other

- The magnitude of the electric field strength in a **uniform** field between two charged parallel plates is defined as:

$$E = \frac{V}{d}$$

- Where:
  - E = electric field strength (V m^{-1})
  - V = potential difference between the plates (V)
  - d = separation between the plates (m)

- The electric field strength is now defined by the units **V m^{-1}**
  - Therefore, the units V m^{-1} are equivalent to the units **N C^{-1}**

- The equation shows:
  - The greater the **voltage** (potential difference) between the plates, the **stronger** the field
  - The greater the **separation** between the plates, the **weaker** the field

- This equation **cannot** be used to find the electric field strength around a point charge (since this would be a radial field)
- The direction of the electric field is from the plate connected to the **positive** terminal of the cell to the plate connected to the **negative** terminal

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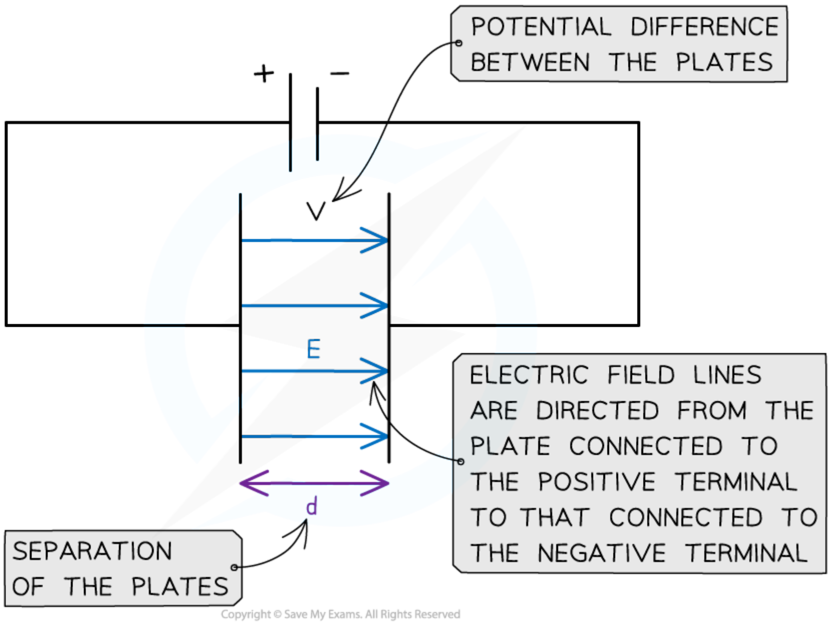

The E field strength between two charged parallel plates is the ratio of the potential difference and separation of the plates

- **Note:** if one of the parallel plates is **earthed**, it has a voltage of 0 V

Radial Fields

- A point charge or mass produces a **radial** field
  - A charged sphere also acts as a point charge
  - A spherical mass also acts as a point mass
- Radial fields always have an inverse square law relationship with distance
  - This means the field strength decreases by a factor of **four** when the distance *r* is **doubled**
- The gravitational force F_G between two masses is defined by:

$$F_G = \frac{Gm_1 m_2}{r^2}$$

- Where:
  - F_G = gravitational force between two masses (N)
  - G = Newton's gravitational constant
  - m_1, m_2 = two points masses (kg)
  - *r* = distance between the centre of the two masses (m)

- The electric field strength *E* at a distance *r* due to a point charge Q in free space is defined by:

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

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- Where:
  - Q = the point charge producing the radial electric field (C)
  - r = distance from the centre of the charge (m)
  - ε_0 = permittivity of free space (F m^{-1}) = ($\varepsilon_0 = 8.85 \times 10^{-12}\ F\ m^{-1}$)

- This equation shows:
  - The electric field strength in a radial field is **not constant**
  - As the distance, r, from the charge increases, E decreases by a factor of $1/r^2$

Gravitational vs Electrostatic Forces

- The similarities and differences between gravitational and electrostatic forces are listed in the table below:

Comparing G and E Fields

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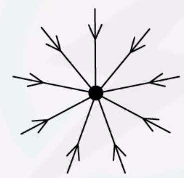
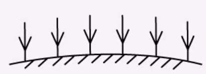
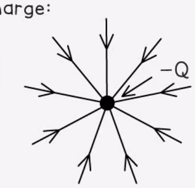
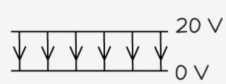
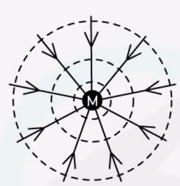
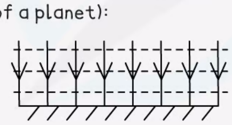
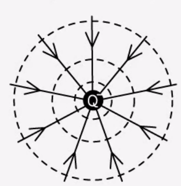
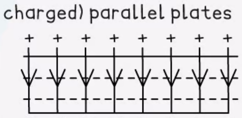
	Gravitational Fields	Electric Fields
Origin of the force	Mass	Charge
Force between two point masses/ charges	$F_G = \frac{GM_1GM_2}{r^2}$	$F_E = \frac{Q_1Q_2}{4\pi\varepsilon_0 r^2}$
Type of Force	Attractive force	Attractive force (opposite charges) Repulsive force (like charges)
Field Strength	$g = \frac{F}{M}$	$E = \frac{F}{Q}$
Field strength due to a point mass/ charge	$g = \frac{GM}{r^2}$	$E = \frac{Q}{4\pi\varepsilon_0 r^2}$
Field Lines	Around a point mass:  In a uniform field (surface of a planet): 	Around a (negative) point charge:  In a uniform field (between charged) parallel plates: 
Potential	$V = -\frac{GM}{r}$	$V = \frac{Q}{4\pi\varepsilon_0 r}$
Equipotential Surfaces	Around a point mass:  In a uniform field (surface of a planet): 	Around a point charge:  In a uniform field (between charged) parallel plates 
Work Done on a Mass or Charge	$\Delta W = M\Delta V$	$\Delta W = Q\Delta V$

- The key similarities are:
  - The magnitude of the gravitational and electrostatic force between two point masses or charges are **inverse square law** relationships
  - The field lines around a **point mass** and **negative point charge** are identical

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- The field lines in a **uniform** gravitational and electric field are identical
- The **gravitational field strength** and **electric field strength** both have a 1 / r^2 relationship in a **radial field**
- The **gravitational potential** and **electric potential** both have a 1 / r relationship
- **Equipotential surfaces** for both gravitational and electric fields are **spherical** around a point mass or charge and **equally spaced** parallel lines in uniform fields
- The work done in each field is either the product of the **mass** and change in potential or **charge** and change in potential

- The key differences are:
  - The gravitational force acts on particles with **mass** whilst the electrostatic force acts on particles with **charge**
  - The gravitational force is **always** attractive whilst the electrostatic force can be attractive **or** repulsive
  - The gravitational potential is **always** negative whilst the electric potential can be either negative **or** positive

Worked Example

Two parallel metal plates are separated by 3.5 cm and have a potential difference of 7.9 kV.

Calculate the electric force acting on a stationary charged particle between the plates that has a charge of 2.6×10^{-15} C.

Step 1: Write down the known values

- Potential difference, $V = 7.9$ kV $= 7.9 \times 10^3$ V
- Distance between plates, $d = 3.5$ cm $= 3.5 \times 10^{-2}$ m
- Charge, $Q = 2.6 \times 10^{-15}$ C

Step 2: Calculate the electric field strength between the parallel plates

$$E = \frac{V}{d} = \frac{7.9 \times 10^3}{3.5 \times 10^{-2}} = 2.257 \times 10^5 \text{ V m}^{-1}$$

Step 3: Write out the equation for electric force on a charged particle

$$F = QE$$

Step 4: Substitute electric field strength and charge into electric force equation

$$F = QE = (2.6 \times 10^{-15}) \times (2.257 \times 10^5) = 5.87 \times 10^{-10} \text{ N} = 5.9 \times 10^{-10} \text{ N (2 s.f.)}$$

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Exam Tip

Remember to use the correct equation depending on whether there is a **uniform** or **radial** field.

For electric fields:

- Uniform fields: parallel plates / capacitors
- Radial fields: around point charges

For gravitational fields:

- Uniform fields: near the Earth's surface
- Radial fields: around masses (e.g. planets and moons)

You should be able to tell the type of field from the **field lines**. Uniform fields have equally spaced, parallel field lines.

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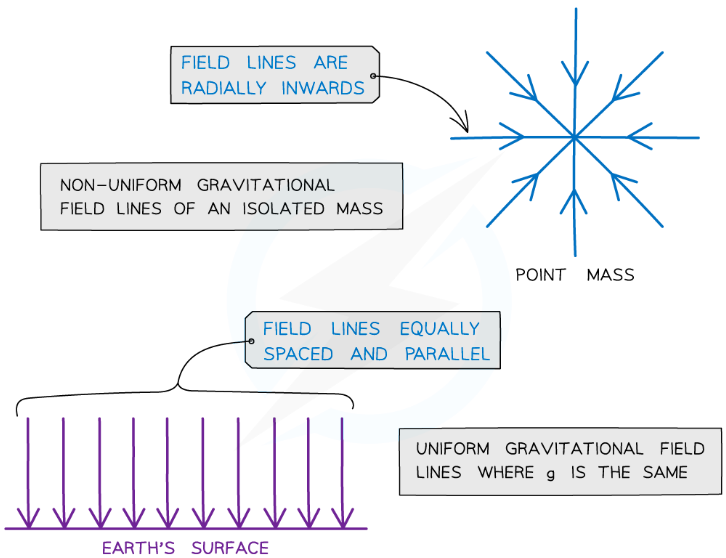
Gravitational & Electrostatic Field Lines

Gravitational Field Lines

- The direction of a gravitational field is represented by gravitational field lines
- The gravitational field lines around a point mass are **radially inwards**
- The gravitational field lines of a uniform field, where the field strength is the same at all points, is represented by **equally spaced parallel lines**
  - For example, the fields lines on the Earth's surface



Gravitational field lines for a point mass and a uniform gravitational field

- Radial fields are considered **non-uniform fields**
  - The gravitational field strength g is different depending on how far you are from the centre
- Parallel field lines on the Earth's surface are considered a **uniform field**
  - The gravitational field strength g is the same throughout

Point Mass Approximation

- For a point outside a uniform sphere, the mass of the sphere may be considered to be a **point mass** at its centre
  - A uniform sphere is one where its mass is **distributed evenly**
  - The gravitational field lines around a uniform sphere are therefore **identical to those around a point mass**
- An object can be regarded as point mass when:

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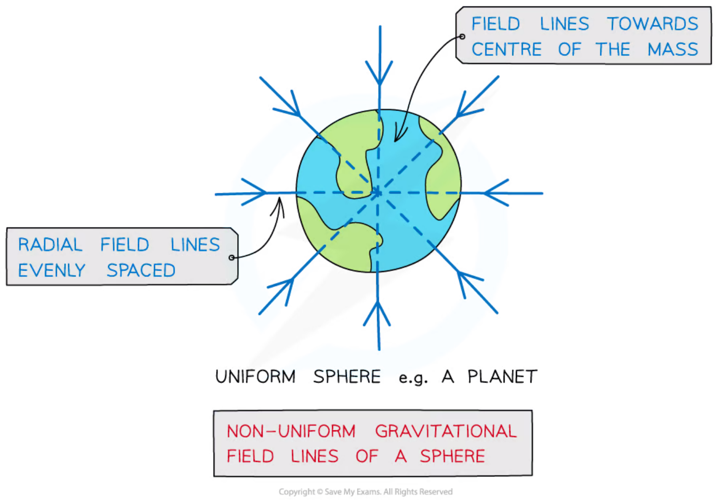
A body covers a very large distance as compared to its size, so, to study its motion, its size or dimensions can be neglected

- An example of this is field lines around planets



Gravitational field lines around a uniform sphere are identical to those on a point mass

- Radial fields are considered **non-uniform** fields
  - So, the gravitational field strength *g* is different depending on how far an object is from the centre of mass of the sphere

Exam Tip

Always label the arrows on the field lines! Gravitational forces are attractive only. Remember:

- For a radial field: it is towards the centre of the sphere or point charge
- For a uniform field: towards the surface of the object e.g. Earth

Representing Electric Fields

- The direction of electric fields is represented by electric field lines
- Electric field lines are directed from positive to negative
  - Therefore, the field lines must be pointed **away** from the **positive** charge and **towards** the **negative** charge
- A radial field spreads uniformly to or from the charge in all directions
  - e.g. the field around a point charge or sphere
- Around a point charge, the electric field lines are directly radially inwards or outwards:
  - If the charge is **positive** (+), the field lines are radially **outwards**
  - If the charge is **negative** (-), the field lines are radially **inwards**

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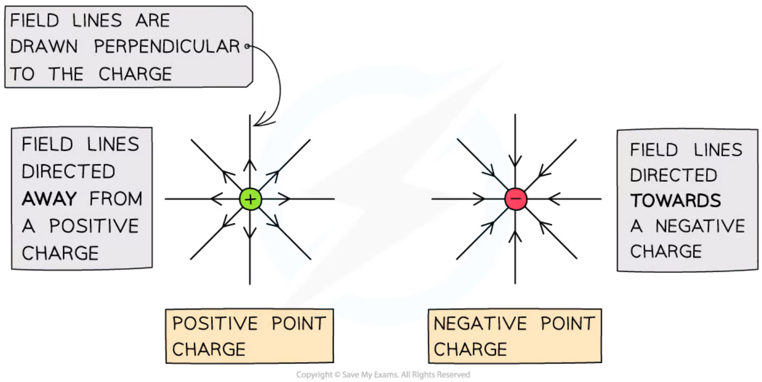
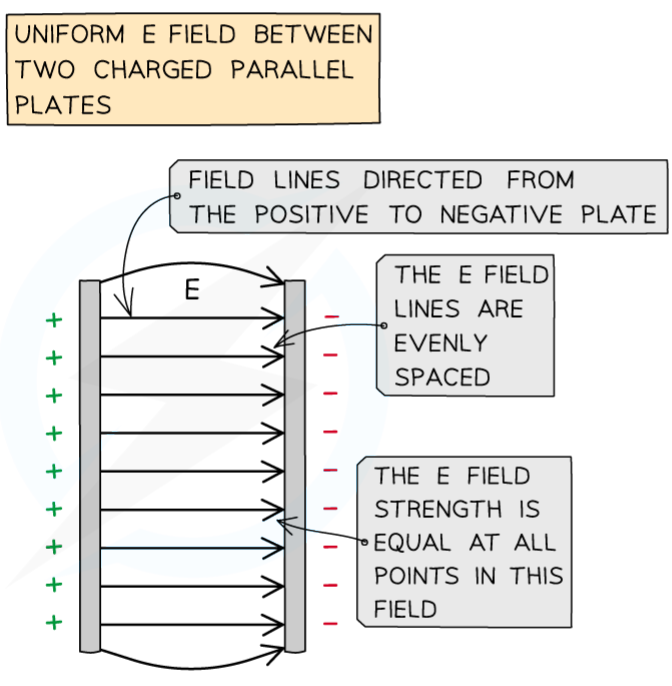


Electric field lines point away from a positive charge and point towards a negative charge

- This shares many similarities to radial gravitational field lines around a point mass
  - Since gravity is only an attractive force, the field lines will look similar to the negative point charge, whilst electric field lines can be in either direction
- A uniform electric field has the same electric field strength throughout the field
  - For example, the field between oppositely charged parallel plates
  - This is represented by **equally spaced** field lines and shares many similarities to uniform gravitational field lines on the surface of a planet
- A **non-uniform** electric field has varying electric field strength throughout
  - The strength of an electric field is represented by the spacing of the field lines:
  - A **stronger** field is represented by the field lines which are **closer** together
  - A **weaker** field is represented by the field lines which are **further** apart
- The electric field lines are directed from the **positive** to the **negative** plate



The electric field between two parallel plates is directed from the positive to the negative plate. A uniform E field has equally spaced field lines

- A radial field is considered a **non-uniform** field
  - Electric field strength *E* varies with distance from a charged particle

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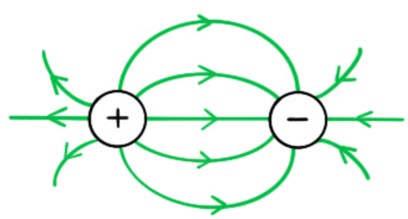
Worked Example

Sketch the electric field lines between the two point charges in the diagram below.

- Electric field lines around point charges have arrows which point radially outwards for positive charges and radially inwards for negative charges
- Arrows (representing force on a positive test charge) point **from the positive charge to the negative charge**



Exam Tip

Always label the arrows on the field lines! The lines must also touch the surface of the source charge or plates.

10.1.3 Gravitational & Electrostatic Potential

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Gravitational & Electrostatic Potential

Gravitational Potential

- The gravitational potential energy (G.P.E) is the energy an object has when lifted off the ground
- G.P.E is given by the familiar equation:

$$GPE = mgh$$

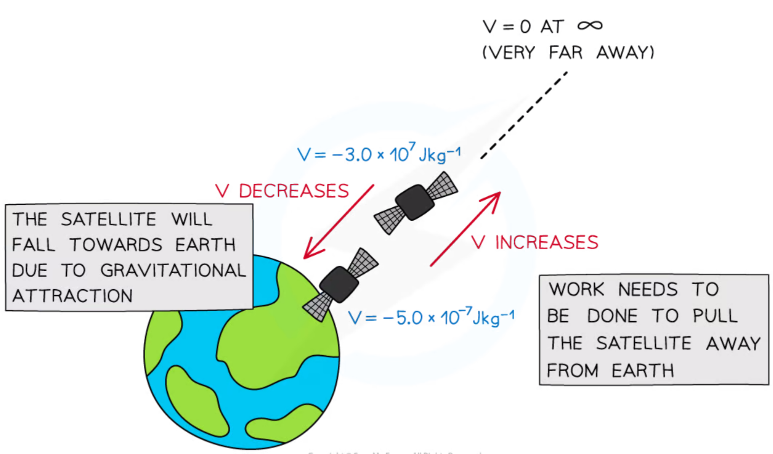
- The G.P.E on the surface of the Earth is taken to be zero
  - This means work is done to **lift** the object
  - The equation is only valid for objects that are **close** to the Earth's surface
- However, outside the Earth's immediate influence, G.P.E can be better defined as:

The energy an object possess due to its position in a gravitational field

- The gravitational potential at a point can, therefore, be defined as:

The work done per unit mass in bringing a test mass from infinity to a defined point

- It is represented by the symbol, V and is measured in J kg^{-1}
- Gravitational potential is always a negative value, this is because:
  - It is defined as having a value of **zero** at **infinity**
  - Since the gravitational force is **attractive**, work must be done **on** a mass to reach infinity
- On the surface of a mass (such as a planet), gravitational potential has a negative value
  - The value becomes less negative, i.e. it increases, with distance from that mass
- Work has to be done **against** the gravitational pull of the planet to take a unit mass away from the planet
- The gravitational potential at a point depends on:
  - The **mass** of the object producing the gravitational field
  - The **distance** the centre of mass to the point



Gravitational potential decreases as the satellite moves closer to the Earth

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Gravitational Potential Difference

- Two points at different distances from a mass will have different gravitational potentials because the gravitational potential increases with distance
- So there will also be a gravitational potential difference between the two points, which is represented by the symbol ΔV
- Gravitational potential difference, ΔV, is given by:

$$\Delta V = V_f - V_i$$

- Where:
  - V_f = final gravitational potential (J kg^{-1})
  - V_i = initial gravitational potential (J kg^{-1})
- A difference in gravitational potential will give a difference in gravitational potential energy, which can also be calculated

Calculating Gravitational Potential

- The equation for gravitational potential V is defined by the mass M and distance r:

$$V = -\frac{G \times M}{r}$$

- Where:
  - V = gravitational potential (J kg^{-1})
  - G = Newton's gravitational constant
  - M = mass of the body producing the gravitational field (kg)
  - r = distance from the centre of the mass to the point mass (m)

- The gravitational potential always is negative near an isolated mass, such as a planet, because:
  - The potential when r is at infinity (∞) is defined as 0
  - Work must be done to move a mass away from a planet (V becomes less negative)

- It is also a scalar quantity, unlike the gravitational field strength which is a vector quantity

- Gravitational forces are always **attractive**, this means as r decreases, positive work is done by the mass when moving from infinity to that point
  - When a mass is closer to a planet, its gravitational potential becomes smaller (more negative)
  - As a mass moves away from a planet, its gravitational potential becomes larger (less negative) until it reaches 0 at infinity

- This means when the distance (r) becomes very large, the gravitational force tends rapidly towards 0 at a point further away from a planet

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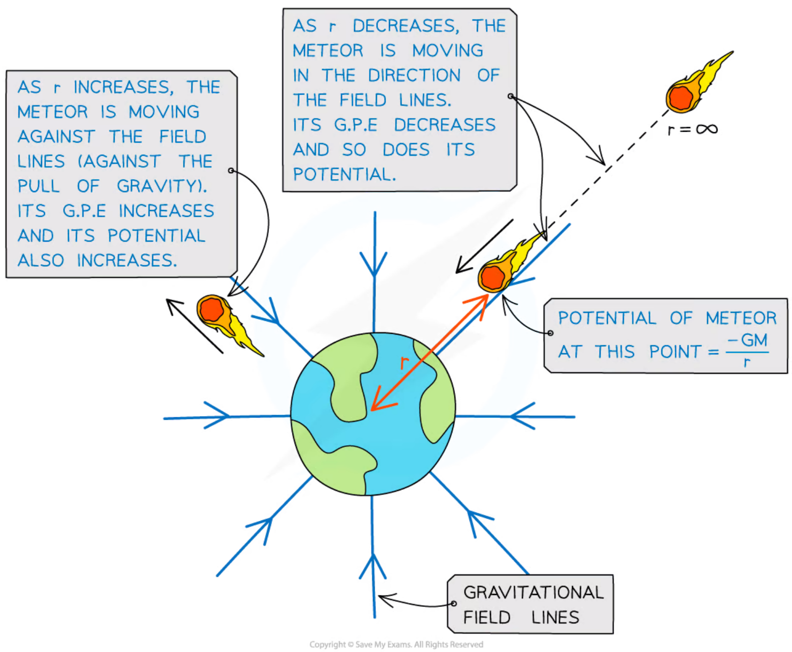

Gravitational potential increases and decreases depending on whether the object is travelling towards or against the field lines from infinity

Worked Example

A planet has a diameter of 7600 km and a mass of 3.5 × 10^{23} kg. A rock of mass 528 kg accelerates towards the planet from infinity.

Calculate the gravitational potential of the rock at a distance of 400 km above the planet's surface.

Step 1: Write the gravitational potential equation

$$V = -\frac{G \times M}{r}$$

Step 2: Determine the value of *r*

- r is the distance from the centre of the planet

Radius of the planet = planet diameter ÷ 2 = 7600 ÷ 2 = 3800 km
r = 3800 + 400 = 4200 km = 4.2×10^6 m

Step 3: Substitute in values

$$V = -\frac{(6.67 \times 10^{-11}) \times (3.5 \times 10^{23})}{4.2 \times 10^6} = -5.6 \times 10^6 \ J\ kg^{-1}$$

Graphical Representation of Gravitational Potential

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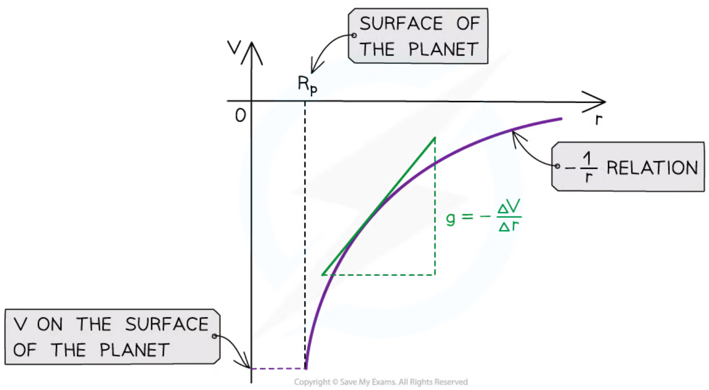
- Gravitational field strength, g and the gravitational potential, V can be graphically represented against the distance from the centre of a planet, r
- g, V and r are related by the equation:

$$g = -\frac{\Delta V}{\Delta r}$$

- Where:
  - g = gravitational field strength (N kg^{-1})
  - ΔV = change in gravitational potential (J kg^{-1})
  - Δr = distance from the centre of a point mass (m)
- The graph of V against r for a planet is:



The gravitational potential and distance graphs follows a -1/r relation

- **The key features of this graph are:**
  - The values for V are all negative
  - As r increases, V against r follows a -1/r relation
  - The **gradient** of the graph at any particular point is the value of g at that point
  - The graph has a shallow increase as r increases
- To calculate g, draw a tangent to the graph at that point and calculate the gradient of the tangent
  - This gives you a graphical representation of the equation:

$$V = -\frac{GM}{r}$$

- Where G and M are constant
- The graph of g against r for a planet is:

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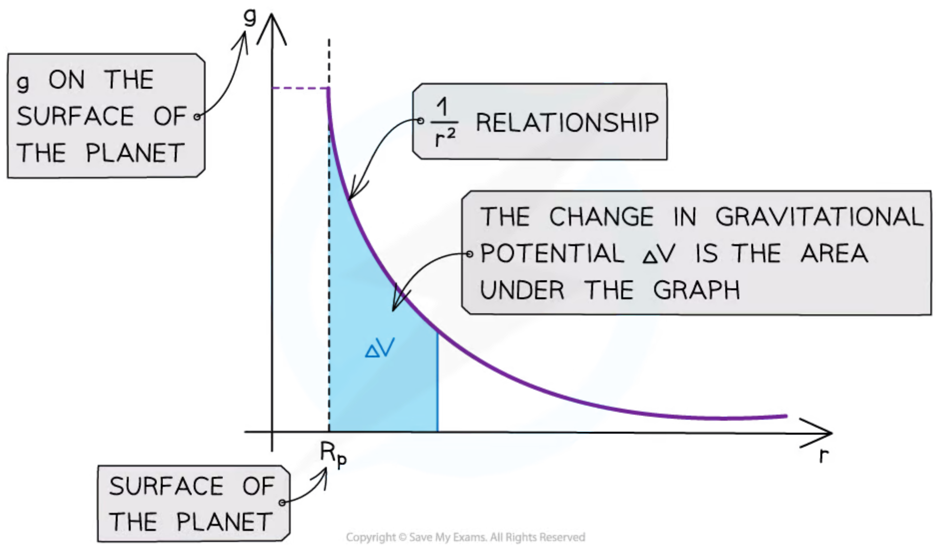

The gravitational field strength and distance graph follows a 1/r^2 relation

- **The key features of this graph are**:
  - The values for *g* are all positive
  - As *r* increases, g against *r* follows a 1/r^2 relation (inverse square law)
  - The **area** under this graph is the change in gravitational potential Δ*V*
  - The graph has a steep decline as *r* increases

- The area under the graph can be estimated by counting squares, if it is plotted on squared paper, or by splitting it into trapeziums and summing the area of each trapezium
  - The inverse square law relation means that as the distance *r* **doubles**, the value of *g* **decreases** by a factor of **four**
- This is a graphical representation of the equation:

$$g = \frac{GM}{r^2}$$

- Where G and *M* are constant

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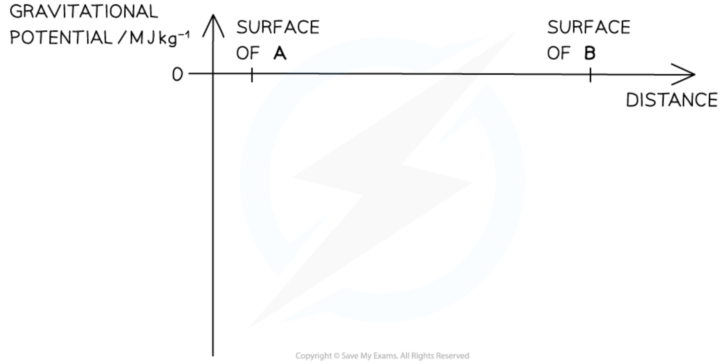
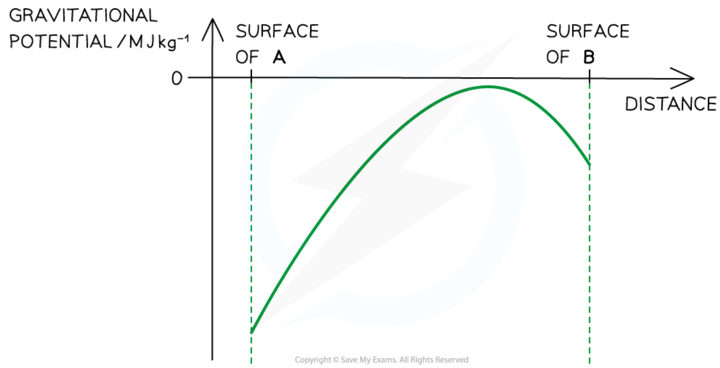
Worked Example

Sketch a graph on the axes below to indicate how the gravitational potential varies with distance along a line outwards from the surface of planet **A** which is 80 times the mass of planet **B**.





- Graph increases from a large negative value for *V* at the surface of **A** as distance increases
- Up to a value close to, but below 0 near the surface of planet **B**
- The graph then falls near the surface of planet **B**
- From a point much closer to planet **B** than **A**

Electric Potential

- In order to move a positive charge closer to another positive charge, work must be done to overcome the force of repulsion between them
  - Similarly, to move a positive charge away from a negative charge, work must be done to overcome the force of attraction between them
- Energy is therefore transferred to the charge that is being pushed upon
  - This means its **potential energy** increases
- If the positive charge is free to move, it will start to move away from the repelling charge
  - As a result, its potential energy decreases back to 0
- This is analogous to the gravitational potential energy of a mass increasing as it is being lifted upwards and decreasing as it falls

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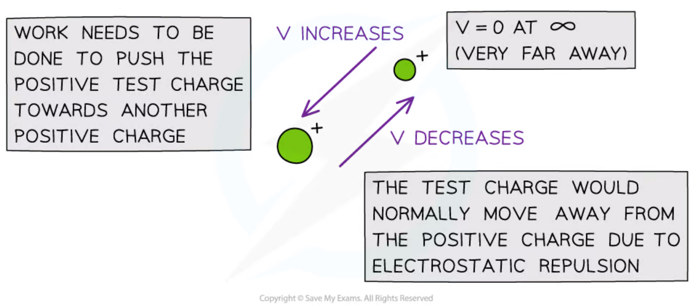
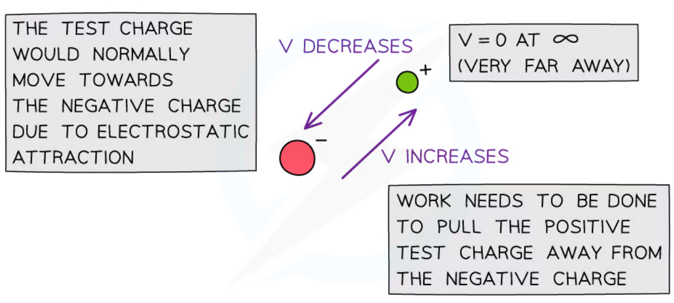
- The electric potential at a point is defined as:

  The work done per unit positive charge in bringing a point test charge from infinity to a defined point

- Electric potential is a **scalar** quantity
  - This means it doesn't have a direction
- However, electric potential can have a positive or negative sign, this is because it is:
  - **Positive** around an isolated positive charge
  - **Negative** around an isolated negative charge
  - **Zero** at infinity
- Positive work is done by the mass from infinity to a point around a positive charge and negative work is done around a negative charge. This means:
  - When a positive **test charge** moves closer to a **negative** charge, its electric potential **decreases**
  - When a positive test charge moves closer to a **positive** charge, its electric potential **increases**
- To find the potential at a point caused by multiple charges, the total potential is the sum of the potential from each charge





The electric potential V decreases in the direction the test charge would naturally move in due to repulsion or attraction

Electric Potential Difference

- Two points at different distances from a charge will have different electric potentials
  - This is because the electric potential increases with distance from a negative charge and decreases with distance from a positive charge
- Therefore, there will be an **electric potential difference** between the two points

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- This is represented by the symbol ΔV

- ΔV is normally given as the equation

$$\Delta V = V_f - V_i$$

- Where:
  - V_f = final electric potential ($J\,C^{-1}$)
  - V_i = initial electric potential ($J\,C^{-1}$)

- A difference in electric potential will give a difference in electric potential energy, which can also be calculated

Graphical Representation of Electric Potential

- **Electric field strength**, E and the **electric potential**, V can be graphically represented against the distance from the centre of a charge, r

- E, V and r are related by the equation:

$$E = \frac{\Delta V}{\Delta r}$$

- Where:
  - E = electric field strength ($V\,m^{-1}$)
  - ΔV = change in potential (V)
  - Δr = displacement in the direction of the field (m)

- An electric field can be defined in terms of the variation of **electric potential** at different points in the field**:**

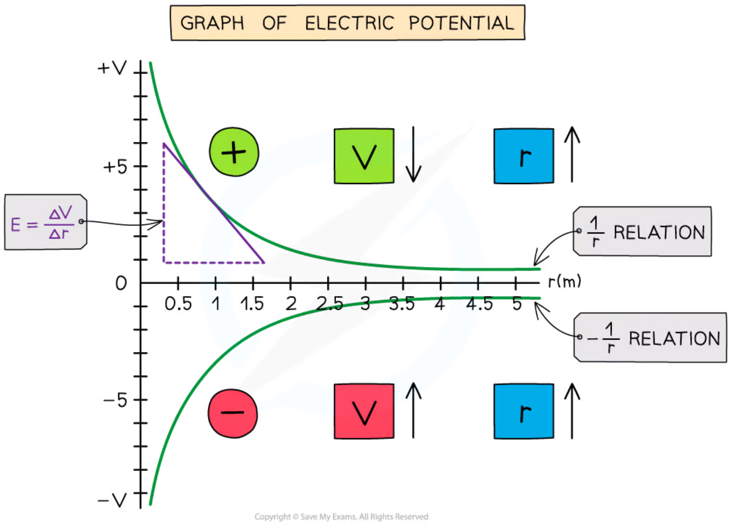
  The electric field at a particular point is equal to the gradient of a potential-distance graph at that point

- The potential gradient in an electric field is defined as:

  The rate of change of electric potential with respect to displacement in the direction of the field

- The graph of potential V against distance r for a negative or positive charge is:

YOUR NOTES



The electric potential around a positive charge decreases with distance and increases with distance around a negative charge

- **The key features of this graph are:**
  - The values for *V* are all negative for a negative charge
  - The values for *V* are all positive for a positive charge
  - As *r* increases, *V* against *r* follows a 1/*r* relation for a positive charge and -1/*r* relation for a negative charge
  - The **gradient** of the graph at any particular point is the value of *E* at that point
  - The graph has a shallow increase (or decrease) as *r* increases

- The electric potential changes according to the charge creating the potential as the distance *r* increases from the centre:
  - If the charge is **positive**, the potential **decreases** with distance
  - If the charge is **negative**, the potential **increases** with distance

- To calculate *E*, draw a tangent to the graph at that point and calculate the gradient of the tangent
- This is a graphical representation of the equation:

$$V = \frac{Q}{4\pi\varepsilon_o r}$$

- Where Q and $4\pi\varepsilon_0$ are constants

- The graph of *E* against *r* for a charge is:

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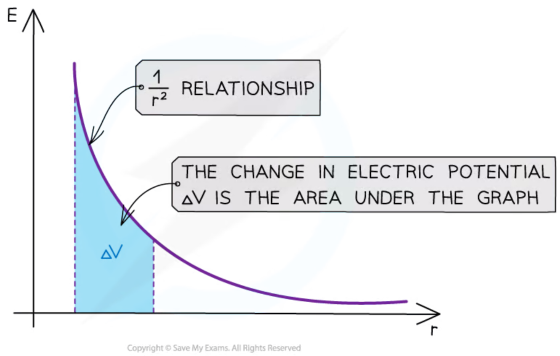


The electric field strength E has a $1/r^2$ relationship

- **The key features of this graph are**:
  - The values for *E* are all positive
  - As *r* increases, *E* against *r* follows a $1/r^2$ relation (inverse square law)
  - The **area** under this graph is the change in electric potential ΔV
  - The graph has a steep decline as *r* increases

- The area under the graph can be estimated by counting squares, if it is plotted on squared paper, or by splitting it into trapeziums and summing the area of each trapezium
- The inverse square law relation means that as the distance *r* doubles, *E* decreases by a factor of **4**

- This is a graphical representation of the equation:

$$E = \frac{Q}{4\pi\varepsilon_o r^2}$$

- Where Q and $4\pi\varepsilon_0$ are constants

Exam Tip

Drawing, interpreting or calculating from inverse square law graphs are common exam questions - there are lots of similarities between gravitational and electric field graphs:

Graphs of field strength against distance should start off steeper and decrease rapidly compared to that of potential graphs against distance, to distinguish it as an inverse square law ($1/r^2$) relation instead of just an inverse relation (1/r)

There are plenty of differences too:

For example, gravitational potential **always increases** with respect to distance whereas electric potential can increase or decrease

One way to remember whether the electric potential increases or decreases with respect to the distance from the charge is by the direction of the electric field lines - the potential always **decreases** in the **same** direction as the field lines and vice versa.

10.1.4 Equipotential Surfaces

YOUR NOTES

Equipotential Surfaces

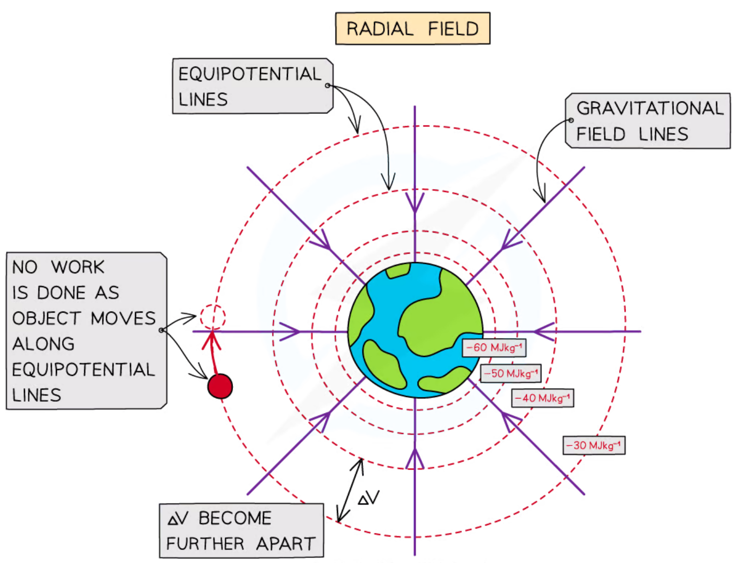
- Equipotential lines (when working in 2D) and surfaces (when working in 3D) join together points that have the same gravitational potential
- These are always:
  - **Perpendicular** to the gravitational field lines in both radial and uniform fields
  - Represented by **dotted** lines (unlike field lines, which are solid lines with arrows)

- In a radial field (eg. a planet), the equipotential lines:
  - Are concentric circles around the planet
  - Become further apart further away from the planet

- In a uniform field (eg. near the Earth's surface), the equipotential lines are:
  - Horizontal straight lines
  - Parallel
  - Equally spaced

- Potential gradient is defined by the **equipotential lines**

- **No work is done** when moving along an equipotential line or surface, only **between** equipotential lines or surfaces
  - This means that an object travelling along an equipotential doesn't lose or gain energy and $\Delta V = 0$



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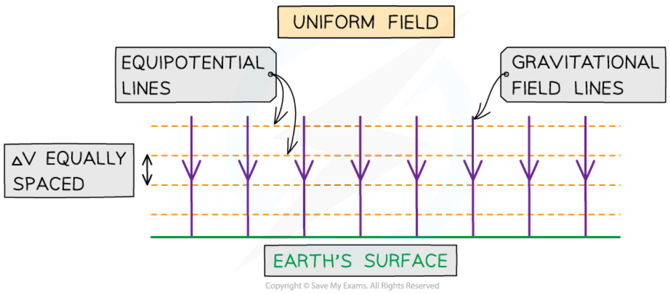
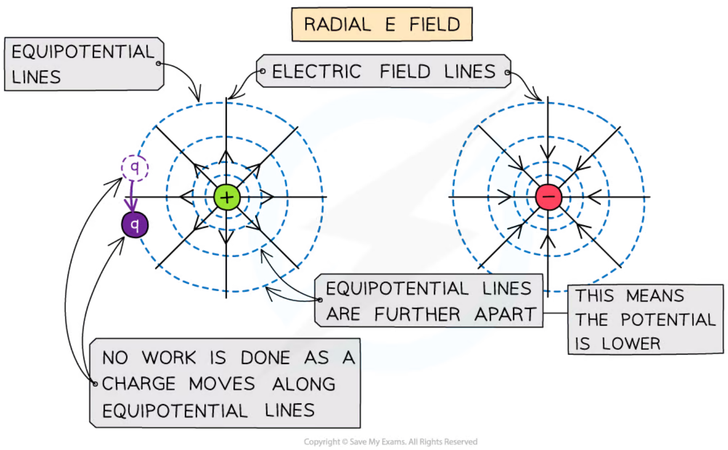
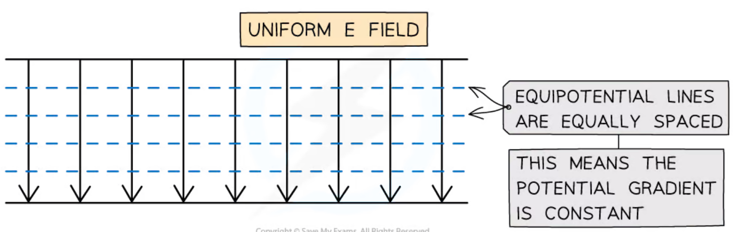


Gravitational equipotential lines in a non-uniform and uniform gravitational field





Equipotential lines around a radial field or uniform field are perpendicular to the electric field lines

- The distinction between radial and uniform fields is an important one
- In a **radial field** (eg. a point charge), the equipotential lines:
  - Are concentric circles around the charge
  - Become further apart further away from the charge
  - Remember: **radial** field is made up of lines which follow the **radius** of a circle
- In a **uniform field** (eg. between charged parallel plates), the equipotential lines are:
  - Horizontal straight lines
  - Parallel
  - Equally spaced
  - Remember: **uniform** field is made up of lines which are a **uniform** distance apart

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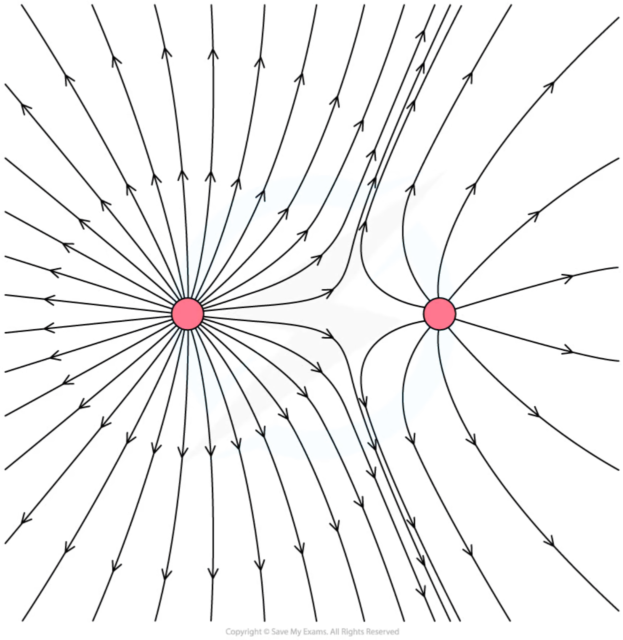
Worked Example

In the following diagram, two electric charges are shown which include the electric field lines



a) Draw the lines of equipotential including at least four lines and at least one that encircles both charges

b) By considering the field lines and equipotentials from part (a), state what can be assumed about the two charges

Part (a)

- The lines of equipotential need to be perpendicular to the field lines at all times
- These lines are almost circular when they are near the charges
- And when moving out further the lines of equipotential cover both charges.

The lines of equipotential can be seen below

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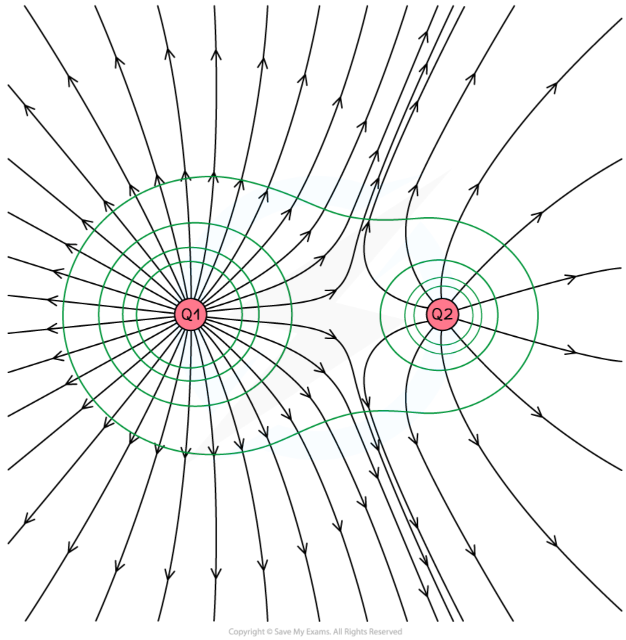

Part (b)

- It can be assumed that both charges are positive since the field lines point outwards.
- It can also be assumed that the charge on the left has a larger charge than the charge on the right since:
  - It has a greater density of field lines
  - It has a larger sphere of influence shown by the lines of equipotential
  - The point of zero electric field strength between the two charges is closer to the right charge

Exam Tip

Remember equipotential lines do **not** have arrows, since they have no particular direction and are not vectors.

Make sure to draw any straight lines with a ruler or a straight edge.

YOUR NOTES
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10.2 Fields at Work

10.2.1 Potential & Potential Energy

Potential & Potential Energy

Work Done on a Mass

- When a mass is moved against the force of gravity, work is done
- The work done in moving a mass m is given by:

$$\Delta W = m \times \Delta V$$

- Where:
  - ΔW = change in work done (J)
  - m = mass (kg)
  - ΔV = change in gravitational potential (J kg^{-1})

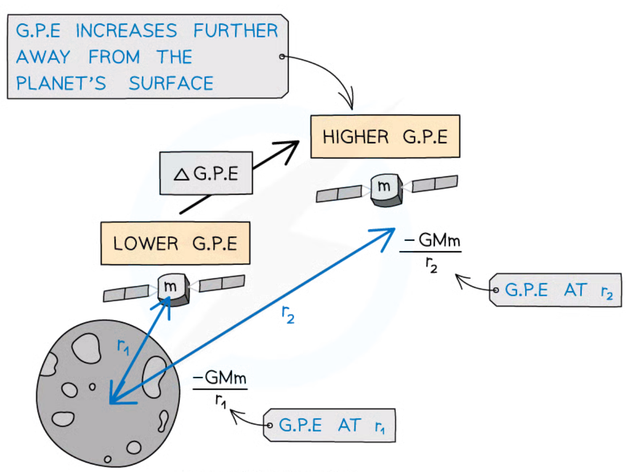
- This change in work done is equal to the change in **gravitational potential energy** (G.P.E)
  - When V = 0, then the G.P.E = 0

- The change in G.P.E, or work done, for an object of mass m at a distance r_1 from the centre of a larger mass M, to a distance of r_2 further away can be written as:

$$\Delta G.P.E = -\frac{GMm}{r_2} - \left(-\frac{GMm}{r_1}\right) = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

  - Where:
    - M = mass that is producing the gravitational field (eg. a planet) (kg)
    - m = mass that is moving in the gravitational field (eg. a satellite) (kg)
    - r_1 = first distance of m from the centre of M (m)
    - r_2 = second distance of m from the centre of M (m)

- Work is done when an object in a planet's gravitational field moves **against** the gravitational field lines i.e.: away from the planet



Gravitational potential energy increases as a satellite leaves the surface of the Moon

YOUR NOTES
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Worked Example

A spacecraft of mass 300 kg leaves the surface of Mars to an altitude of 700 km. Calculate the work done by the spacecraft. The radius of Mars = 3400 km, Mass of Mars = 6.40×10^{23} kg

Step 1: Write down the work done (or change in G.P.E) equation

$$\Delta G.P.E = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Step 2: **Determine values for r_1 and r_2**

r_1 is the radius of Mars = 3400 km = 3400×10^3 m

r_2 is the radius + altitude = 3400 + 700 = 4100 km = 4100×10^3 m

Step 3: Substitute in values

$$\Delta G.P.E = (6.67 \times 10^{-11}) \times (6.40 \times 10^{23}) \times 300 \times \left(\frac{1}{3400 \times 10^3} - \frac{1}{4100 \times 10^3}\right)$$

$$\Delta G.P.E = 643.076 \times 10^6 = 640\ MJ$$

Work Done on a Charge

- When a mass with charge moves through an electric field, work is done
- The work done in moving a charge q is given by:

$$\Delta W = q \times \Delta V$$

- Where:
  - ΔW = change in work done (J)
  - q = charge (C)
  - ΔV = change in **electric potential** (J C^{-1})
- This change in work done is equal to the change in **electric potential energy** (E.P.E)
  - When V = 0, then the E.P.E = 0
- The change in E.P.E, or work done, for a point charge q at a distance r_1 from the centre of a larger charge Q, to a distance of r_2 further away can be written as:

$$\Delta E.P.E = \frac{Qq}{4\pi\varepsilon_o}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

  - Where:
    - Q = charge that is producing the electric field (C)
    - q = charge that is moving in the electric field (C)

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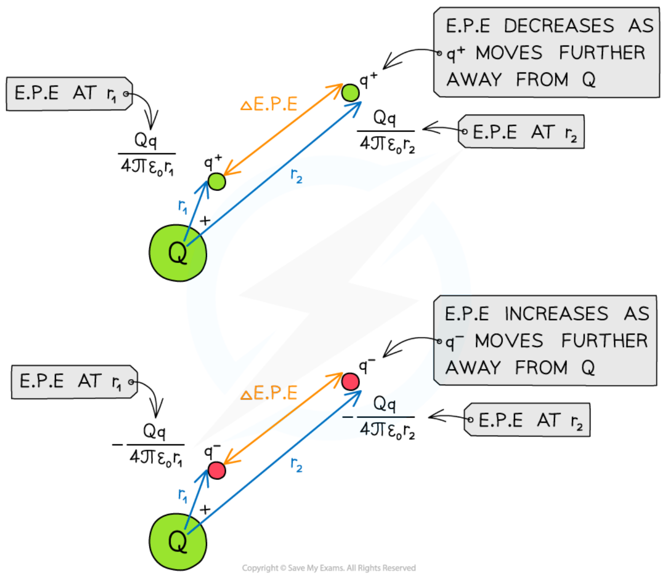
- r_1 = first distance of q from the centre of Q (m)
- r_2 = second distance of q from the centre of Q (m)

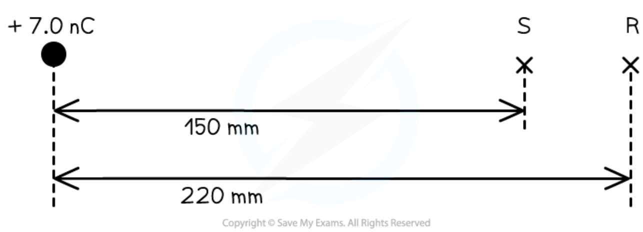


Work is done when moving a point charge away from another charge

- Work is done when a positive charge in an electric field moves **against** the electric field lines or when a negative charge moves **with** the electric field lines

Worked Example

The potentials at points **R** and **S** due to the +7.0 nC charge are 675 V and 850 V respectively.



Calculate how much work is done when a +3.0 nC charge is moved from **R** to **S**.

Step 1: Write down the known quantities

- p.d. at **R**, V_1 = 675 V
- p.d. at **S**, V_2 = 850 V
- Charge, q = +3.0 nC = +3.0 × 10^{-9} C

Step 2: Write down the work done equation

$$W = q \times \Delta V$$

Step 3: Substitute in the values into the equation

$W = (3.0 \times 10^{-9}) \times (850 - 675) = 5.3 \times 10^{-7}$ J

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Exam Tip

Make sure to not confuse the ΔG.P.E equation with ΔG.P.E = mgΔh, they look similar but refer to quite different situations.

The more familiar equation is only relevant for an object lifted in a uniform gravitational field, meaning very close to the Earth's surface, where we can model the field as uniform.

The new equation for G.P.E does not include g. The gravitational field strength, which is different on different planets, does not remain constant as the distance from the surface increases. Gravitational field strength falls away according to the inverse square law.

Remember that *q* in the work done equation is the charge that is being moved, whilst Q is the charge which is producing the potential. Make sure not to get these two mixed up. It is common for both to be given in the question, as in our worked example. You are expected to choose the correct one.

10.2.2 Potential Energy Calculations

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Potential Energy Calculations

Electric Potential Energy of Two Point Charges

- The electric potential energy E_p at point in an electric field is defined as:

The work done in bringing a charge from infinity to that point

- The electric potential energy of a pair of point charges $Q_1$and Q_2 is defined by:

$$E_p = \frac{Q_1 \times Q_2}{4 \times \pi \times \varepsilon_0 \times r}$$

- Where:
  - E_p = electric potential energy (J)
  - r = separation of the charges Q_1 and Q_2 (m)
  - ε_0 = **permittivity of free space** ($F\,m^{-1}$)

- The potential energy equation is defined by the work done in moving point charge Q_2 from infinity towards a point charge Q_1.

- The work done is equal to:

$$W = q \times \Delta V_e$$

  - Where:
    - W = work done (J)
    - V = **electric potential** due to a point charge (V)
    - Q = Charge producing the potential (C)

- This equation is relevant to calculate the work done due on a charge in a uniform field
  - Unlike the electric potential, the potential energy will always be positive
  - Recall that at infinity, V = 0 therefore E_p = 0

- It is more useful to find the **change** in potential energy, for example, as one charge moves away from another

  - The change in potential energy from a charge Q_1 at a distance r_1 from the centre of charge Q_2 to a distance r_2 is equal to:

$$\Delta E_p = \frac{Q_1 \times Q_2}{4 \times \pi \times \varepsilon_0} \times \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

  - The change in electric potential ΔV is the same, without the charge Q_2

$$\Delta V = \frac{Q}{4 \times \pi \times \varepsilon_0} \times \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

- There is another similarity with gravitational potential, as both equations are very similar to the change in gravitational potential between two points near a point mass

YOUR NOTES
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Worked Example

An α-particle ${}^{4}_{2}He$ is moving directly towards a stationary gold nucleus ${}^{197}_{79}Au$.

At a distance of 4.7×10^{-15} m, the α-particle momentarily comes to rest.

Calculate the electric potential energy of the particles at this instant.

Step 1: Write down the known quantities

- Distance, $r = 4.7 \times 10^{-15}$ m
- The charge of one proton, $q = +1.60 \times 10^{-19}$ C

An alpha particle (Helium nucleus) has 2 protons

- Charge of alpha particle, $Q_1 = 2 \times 1.60 \times 10^{-19} = +3.2 \times 10^{-19}$ C

The gold nucleus has 79 protons

- Charge of gold nucleus, $Q_2 = 79 \times 1.60 \times 10^{-19} = +1.264 \times 10^{-17}$ C

Step 2: Write down the equation for electric potential energy

$$E_p = \frac{Q_1 \times Q_2}{4 \times \pi \times \varepsilon_0 \times r}$$

Step 3: Substitute values into the equation

$$E_p = \frac{(3.2 \times 10^{-19}) \times (1.264 \times 10^{-17})}{(4\pi \times 8.85 \times 10^{-12}) \times (4.7 \times 10^{-15})} = 7.7 \times 10^{-12}\ J$$

Gravitational Potential Energy Between Two Point Masses

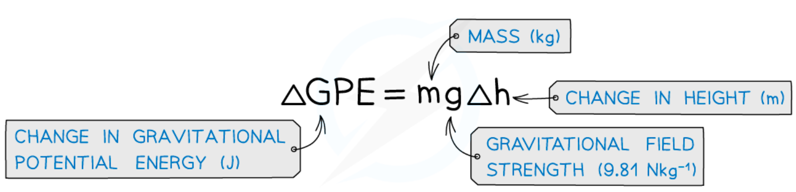
- The gravitational potential energy (G.P.E) at point in a gravitational field is defined as:

The work done in bringing a mass from infinity to that point

- The equation for G.P.E of two point masses m and M at a distance r is:

$$G.P.E = -\frac{G \times M \times m}{r}$$

- G.P.E is calculated using mgh, but recall that at infinity, $g = 0$ and therefore $G.P.E = 0$



- It is more useful to find the **change** in $G.P.E$ for example for a satellite which is lifted into space from the Earth's surface

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- The change in G.P.E from for an object of mass m at a distance r_1 from the centre of mass M, to a distance of r_2 further away is:

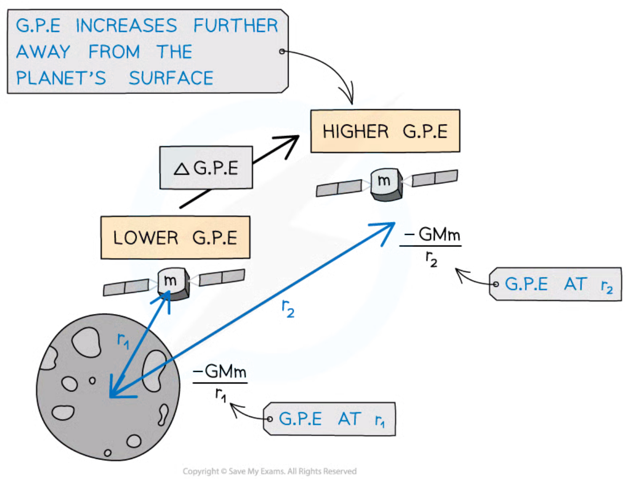
$$\Delta G.P.E = -\frac{G \times M \times m}{r_2} - \left(-\frac{G \times M \times m}{r_1}\right) = G \times M \times m \times \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Change in gravitational potential energy between two points

- The change in potential Δg is the same, without the mass of the object m:

$$\Delta g = -\frac{G \times M}{r_2} - \left(-\frac{G \times M}{r_1}\right) = G \times M \times \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Change in gravitational potential between two points



Gravitational potential energy increases as a satellite leaves the surface of the Moon

Worked Example

Calculate the difference in potential energy when a satellite of mass 1450 kg when it is moved from a distant orbit of 980 km above Earth's surface to a closer orbit of 480 km above the Earth's surface. Assume the Earth's mass to be 5.97×10^{24} kg and the radius of the Earth to be 6.38×10^6 m.

Step 1: Write down the known quantities

- Initial distance of orbit above Earth's surface: 980 km
- Final distance of orbit above Earth's surface: 480 km
- Mass of the satellite: m = 1450 kg
- Earth's mass: $M = 5.97 \times 10^{24}$ kg
- Radius of the Earth: 6.38×10^6 m

Step 2: Write down the equation for change in gravitational potential energy

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$$\Delta G.P.E = -\frac{G \times M \times m}{r_2} - \left(-\frac{G \times M \times m}{r_1}\right) = G \times M \times m \times \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Step 3: Convert distances into standard units and include Earth radius

- Distance from centre of Earth to orbit 1:

$$9.8 \times 10^5 \text{ m} + 6.38 \times 10^6 \text{ m} = 7.36 \times 10^6 \text{ m}$$

- Distance from centre of Earth to orbit 2:

$$4.8 \times 10^5 \text{ m} + 6.38 \times 10^6 \text{ m} = 6.86 \times 10^6 \text{ m}$$

Step 4: Substitute values into the equation

$$\Delta G.P.E = 6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 1450 \times \left(\frac{1}{7.36 \times 10^6} - \frac{1}{6.86 \times 10^6}\right) = 5.72 \times 10^9 \ J$$

Step 5: State final answer

- The difference in gravitational potential energy between the two orbits is: **5.72×10^9 *J***

10.2.3 Potential Gradient & Difference

YOUR NOTES
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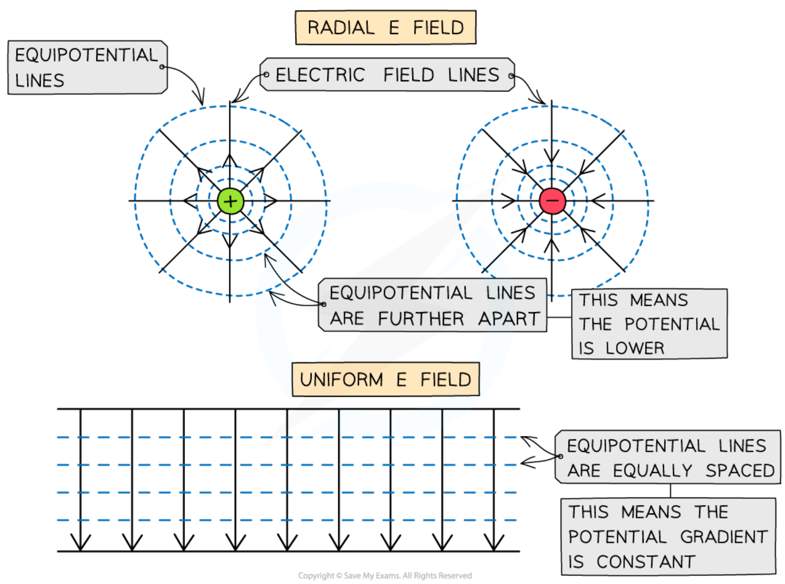
Potential Gradient

Electric Potential Gradient

- An electric field can be defined in terms of the variation of electric potential at different points in the field:

  The electric field at a particular point is equal to the negative gradient of a potential-distance graph at that point

- The potential gradient is defined by the **equipotential lines**
  - These demonstrate the electric potential in an electric field and are always drawn **perpendicular** to the field lines



Equipotential lines around a radial field or uniform field are perpendicular to the electric field lines

- Equipotential lines are lines of **equal electric potential**
  - Around a radial field, the equipotential lines are represented by concentric circles around the charge with increasing radius
  - The equipotential lines become further away from each other
  - In a uniform electric field, the equipotential lines are equally spaced

- The potential gradient in an electric field is defined as:

  The rate of change of electric potential with respect to displacement in the direction of the field

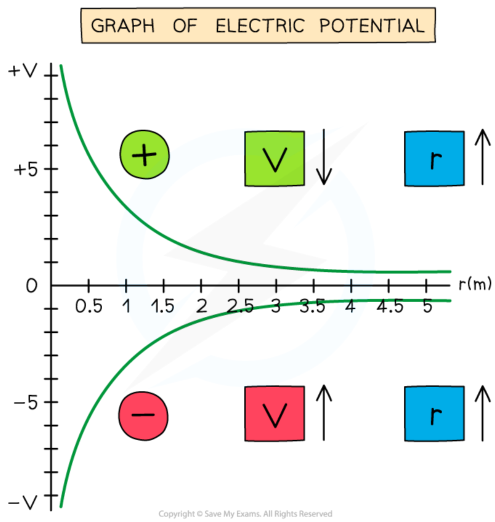
- The electric field strength is equivalent to this, except with a negative sign:

$$E = -\frac{\Delta V}{\Delta r}$$

YOUR NOTES

- Where:
  - E = electric field strength (V m^{-1})
  - ΔV = change in potential (V)
  - Δr = displacement in the direction of the field (m)

- The minus sign is important to obtain an **attractive field** around a **negative charge** and a **repulsive field** around a **positive charge**



The electric potential around a positive charge decreases with distance and increases with distance around a negative charge

- The electric potential changes according to the charge creating the potential as the distance *r* increases from the centre:
  - If the charge is **positive**, the potential **decreases** with distance
  - If the charge is **negative**, the potential **increases** with distance

- This is because the test charge is positive

Gravitational Potential Gradient

- A gravitational field can be defined in terms of the variation of gravitational potential at different points in the field**:**

  The gravitational field at a particular point is equal to the negative gradient of a potential-distance graph at that point

- The potential gradient is defined by the **equipotential lines**
  - These demonstrate the gravitational potential in a gravitational field and are always drawn **perpendicular** to the field lines
- The potential gradient in a gravitational field is defined as:

  The rate of change of gravitational potential with respect to displacement in the direction of the field

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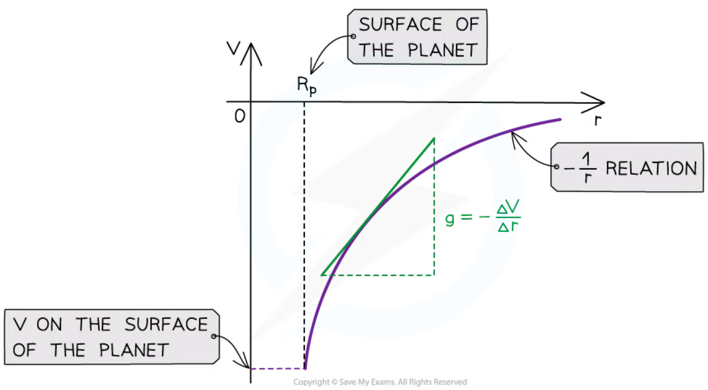
- Gravitational field strength, *g* and the gravitational potential, *V* can be graphically represented against the distance from the centre of a planet, *r*

$$g = -\frac{\Delta V}{\Delta r}$$

- Where:
  - g = gravitational field strength (N kg^{-1})
  - ΔV = change in gravitational potential (J kg^{-1})
  - Δr = distance from the centre of a point mass (m)

- The graph of *V* against *r* for a planet is:



The gravitational potential and distance graphs follows a -1/r relation

- **The key features of this graph are:**
  - The values for *V* are all negative
  - As *r* increases, *V* against *r* follows a -1/r relation
  - The **gradient** of the graph at any particular point is the value of *g* at that point
  - The graph has a shallow increase as *r* increases

- To calculate *g*, draw a tangent to the graph at that point and calculate the gradient of the tangent
- This is a graphical representation of the equation:

$$V = -\frac{G \times M}{r}$$

where *G* and *M* are constant

Worked Example

Determine the change in gravitational potential when travelling from 3 Earth radii (from Earth's centre) to the surface of the Earth.

Assume that the mass of the Earth is 5.97×10^{24} kg and the radius of the Earth is 6.38×10^{6} m

Step 1: List the known quantities

- Earth's mass, $M_E = 5.97 \times 10^{24}$ kg
- Radius of the Earth, $r_E = 6.38 \times 10^{6}$ m

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- Initial distance, $r_1 = 3 \times r_E = 3 \times (6.38 \times 10^6)$ m = 1.914×10^7 m
- Final distance, $r_2 = 1 \times r_E = 6.38 \times 10^6$ m
- Gravitational constant, $G = 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2}

Step 2: Write down the equation for potential difference

$$\Delta V = -G \times M_E \times \left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

Step 3: Substitute in the values:

$$\Delta V = -(6.67 \times 10^{-11}) \times (5.97 \times 10^{24}) \times \left(\frac{1}{6.38 \times 10^6} - \frac{1}{1.914 \times 10^7}\right)$$

$$\Delta V = -4.16 \times 10^7 \text{ J kg}^{-1}$$

Step 4: State the final answer:

- When travelling from 3 Earth radii to one Earth radii the potential difference is: **-4.16×10^7 J kg^{-1}**

Exam Tip

One way to remember whether the electric potential increases or decreases with respect to the distance from the charge is by the direction of the electric field lines. The potential always **decreases** in the **same** direction as the field lines and vice versa.

YOUR NOTES
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Potential Difference

- The potential difference is defined as:

  The work done by moving a positive test charge from one point to another in an electric field

- The units for potential difference are:
  - *Joules per Coulomb* for electrostatic potential difference
  - *Joules per Kilogram* for gravitational potential difference
- This can be related to the concept of equipotentials where the movement of a small test mass or positive test charge from one equipotential to another visually represents a potential difference
  - Further, potential difference also occurs across electrical components which is also known as voltage

Uniform Electric Field Strength

- The magnitude of the electric field strength in a **uniform** field between two charged parallel plates is defined as:

$$E = \frac{V}{d}$$

- Where:
  - E = electric field strength (V m^{-1})
  - V = potential difference between the plates (V)
  - d = separation between the plates (m)

- **Note:** the electric field strength is now also defined by the units **V m^{-1}**
- The equation shows:
  - The greater the **voltage** between the plates, the **stronger** the field
  - The greater the **separation** between the plates, the **weaker** the field

- This equation cannot be used to find the electric field strength around a point charge (since this would be a radial field)
- The direction of the electric field is from the plate connected to the **positive** terminal of the cell to the plate connected to the **negative** terminal

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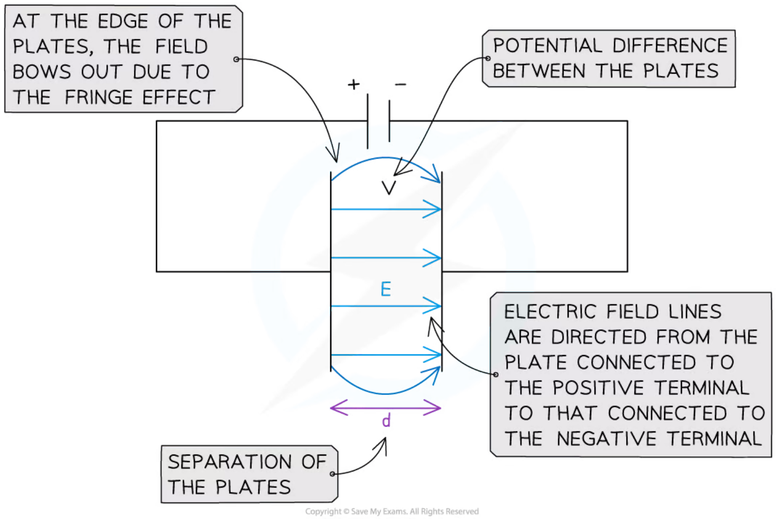
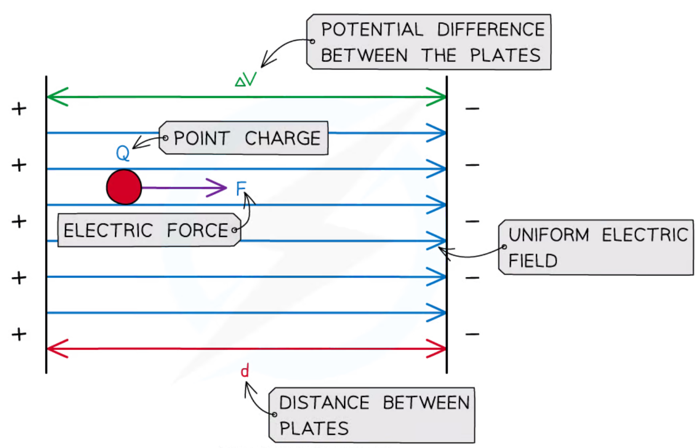


The E field strength between two charged parallel plates is the ratio of the potential difference and separation of the plates

- **Note:** if one of the parallel plates is **earthed**, it has a voltage of 0 V

Derivation of Electric Field Strength Between Plates

- When two points in an electric field have a different potential, there is a **potential difference** between them
  - To move a charge across that potential difference, **work** needs to be done
  - Two parallel plates with a potential difference ΔV across them create a uniform electric field



The work done on the charge depends on the electric force and the distance between the plates

- Potential difference is defined as the energy, W, transferred per unit charge, Q, this can also be written as:

$$\Delta V = \frac{W}{Q}$$

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- Therefore, the work done in transferring the charge is equal to:

$$W = \Delta V \times Q$$

- When a charge Q moves from one plate to the other, its work done is:

$$W = F \times d$$

- Where:
  - W = work done (J)
  - F = force (N)
  - d = distance (m)
- Equate the expressions for work done:

$$F \times d = \Delta V \times Q$$

- Rearranging the fractions by dividing by Q and d on both sides gives:

$$\frac{F}{Q} = \frac{\Delta V}{d}$$

- Since $E = \frac{F}{Q}$ the electric field strength between the plates can be written as:

$$E = \frac{F}{Q} = \frac{\Delta V}{d}$$

Worked Example

Two parallel metal plates are separated by 3.5 cm and have a potential difference of 7.9 kV.

Calculate the magnitude of the electric force acting on a stationary charged particle between the plates that has a charge of 2.6×10^{-15} C.

Step 1: Write down the known values

- Potential difference, V = 7.9 kV = 7.9×10^3 V
- Distance between plates, d = 3.5 cm = 3.5×10^{-2} m
- Charge, Q = 2.6×10^{-15} C

Step 2: Write down the equation for the electric field strength between the parallel plates

$$E = \frac{\Delta V}{d} = \frac{F}{Q}$$

Step 3: Rearrange for electric force, *F*

$$F = \frac{Q \Delta V}{d}$$

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Step 4: Substitute the values into the electric force equation

$$F = \frac{(2.6 \times 10^{-15}) \times (7.9 \times 10^{3})}{(3.5 \times 10^{-2})} = 5.869 \times 10^{-10}\ \text{N}$$

Step 5: State the final answer

- The magnitude of the electric force acting on this charged particle is 5.9×10^{-10} N

Exam Tip

Remember the equation for electric field strength with V and d is only used for parallel plates, and not for point charges (where you would use $E = \frac{F}{Q}$)

10.2.4 Potential in a Charged Sphere

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Potential in a Charged Sphere

- The **electric potential** in the field due to a **point charge** is defined as:

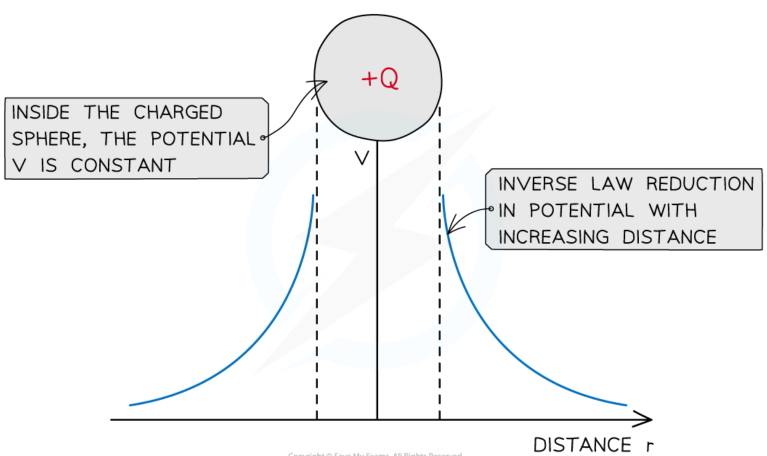
$$V = \frac{Q}{4 \times \pi \times \varepsilon_0 \times r}$$

  - Where:
    - V = the electric potential (V)
    - Q = the point charge producing the potential (C)
    - ε_0 = permittivity of free space (F m^{-1})
    - r = distance from the centre of the point charge (m)

- This equation shows that for a positive (+) charge:
  - As the distance from the charge r **decreases**, the potential V **increases**
  - This is because more work has to be done on a positive test charge to overcome the repulsive force

- For a negative (–) charge:
  - As the distance from the charge r **decreases**, the potential V **decreases**
  - This is because less work has to be done on a positive test charge thanks to the effect of the attractive force

- Unlike the **gravitational potential** equation, the minus sign in the electric potential equation will be included in the charge
- The electric potential changes according to an **inverse square law** with distance



The potential changes as an inverse law with distance near a charged sphere

- **Note:** this equation still applies to a conducting sphere. The charge on the sphere is treated as if it concentrated at a point in the sphere from the point charge approximation

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Worked Example

A Van de Graaf generator has a spherical dome of radius 15 cm. It is charged up to a potential of 240 kV. Calculate:

a) The charge stored on the dome

b) The potential at a distance of 30 cm from the dome

Part (a)

Step 1: Write down the known quantities

- Radius of the dome, r = 15 cm = 15 × 10^{-2} m
- Potential difference, V = 240 kV = 240 × 10^{3} V

Step 2: Write down the equation for the electric potential due to a point charge

$$V = \frac{Q}{4 \times \pi \times \varepsilon_0 \times r}$$

Step 3: Rearrange for charge Q

$$Q = V \times 4 \times \pi \times \varepsilon_0 \times r$$

Step 4: Substitute in values

$$Q = (240 \times 10^3) \times (4 \times \pi \times 8.85 \times 10^{-12}) \times (15 \times 10^{-2}) = 4.0 \times 10^{-6}\ C = 4.0\ \mu C$$

Part (b)

Step 1: Write down the known quantities

- Q = charge stored in the dome = 4.0 μC = 4.0 × 10^{-6} C
- r = radius of the dome + distance from the dome = 15 + 30 = 45 cm = 45 × 10^{-2} m

Step 2: Write down the equation for electric potential due to a point charge

$$V = \frac{Q}{4 \times \pi \times \varepsilon_0 \times r}$$

Step 3: Substitute in values

$$V = \frac{(4.0 \times 10^{-6})}{(4 \times \pi \times 8.85 \times 10^{-12}) \times (45 \times 10^{-2})} = 79.93 \times 10^3 = 80\ kV$$

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Worked Example

A metal sphere of diameter 15 cm is negatively charged. The electric field strength at the surface of the sphere is 1.5×10^5 V m^{-1}. Determine the total surface charge of the sphere.

Step 1: Write down the known values

- Electric field strength, $E = 1.5 \times 10^5$ V m^{-1}
- Radius of sphere, $r = 15 / 2 = 7.5$ cm $= 7.5 \times 10^{-2}$ m

Step 2: Write out the equation for electric field strength

$$V = \frac{Q}{4 \times \pi \times \varepsilon_0 \times r}$$

Step 3: Rearrange for charge Q

$$Q = V \times 4 \times \pi \times \varepsilon_0 \times r$$

Step 4: Substitute in values

$$Q = (4 \times \pi \times 8.85 \times 10^{-12}) \times (1.5 \times 10^5) \times (7.5 \times 10^{-2})^2 = 9.38 \times 10^{-8}\ C\ =\ 94\ nC$$

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Escape Speed

- To escape a gravitational field, a mass must travel at the **escape velocity**
  - This is dependent on the mass and radius of the object creating the gravitational field, such as a planet, a moon or a black hole
- Escape velocity is defined as:

  The minimum speed that will allow an object to escape a gravitational field with no further energy input

- It is the same for all masses in the same gravitational field ie. the escape velocity of a rocket is the same as a tennis ball on Earth

- An object reaches escape velocity when all its kinetic energy has been transferred to gravitational potential energy
- This is calculated by equating the equations:

$$\frac{1}{2} \times m \times v^2 = \frac{G \times M \times m}{r}$$

- Where:
  - m = mass of the object in the gravitational field (kg)
  - v = escape velocity of the object (m s^{-1})
  - G = Newton's Gravitational Constant
  - M = mass of the object to be escaped from (ie. a planet) (kg)
  - r = distance from the centre of mass M (m)

- Since mass m is the same on both sides of the equations, it can cancel on both sides of the equation:

$$\frac{1}{2} \times v^2 = \frac{G \times M}{r}$$

- Multiplying both sides by 2 and taking the square root gives the equation for escape velocity, v:

$$v = \sqrt{\frac{2 \times G \times M}{r}}$$

- This equation is **not** given on the datasheet. Be sure to memorise how to derive it

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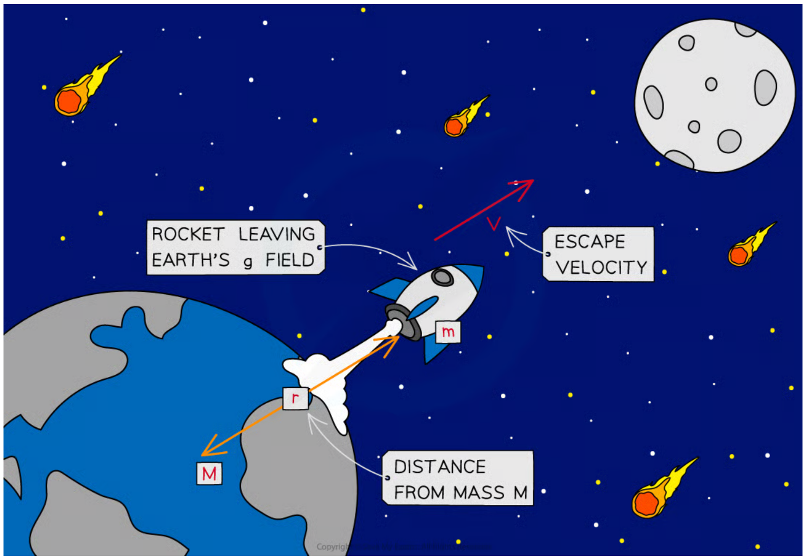

For an object to leave the Earth's gravitational field, it will have to travel at a speed greater than the Earth's escape velocity, v

- Rockets launched from the Earth's surface do **not** need to achieve escape velocity to reach their orbit around the Earth
- This is because:
  - They are continuously given energy through fuel and thrust to help them move
  - Less energy is needed to achieve orbit than to escape from Earth's gravitational field

- The escape velocity is **not** the velocity needed to escape the planet but to escape the planet's **gravitational field** altogether
  - This could be quite a large distance away from the planet

Worked Example

Calculate the escape velocity at the surface of the Moon given that its density is 3340 kg m^{-3} and has a mass of 7.35 × 10^{22} kg. Newton's Gravitational Constant = 6.67 × 10^{-11} N m^2 kg^{-2}

Step 1: Rearrange the density equation for radius:

$$\rho = \frac{M}{V} \text{ and } V = \frac{4}{3}\pi r^3$$

$$\rho = \frac{M}{\frac{4}{3}\pi r^3} = \frac{3M}{4\pi r^3}$$

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$$r = \sqrt[3]{\frac{3M}{4\pi\rho}}$$

Step 2: Calculate the radius by substituting in the values:

- $M = 7.35 \times 10^{22}$ kg
- $\rho = 3\,340$ kg m^{-3}

$$r = \sqrt[3]{\frac{3 \times (7.35 \times 10^{22})}{4\pi \times 3340}} = 1.7384 \times 10^{6} \text{ m}$$

Step 3: Substitute r into the escape velocity equations:

$$V = \sqrt{\frac{2GM}{r}} = \sqrt{\frac{2 \times (6.67 \times 10^{-11}) \times (7.35 \times 10^{22})}{1.7384 \times 10^{6}}}$$

V = 2.37 km s^{-1}

Exam Tip

When writing the definition of **escape velocity**, avoid terms such as 'gravity' or the 'gravitational pull / attraction' of the planet. It is best to refer to its **gravitational field.**

10.2.6 Orbital Motion, Speed & Energy

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Orbital Motion, Speed & Energy

- Since most planets and satellites have a near circular orbit, the gravitational force F_G between the sun or another planet provides the centripetal force needed to stay in an orbit
  - Both the gravitational force and centripetal force are **perpendicular** to the direction of travel of the planet
- Consider a satellite with mass *m* orbiting Earth with mass *M* at a distance *r* from the centre travelling with linear speed *v*

$$F_G = F_{circ}$$

- Equating the gravitational force to the centripetal force for a planet or satellite in orbit gives:

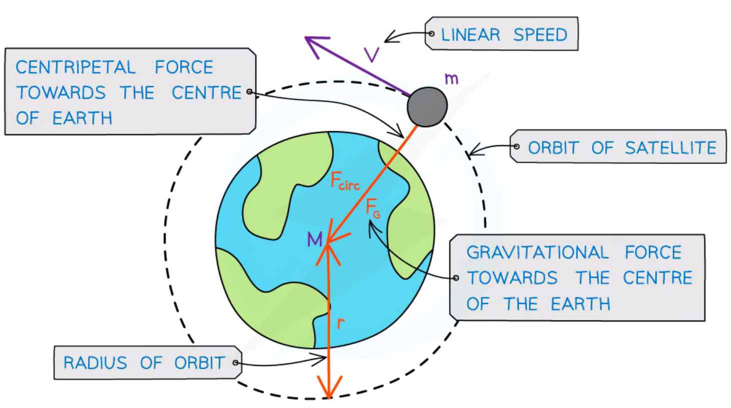
$$\frac{G \times M \times m}{r^2} = \frac{m \times v^2}{r}$$

- The mass of the satellite *m* will cancel out on both sides to give:

$$v^2 = \frac{G \times M}{r}$$

- Where:
  - v = linear speed of the mass in orbit (m s^{-1})
  - G = Newton's Gravitational Constant
  - M = mass of the object being orbited (kg)
  - r = orbital radius (m)

- This means that all satellites, ***whatever their mass***, will travel at the same speed *v* in a particular orbit radius *r*
  - Since the direction of a planet orbiting in circular motion is constantly changing, the **centripetal acceleration** acts towards the planet



A satellite in orbit around the Earth travels in circular motion

Time Period & Orbital Radius Relation

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- Since a planet or a satellite is travelling in circular motion when in order, its orbital time period T to travel the circumference of the orbit $2\pi r$, the linear speed v is:

$$v = \frac{2 \times \pi \times r}{T}$$

- This is a result of the well-known equation, speed = distance / time and first introduced in the circular motion topic
- Substituting the value of the linear speed v from equating the gravitational and centripetal force into the above equation gives:

$$v^2 = \left(\frac{2 \times \pi \times r}{T}\right)^2 = \frac{G \times M}{r}$$

- Squaring out the brackets and rearranging for T^2 gives the equation relating the time period T and orbital radius r:

$$T^2 = \frac{4 \times \pi^2 \times r^3}{G \times M}$$

- Where:
  - T = time period of the orbit (s)
  - r = orbital radius (m)
  - G = Newton's Gravitational Constant
  - M = mass of the object being orbited (kg)

- The equation shows that the orbital period T is related to the radius r of the orbit. This is also known as Kepler's third law:

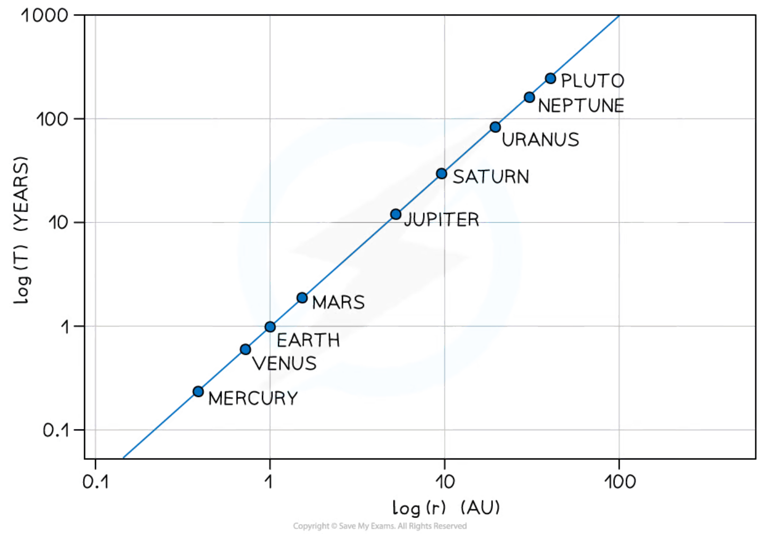
  For planets or satellites in a circular orbit about the same central body, the square of the time period is proportional to the cube of the radius of the orbit

- Kepler's third law can be summarised as:

$$T^2 \ \alpha \ r^3$$

Graphical Representation of $T^2 \propto r^3$

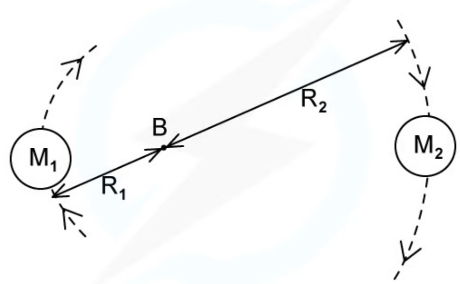
- The relationship between T and r can be shown using a logarithmic plot
  - $T^2 \propto r^3$
  - $2 \times \log(T) \propto 3 \times \log(r)$
- The graph of log(T) in years against log(r) in AU (astronomical units) for the planets in our solar system is a straight line graph:



- The graph does not go through the origin since it has a negative y-intercept
- Only the log of both *T* and *r* will produce a straight line graph

Worked Example

A binary star system constant of two stars orbiting about a fixed point **B**. The star of mass M_1 has a circular orbit of radius R_1 and mass M_2 has a radius of R_2. Both have linear speed *v* and an angular speed ω about **B**.



State the following formula, in terms of G, M_2, R_1 and R_2

(i) The angular speed ω of M_1

(ii) The time period *T* for each star in terms of angular speed ω

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(i) The angular speed ω of M_1

Step 1: Equate the centripetal force to the gravitational force

$$M_1R_1\omega^2 = \frac{GM_1M_2}{(R_1+R_2)^2}$$

Step 2: M_1 cancels on both sides

$$R_1\omega^2 = \frac{GM_2}{(R_1+R_2)^2}$$

Step 3: Rearrange for angular velocity ω

$$\omega^2 = \frac{GM_2}{R_1(R_1+R_2)^2}$$

Step 4: Square root both sides

$$\omega = \sqrt{\frac{GM_2}{R_1(R_1+R_2)^2}}$$

(ii) The time period T for each star in terms of angular speed ω

Step 1: Write down the angular speed ω equation with time period T

$$\omega = \frac{2\pi}{T}$$

Step 2: Rearrange for T

$$T = \frac{2\pi}{\omega}$$

Step 3: Substitute in ω from part (i)

$$T = 2\pi \div \sqrt{\frac{GM_2}{R_1(R_1+R_2)^2}} = 2\pi\sqrt{\frac{R_1(R_1+R_2)^2}{GM_2}}$$

Energy of an Orbiting Satellite

- An orbiting satellite follows a circular path around a planet
- Just like an object moving in circular motion, it has both kinetic energy (KE) **and** gravitational potential energy (GPE) and its **total** energy is always **constant**
- An orbiting satellite's total energy is calculated by:

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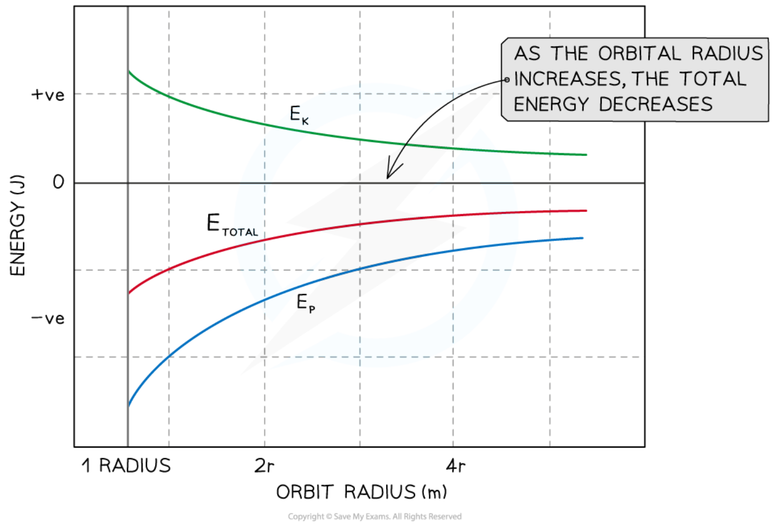
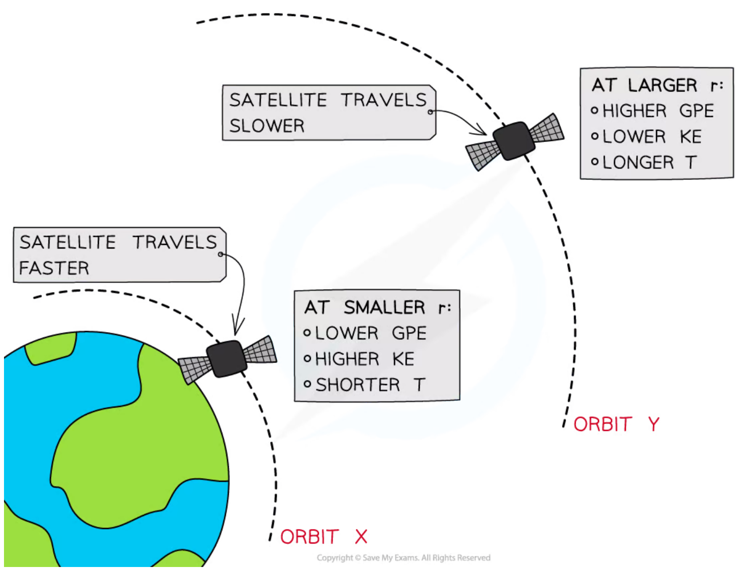


A graph showing the kinetic, potential and total energy for a mass at varying orbital distances from a massive body

Total energy = Kinetic energy + Gravitational potential energy

- This means that the satellite's KE and GPE are also both constant in a particular orbit
  - If the orbital radius of a satellite **decreases** its KE **increases** and its GPE **decreases**
  - If the orbital radius of a satellite **increases** its KE **decreases** and its GPE **increases**



At orbit Y, the satellite has greater GPE and less KE than at at orbit X

- A satellite is placed in two orbits, X and Y, around Earth
- At orbit X, where the radius of orbit *r* is smaller, the satellite has a:
  - Larger gravitational force on it
  - Higher speed
  - Higher KE
  - Lower GPE

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- Shorter orbital time period, T

- At orbit Y, where the radius of orbit r is larger, the satellite has a:
  - Smaller gravitational force on it
  - Smaller speed
  - Lower KE
  - Higher GPE
  - Longer orbital time period, T

Worked Example

Two satellites A and B, of equal mass, orbit a planet at radii R and $3R$ respectively. Which one of the following statements is incorrect?

A A has more kinetic energy and less potential energy than B

B A has a shorter time period and travels faster than B

C B has less kinetic energy and more potential energy than A

D B has a longer time period and travels faster than A

ANSWER: D

- Since B is at a larger orbital radius ($3R$ instead of R) it has a longer time period since $T^2 \propto R^3$ for an orbiting satellite
- However, satellite B will travel much slower than A
- Its larger orbital radius means the force of gravity will be much lower for B than for A

Exam Tip

If you can't remember which way around the kinetic and potential energy increases and decreases, think about the velocity of a satellite at different orbits. When it is orbiting close to a planet, it experiences a larger gravitational pull and therefore orbits faster. Since the kinetic energy is proportional to v^2, it, therefore, has higher kinetic energy closer to the planet. To keep the total energy constant, the potential energy must decrease too.

Orbital Energy Calculations

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- A synchronous orbit is:

  When an orbiting body has a time period equal to that of the body being orbited and in the same direction of rotation as that body

- These usually refer to **satellites** (the orbiting body) around **planets** (the body being orbited)
- The orbit of a synchronous satellite can be above any point on the planet's surface and in any plane
  - When the plane of the orbit is directly above the equator, it is known as a **geosynchronous** orbit

Geostationary Orbit

- Many communication satellites around Earth follow a **geostationary orbit**
  - This is sometimes referred to as a **geosynchronous** orbit

- This is a specific type of orbit in which the satellite:
  - Remains directly **above the equator**
  - Is in the **plane of the equator**
  - Always orbits at the **same point** above the Earth's surface
  - Moves from **west to east** (same direction as the Earth spins)
  - Has an orbital time period equal to Earth's rotational period of **24 hours**

- Geostationary satellites are used for **telecommunication** transmissions (e.g. radio) and television broadcast
- A base station on Earth sends the TV signal up to the satellite where it is amplified and broadcast back to the ground to the desired locations
- The satellite receiver dishes on the surface must point towards the same point in the sky
  - Since the geostationary orbits of the satellites are fixed, the receiver dishes can be fixed too

Low Orbits

- Some satellites are in low orbits, which means their altitude is closer to the Earth's surface
- One example of this is a **polar** orbit, where the satellite orbits around the north and south pole of the Earth
- Low orbits are useful for taking high-quality photographs of the Earth's surface. This could be used for:
  - Weather
  - Military applications

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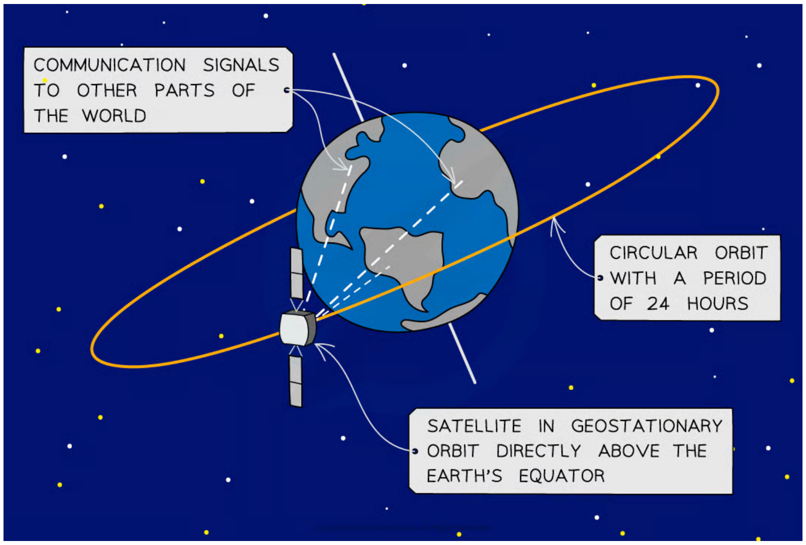
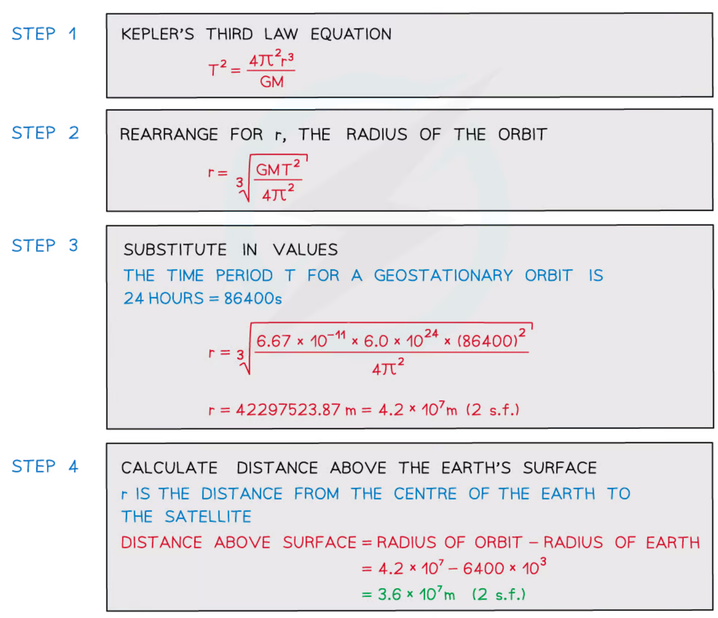


Geostationary satellite in orbit

Worked Example

Calculate the distance above the Earth's surface that a geostationary satellite will orbit. Mass of the Earth = 6.0 × 10^{24} *kg*, Radius of the Earth = 6400 *km*



Orbital Motion

- When a satellite is moving in orbital motion, the velocity of the satellite will be given by:

$$v_{satellite} = \sqrt{\frac{GM}{r}}$$

- Where

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- $v_{satellite}$ = velocity of the satellite (m s^{-1})
- M = mass of the Earth (or large body being orbited) (kg)
- G = The gravitational constant = 6.67 x 10^{-11} Nm^{2}kg^{-2}
- r = the orbital radius of the satellite (m)
- Therefore the kinetic energy of the satellite will be:

$$E_K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\sqrt{\frac{GM}{r}}\right)^2 = \frac{GMm}{2r}$$

- Where
  - E_K = kinetic energy (J)
  - v = velocity (m s^{-1})
  - M = mass of the Earth (or large body being orbited) (kg)
  - m = mass of satellite or orbiting object (kg)
  - G = The gravitational constant = 6.67 x 10^{-11} Nm^{2}kg^{-2}
  - r = the orbital radius of the satellite (m)
- The gravitational potential energy of an orbiting object is:

$$E_P = -\frac{GMm}{r}$$

- Where
  - E_P = potential energy (J)
  - M = mass of the Earth (or large body being orbited) (kg)
  - m = mass of satellite or orbiting object (kg)
  - G = The gravitational constant = 6.67 x 10^{-11} Nm^{2}kg^{-2}
  - r = the orbital radius of the satellite (m)

- The total energy for any satelite orbiting a massive body is found from the combination of potential energy and kinetic energy
  - The result is show in the equation below:

$$E_T = E_K + E_P = \frac{GMm}{2r} - \frac{GMm}{r} = -\frac{GMm}{2r} = -\frac{1}{2} \times m \times v^2$$

- Where
  - E_T = The total energy of the satellite (J)
  - E_P = The potential energy of the satellite (J)
  - E_K = The kinetic energy of the satellite (J)
  - v = the velocity of the satellite (m s^{-1})
  - M = mass of the Earth (or large body being orbited) (kg)
  - m = mass of satellite or orbiting object (kg)
  - G = The gravitational constant = 6.67 x 10^{-11} Nm^{2}kg^{-2}
  - r = the orbital radius of the satellite (m)

10.2.7 Forces & Inverse-Square Law Behaviour

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Forces & Inverse-Square Law Behaviour

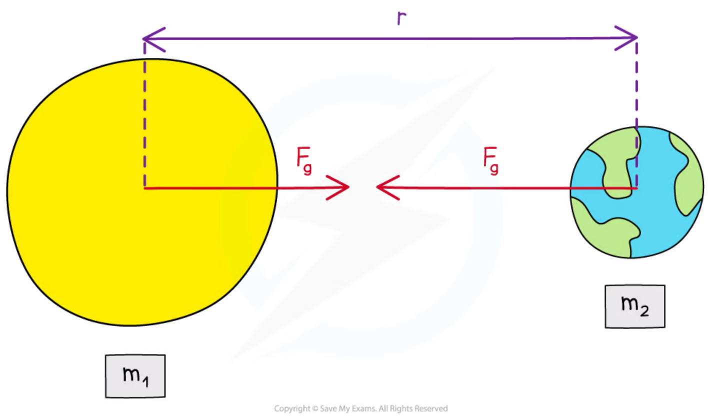
- The gravitational force between two bodies outside a uniform field (for example, between the Earth and the Sun) is defined by Newton's Law of Gravitation which states that:

    The gravitational force between two point masses is proportional to the product of the masses and inversely proportional to the square their separation

- In equation form, this can be written as:

$$F_G = \frac{Gm_1 m_2}{r^2}$$

- Where:
    - F_G = gravitational force between two masses (N)
    - G = Newton's gravitational constant
    - m_1, m_2 = two points masses (kg)
    - r = distance between the centre of the two masses (m)



The gravitational force between two masses outside a uniform field is defined by Newton's Law of Gravitation

- The mass of a uniform sphere can be considered to be a **point mass** at its centre
    - The point mass approximation is a valid assumption if the separation between two objects is much larger than their radii
    - This is why Newton's law of gravitation applies to planets orbiting the Sun
- The $1/r^2$ relation is called the 'inverse square law'
    - This means that when a mass is twice as far away from another, its force due to gravity reduces by $(½)^2$ = ¼

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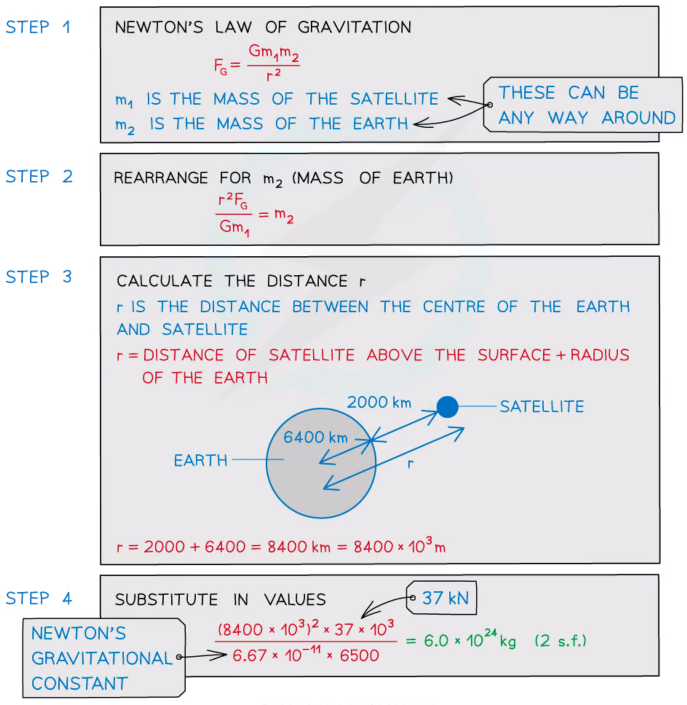
Worked Example

A satellite with a mass of 6500 kg is orbiting the Earth at 2000 km above the Earth's surface. The gravitational force between them is 37 kN.

Calculate the mass of the Earth.

Radius of the Earth = 6400 km.



Coulomb's Law

- All charged particles produce an electric field around it
  - This field exerts a force on any other charged particle within range
- The electrostatic force between two charges is defined by **Coulomb's Law**
  - A charge of a uniform spherical conductor can also be considered as a point charge at its centre
- Coulomb's Law states that:

  The electrostatic force between two point charges is proportional to the product of the charges and inversely proportional to the square of their separation

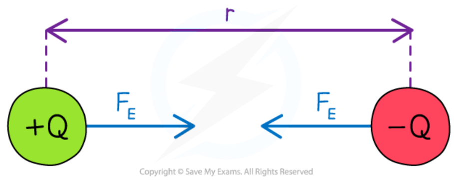
- The Coulomb equation is defined as:

$$F_e = k\frac{Qq}{r^2} = \frac{Qq}{4\pi\varepsilon_0 r^2}$$

- Where:

YOUR NOTES

- F_e = electrostatic force between two charges (*N*)
- *Q* (or Q_1) and *q* (or Q_2) = two point charges (*C*)
- *k* = Coulomb's constant (8.99×10^9 $N\,m^2\,C^{-2}$)
- ε_0 = permittivity of free space
- *r* = distance between the centre of the charges (m)



The electrostatic force between two charges is defined by Coulomb's Law

- The $1/r^2$ relation is called the inverse square law
- This means that when a charge is twice as far as away from another, the electrostatic force between them reduces by $(½)^2 = ¼$
- If there is a positive and negative charge, then the electrostatic force is negative, this can be interpreted as an **attractive force**
- If the charges are the same, the electrostatic force is positive, this can be interpreted as a **repulsive force**
  - Since uniformly charged spheres can be considered as point charges, Coulomb's law can be applied to find the electrostatic force between them as long as the separation is taken from the **centre** of both spheres

Worked Example

An alpha particle is situated 2.0 mm away from a gold nucleus in a vacuum. Assuming them to be point charges, calculate the magnitude of the electrostatic force acting on each of the charges. Atomic number of helium = 2, Atomic number of gold = 79, Charge of an electron = -1.60×10^{-19} C.

Step 1: Write down the known quantities

- Distance, r = 2.0 mm = 2.0×10^{-3} m
- The charge of one proton = $+1.60 \times 10^{-19}$ C
- An alpha particle (helium nucleus) has 2 protons
- Charge of alpha particle, $Q_1 = 2 \times 1.60 \times 10^{-19} = +3.2 \times 10^{-19}$ C
- The gold nucleus has 79 protons
- Charge of gold nucleus, $Q_2 = 79 \times 1.60 \times 10^{-19} = +1.264 \times 10^{-17}$ C

Step 2: The electrostatic force between two point charges is given by Coulomb's Law

$$F_e = \frac{k \times Q \times q}{r^2} = \frac{Q \times q}{4 \times \pi \times \varepsilon_0 \times r^2}$$

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Step 3: Substitute values into Coulomb's Law

$$F_e = \frac{(3.2 \times 10^{-19}) \times (1.264 \times 10^{-17})}{(4 \times \pi \times 8.85 \times 10^{-12}) \times (2.0 \times 10^{-3})^2} = 9.092 \times 10^{-21} = 9.1 \times 10^{-21}\ N$$

Exam Tip

A common mistake in exams is to forget to **add together** the radius of the planet (which is the **distance from the centre of mass** to the surface) and then, the **height above the surface** of the planet.

Sketching a diagram will remind you of these two distances and is really a few seconds well spent!

10.2.8 Forces on Charges & Masses

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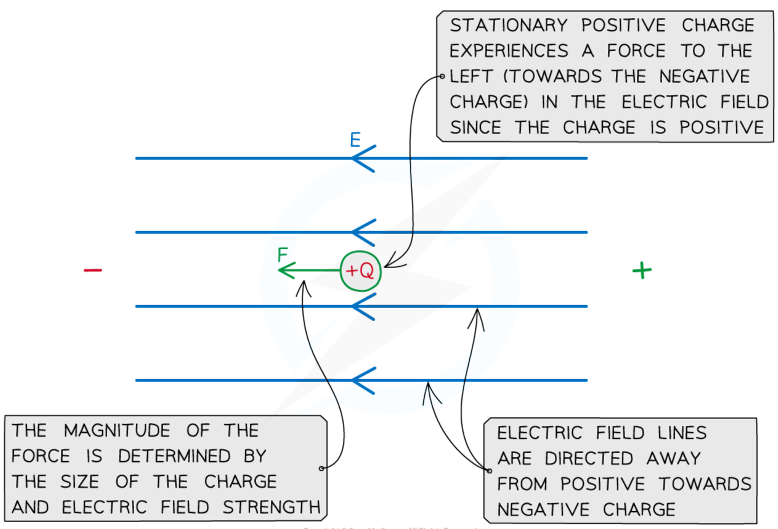
Forces on Charges & Masses

- The electric field strength equation can be rearranged for the force *F* on a charge Q in an electric field *E*:

$$F = Q \times E$$

- Where:
  - F = electrostatic force on the charge (N)
  - Q = charge (C)
  - E = electric field strength (N C^{-1})
- The direction of the force is determined by the charge:
  - If the charge is **positive** (+) the force is in the **same** direction as the E field
  - If the charge is **negative** (-) the force is in the **opposite** direction to the E field
- The force on the charge will cause the charged particle to **accelerate** if its in the same direction as the E field, or **decelerate** if in the opposite



An electric field strength E exerts a force F on a charge +Q in a uniform electric field

- **Note:** the force will always be **parallel** to the electric field lines

Motion of Charged Particles

- A charged particle in an electric field will experience a force on it that will cause it to move
- If a charged particle remains still in a uniform electric field, it will move parallel to the electric field lines (along or against the field lines depending on its charge)
- If a charged particle is in **motion** through a uniform electric field (e.g. between two charged parallel plates), it will experience a constant electric force and travel in a **parabolic trajectory**

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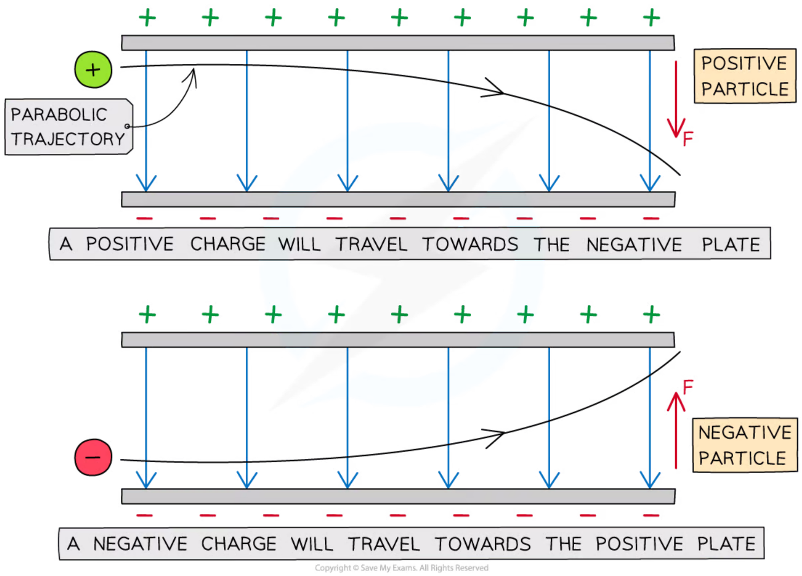
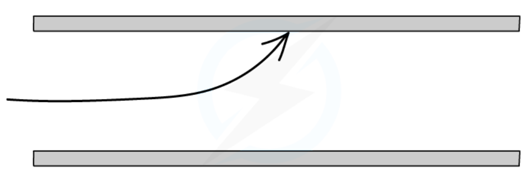


The parabolic path of charged particles in a uniform electric field

- The direction of the parabola will depend on the charge of the particle
  - A **positive** charge will be deflected towards the **negative** plate
  - A **negative** charge will be deflected towards the **positive** plate
- The force on the particle is the same at all points and is always in the same direction
- **Note:** an uncharged particle, such as a neutron experiences no force in an electric field and will therefore travel straight through the plates undeflected
- The amount of deflection depends on the following properties of the particles:
  - **Mass** – the greater the mass, the smaller the deflection and vice versa
  - **Charge** – the greater the magnitude of the charge of the particle, the greater the deflection and vice versa
  - **Speed** – the greater the speed of the particle, the smaller the deflection and vice versa

Worked Example

A single proton travelling with a constant horizontal velocity enters a uniform electric field between two parallel charged plates.The diagram shows the path taken by the proton.



Draw the path taken by a boron nucleus that enters the electric field at the same point and with the same velocity as the proton.Atomic number of boron = 5

Mass number of boron = 11

Step 1: Compare the charge of the boron nucleus to the proton

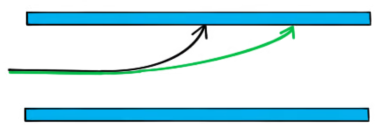
- Boron has 5 protons, meaning it has a charge 5 × greater than the proton
- The force on boron will therefore be 5 × greater than on the proton

Step 2: Compare the mass of the boron nucleus to the proton

- The boron nucleus has a mass of 11 nucleons meaning its mass is 11 × greater than the proton
- The boron nucleus will therefore be less deflected than the proton

Step 3: Draw the trajectory of the boron nucleus

- Since the mass comparison is much greater than the charge comparison, the boron nucleus will be **much less deflected** than the proton
- The nucleus is positively charged since the neutrons in the nucleus have no charge
  - Therefore, the shape of the path will be the same as the proton



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