5.3 Integration

Question Paper

Course	DP IB Maths
Section	5. Calculus
Topic	5.3 Integration
Difficulty	Hard

Time allowed: 100

Score: /79

Percentage: /100

Question 1a

A function f is defined by the equation $f(x) = -3x + 35$.

(a) Sketch the graph of $y = f(x)$ in the interval $0 \leq x \leq 10$.

[2 marks]

Question 1b

(b) Use your sketch from part (a), along with relevant area formulae, to work out the value of the integral

$$\int_1^9 (-3x + 35)\, dx$$

You should *not* use your GDC to find the value of the integral.

[3 marks]

Question 2

The derivative of the function f is given by
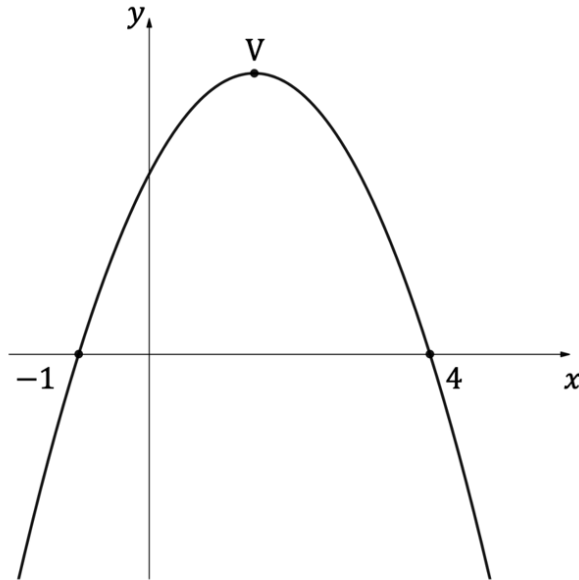
$$f'(x) = \frac{9}{2}x^2 + 7x - 2$$

and the curve $y = f(x)$ passes through the point $\left(-3, -\frac{11}{2}\right)$.

Find an expression for f.

[6 marks]

Question 3a

A curve $y = f(x)$ has the gradient function $f'(x) = 4ax + 6,$ where $a \in \mathbb{R}$ is a constant. The diagram below shows part of the curve, with the x and y intercepts labelled and where V represents the vertex of the curve.

(a) Find

    (i)   the value of a

    (ii) the equation of the curve $y = f(x)$

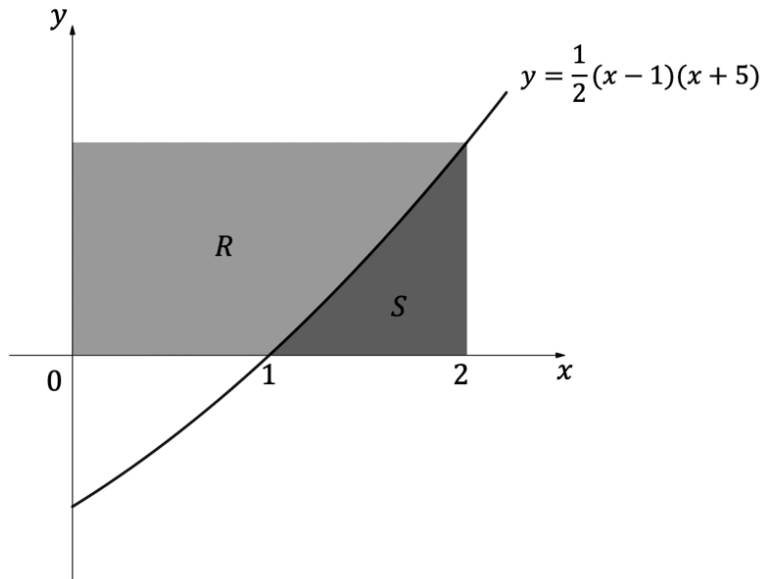   (iii) the coordinates of V.

[5 marks]

Question 3b

(b) Find the area between the curve and the x-axis.

[3 marks]

Question 4a

A section of the curve with equation $y = \frac{1}{2}(x-1)(x+5)$ is shown below:

The shaded region S in the diagram is bounded by the curve, the x-axis and the line $x = 2$.

(a) (i) Write down an integral for the area of the shaded region S.

(ii) Find the area of S. Give your answer as a fraction.

[3 marks]

Question 4b

The shaded region R in the diagram is bounded on three sides by the curve, the x-axis and the y-axis. The boundary on the fourth side is a straight line parallel to the x-axis, and that line, the curve and the line $x = 2$ all intersect at a single point.

(b) Find the area of region R. Give your answer as a fraction.

[3 marks]

Question 5a

A company is designing a plastic piece for a new game. The piece is to be in the form of a prism, with a cross-sectional area as indicated by the shaded region R in the following diagram:
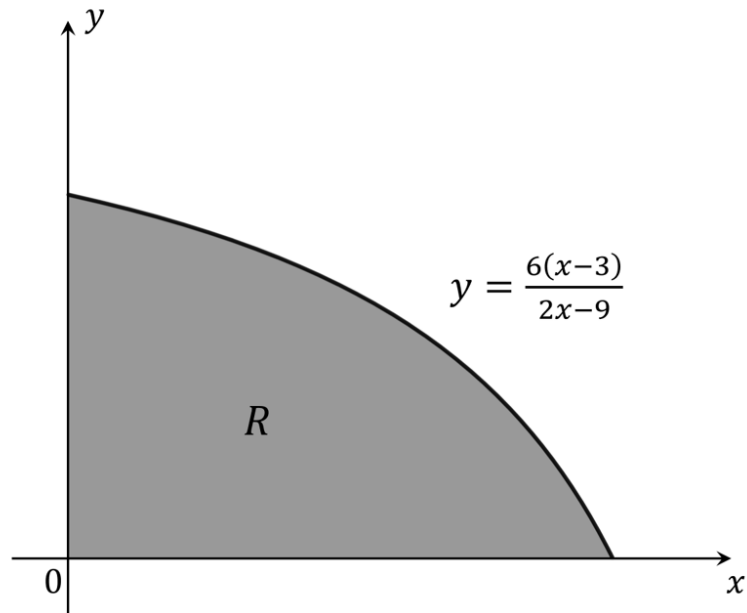
Region R is bounded, as shown, by the positive x- and y-axes and the curve with equation $y = \dfrac{6(x-3)}{2x-9}$. All units are in centimetres.

(a) Using technology, or otherwise, find the coordinates of the points of intersection of the curve with the x- and y-axes.
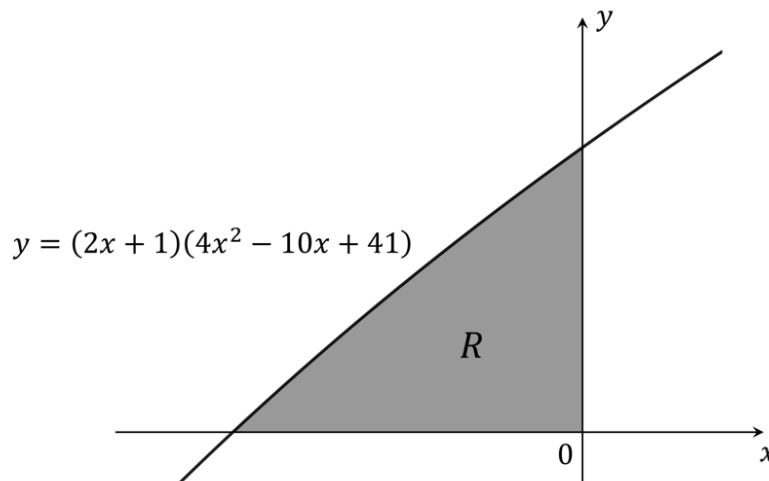
[2 marks]

Question 5b

The volume of the puzzle piece is to be 30 cm^3.

(b) Find the length of the puzzle piece, giving your answer correct to 3 significant figures.

[4 marks]

Question 6a

The following diagram shows part of the graph of $f(x) = (2x + 1)(4x^2 - 10x + 41)$, $x \in \mathbb{R}$. The shaded region R is bounded by the x-axis, the y-axis and the graph of f.
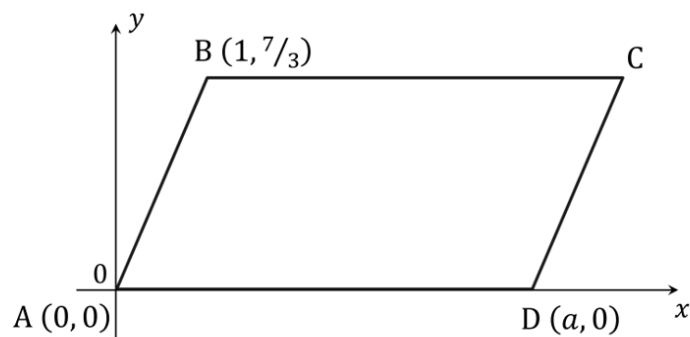

(a) (i) Write down an integral for the area of region R.

    (ii) Find the area of region R.

[3 marks]

Question 6b

ABCD is a parallelogram with vertices A(0,0), B $\left(1, \frac{7}{3}\right)$, C and D$(a, 0)$, as shown in the diagram below. The area of ABCD is equal to the area of region R above.

(b) By first finding the value of a, the x-coordinate of point D, determine the coordinates of point C. The coordinates should be given as exact fractions.

[3 marks]

Question 7a

A curve has the equation $y = -x^3 + 8x^2 - 13x + 6.$ Consider the area enclosed by the curve and the positive x-axis.

(a) Sketch the curve, shading the area indicated above.

[3 marks]

Question 7b

(b) Using the trapezoidal rule with 5 strips, determine an approximation for the shaded area.

[5 marks]

Question 7c

(c) Explain, using your sketch from part (a), why it is not possible to determine immediately whether your approximation will be an underestimate or an overestimate.

[2 marks]

Question 7d

(d) Using integration, determine the exact value of the shaded area.
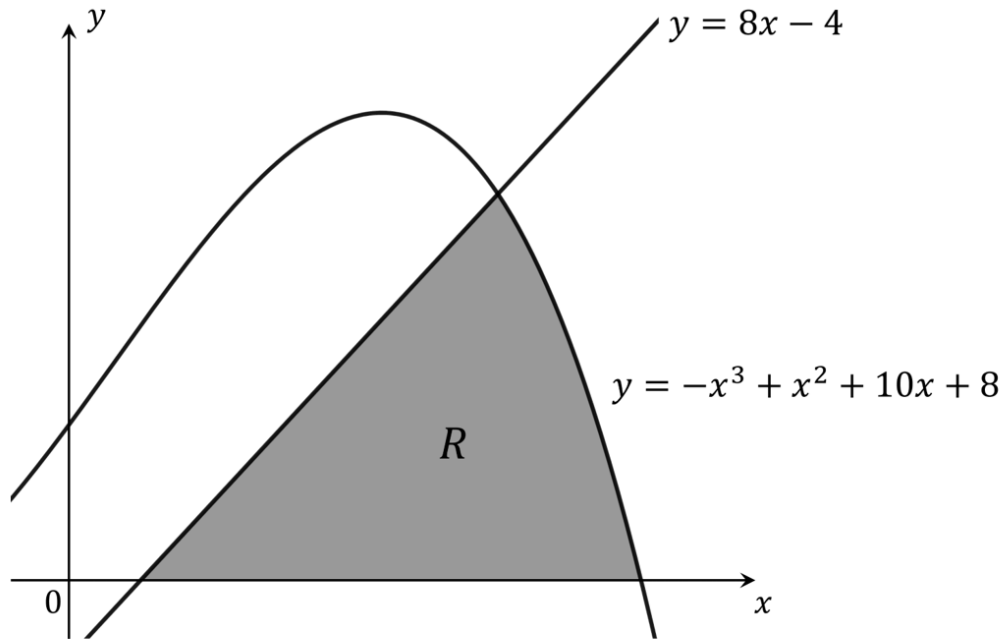
[3 marks]

Question 7e

(e) Find the percentage error of the approximation found in part (b), compared with the exact value.

[2 marks]

Question 8a

The shaded region R in the following diagram is bounded by the x-axis, the line $y = 8x - 4$ and the curve $y = -x^3 + x^2 + 10x + 8$.

(a) Using technology, or otherwise, find the coordinates of

   (i)   the point of intersection between the curve and the line

   (ii) the point of intersection between the line and the x-axis

   (iii) the point of intersection between the curve and the x-axis that is shown in the diagram.

[3 marks]

Question 8b

(b) Show that the area of region R is equal to exactly $\dfrac{439}{12}$ units^2. Be sure to show all of your working.

[6 marks]

Question 9a

For a particle P travelling in a straight line, the velocity, v m/s, of the particle at time t seconds is given by the equation

$$v(t) = 2t^2 - 8t + 9, \quad t \geq 0$$

(a) Sketch the graph of $v(t)$ in the interval $0 \leq t \leq 5$.

[2 marks]

Question 9b

The distance travelled between times t_1 and t_2 by a particle moving in a straight line may be found by finding the area beneath the particle's velocity-time graph between those two times.

(b) Find the distance travelled by the particle P between the times $t = 1$ and $t = 4.5$.

[3 marks]

Question 10a

After analysing several years of company data, a fast food company has determined that the rate of change of its sales figures can be modelled by the equation

$$\frac{dM}{dx} = -0.068x^3 + 0.72x^2 - 0.88x - 1.9, \quad 0 \le x \le 10$$

where M represents the number of meals sold in a week (in thousands of meals sold), and x represents the amount spent on advertising during the preceding week (in thousands of euros).

It is known as well that 5988 meals are sold in a week where 2000 euros had been spent on advertising during the preceding week.

(a) Find an expression for $M(x)$.

[6 marks]

Question 10b

(b) Find the maximum number of meals that the company can expect to sell in a week, and the amount of money that the company should spend on advertising during the preceding week to bring about that level of sales. Give your answers to the nearest meal sold and the nearest euro, respectively. Be sure to justify that the value you find is indeed a maximum.

[7 marks]