3.11 Vector Planes

Question Paper

Course	DP IB Maths
Section	3. Geometry & Trigonometry
Topic	3.11 Vector Planes
Difficulty	Medium

Time allowed: 90

Score: /69

Percentage: /100

Question 1a

A plane Π contains the point $\mathrm{A}(3, 9, -1)$ and has a normal vector $\begin{pmatrix} 4 \\ -2 \\ 2 \end{pmatrix}$.

a)
Find the equation of the plane in its Cartesian form.

[2 marks]

Question 1b

A second point B has coordinates $(-4, 1, -3)$.

b)
Determine whether point B lies on the same plane.

[2 marks]

Question 2a

A plane Π has equation $\boldsymbol{r} = \begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 5 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 2 \\ 7 \end{pmatrix}$.

A line with equation $\boldsymbol{r} = \begin{pmatrix} 6 \\ -2 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix}$ intersects Π at a point Q.

a)
Write down the equations of the line and the plane in their parametric forms.

[3 marks]

Question 2b

b)

Given that the coordinates of Q are $(10, -2, 4)$, find the values for β, λ and μ at the point of intersection.

[5 marks]

Question 3a

Consider the two planes Π_1 and Π_2 which can be defined by the equations

$$\Pi_1: x + 2y - z = 5$$

$$\Pi_2: -3x - y + 8z = 1$$

a)

Write down expressions for the normal vectors of each of the two planes.

[2 marks]

Question 3b

b)
Hence find the angle between the two planes. Give your answer in radians.

[5 marks]

Question 4a

The points A, B and C have position vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ respectively, relative to the origin O.

The position vectors are given by

$$\boldsymbol{a} = 2\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k}$$

$$\boldsymbol{b} = -\boldsymbol{i} + 2\boldsymbol{j} + 2\boldsymbol{k}$$

$$\boldsymbol{c} = \boldsymbol{i} - 4\boldsymbol{j} + 3\boldsymbol{k}$$

a)
Find the direction vectors $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{AC}}$.

[2 marks]

Question 4b

Points A, B and C all lie on a single plane.

b)
Use the results from part (a) to write down the vector equation of the plane.

[2 marks]

Question 4c

c)
Find the Cartesian equation of the plane.

[4 marks]

Question 5a

A plane lies parallel to the line with equation $\boldsymbol{r} = \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix} + \beta \begin{pmatrix} 3 \\ 9 \\ 1 \end{pmatrix}$ and contains the points P and X with coordinates $(5, 4, 5)$ and $(-2, 2, 0)$ respectively.

a)
Find the vector $\overrightarrow{\mathrm{PX}}$.

[2 marks]

Question 5b

b)
By appropriate use of the vector product, find the normal to the plane.

[2 marks]

Question 5c

c)
Hence find the Cartesian equation of the plane.

[2 marks]

Question 6a

Consider the plane defined by the Cartesian equation $5x - 3y - z = 13$.

a)
Show that the line with equation $\boldsymbol{r} = \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 4 \\ -7 \end{pmatrix}$ lies in the plane.

[3 marks]

Question 6b

b)

Show that the line with Cartesian equation $x-2=\frac{y-6}{2}=2-z$ is parallel to the plane but does not lie in the plane.

[3 marks]

Question 7a

Consider the planes Π_1, Π_2 and Π_3, which are defined by the equations

$$\Pi_1: 3x-5y+z=27$$

$$\Pi_2: -4x+y+2z=-10$$

$$\Pi_3: -2x-y-z=-1$$

a)

By solving the system of equations represented by the three planes show that the system of equations has a unique solution.

[3 marks]

Question 7b

b)

Hence write down the coordinates of any point(s) where all three planes intersect.

[1 mark]

Question 8a

Consider the line L with vector equation $\boldsymbol{r} = (1-\lambda)\boldsymbol{i} + (\lambda-2)\boldsymbol{j} + (3+2\lambda)\boldsymbol{k}$ and the plane Π with Cartesian equation $3x - 2y + z = 11$.

a)
Find the angle in radians between the line L and the normal to the plane Π.

[4 marks]

Question 8b

b)
Hence find the angle in radians between the line L and the plane Π.

[2 marks]

Question 9a

Two planes Π_1 and Π_2 are defined by the equations

$$\Pi_1 : 3x - 2y + 4z = 18$$

$$\Pi_2 : -2x + y + 2z = 7$$

a)
Write down expressions for the normal vectors of each of the two planes.

[2 marks]

Question 9b

b)
Find the cross product of the two normal vectors.

[2 marks]

Question 9c

c)
Find the coordinates of a point that lies on both planes.

[3 marks]

Question 9d

d)
Hence find a vector equation of the line of intersection of the two planes.

[2 marks]

Question 10a

A line L_1 is defined by the Cartesian equation $\frac{x}{3d+1} = \frac{y-3}{4} = 5 - z$ and a plane Π is defined by the Cartesian equation $-x + dy - 4z = -29$, where d is a real constant.

The line L_1 lies in the plane Π.

a)
Use the fact that the line L_1 lies in the plane Π to find the value of the constant d.

[4 marks]

Question 10b

Another line, L_2, passes through the origin and is perpendicular to the plane Π.

b)
Write down the equation of line L_2 in vector form.

[2 marks]

Question 10c

c)
By considering the parametric form of the equation for L_2, or otherwise, determine the point of intersection between line L_2 and the plane Π.

[3 marks]

Question 10d

d)
Hence determine the minimum distance between the plane Π and the origin.

[2 marks]