2.3 Functions Toolkit

Question Paper

Course	DP IB Maths
Section	2. Functions
Topic	2.3 Functions Toolkit
Difficulty	Medium

Time allowed: 120

Score: /92

Percentage: /100

Question 1a

The functions f and g are defined such that $f(x) = 4x - 10$ and $g(x) = \frac{x+8}{2}$.

(a) Show that $(g \circ f)(x) = 2x - 1$.

[2 marks]

Question 1b

(b) Given that $(g \circ f)(a) = 27$, find the value of a.

[2 marks]

Question 1c

(c) Show that $(f \circ g)(x) = 2x + 6$.

[2 marks]

Question 1d

(d) Given that $(f \circ g)(b) = 44$, find the value of b.

[2 marks]

Question 2a

The functions $f(x)$ and $g(x)$ are defined as follows

$$f(x) = x^2 \qquad x \in \mathbb{R}$$
$$g(x) = 4x - 3 \qquad x \in \mathbb{R}$$

(a) Write down the range of $f(x)$.

[1 mark]

Question 2b

(b) Find

(i) $(f \circ g)(x)$

(ii) $(g \circ f)(x)$

[4 marks]

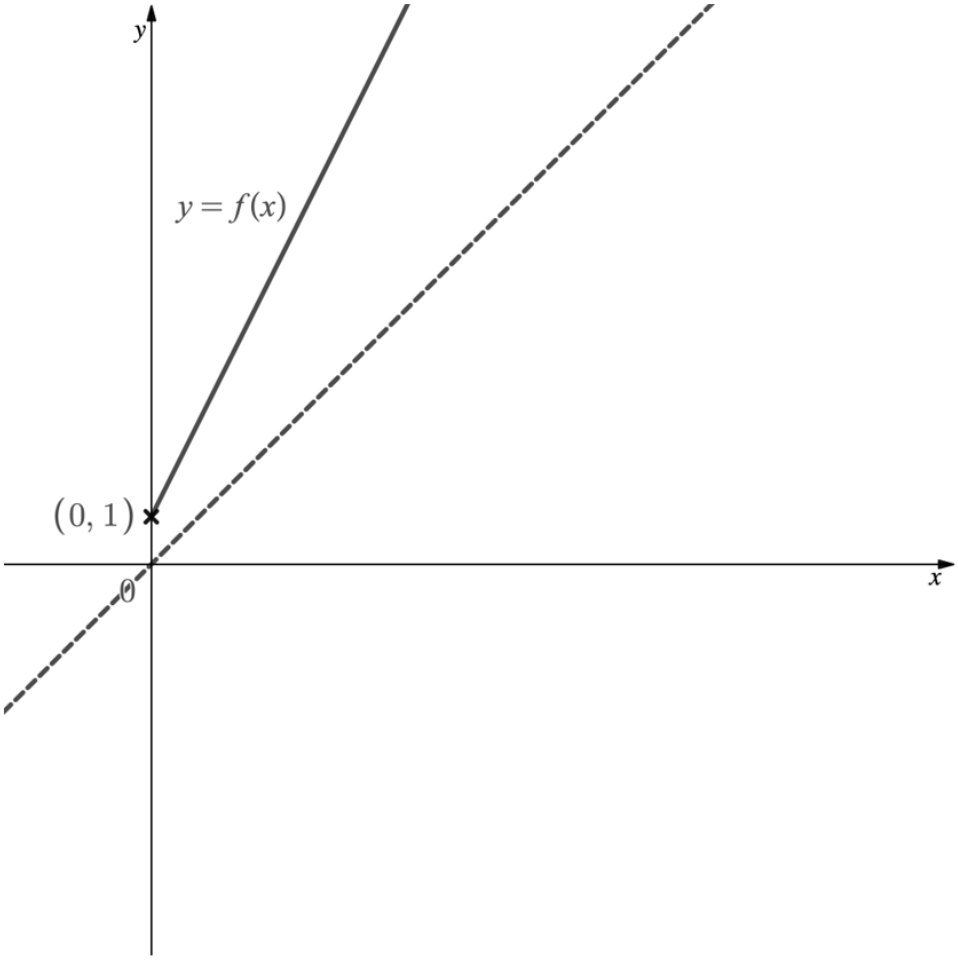
Question 2c

(c) Solve the equation $f(x) = g(x)$.

[2 marks]

Question 3a

The graph of $y = f(x)$ is shown below.



(a) (i) Use the graph to write down the domain and range of $f(x)$.

(ii) Given that the point (1, 1) lies on the dotted line, write down the equation of the line.

[3 marks]

Question 3b

(b) On the diagram above sketch the graph of $y = \mathrm{f}^{-1}(x)$.

[2 marks]

Question 4a

The function $\mathrm{f}(x)$ is defined as

$$\mathrm{f}: x \mapsto \frac{x^2+1}{x^2} \qquad x \in \mathbb{R},\ x \neq 0$$

(a) Show that $\mathrm{f}(x)$ can be written in the form

$$\mathrm{f}: x \mapsto 1 + \frac{1}{x^2}$$

[2 marks]

Question 4b

(b) Explain why the inverse of $\mathrm{f}(x)$ does not exist and suggest an adaption to its domain so the inverse does exist.

[2 marks]

Question 4c

(c) The domain of $f(x)$ is changed to $x > 0$.
Find an expression for $f^{-1}(x)$ and state its domain and range.

[4 marks]

Question 5a

The functions $f(x)$ and $g(x)$ are defined as follows

$$f(x) = \frac{1}{2}(4x - 3) \qquad x \in \mathbb{R}$$
$$g(x) = 0.5x + 0.75 \qquad x \in \mathbb{R}$$

(a) Find (i) $fg(x)$
(ii) $gf(x)$

[3 marks]

Question 5b

(b) Write down $f^{-1}(x)$ and state its domain and range.

[3 marks]

Question 6a

A function is defined by $f(x) = 54x - 13, \quad -2 < x < 20.$

(a) Find the value of $f\left(\frac{5}{2}\right)$.

[1 mark]

Question 6b

(b) Write down the range of $f(x)$.

[2 marks]

Question 6c

(c) Find the inverse function $f^{-1}(x)$.

[2 marks]

Question 6d

(d) Write down the range of the inverse function.

[1 mark]

Question 7a

Consider the function $f(x) = -6x - 3$. The domain of $f(x)$ is $-5 \leq x \leq 3$.

(a) Find

(i) $f(2)$

(ii) x when $f(x) = 15$.

[2 marks]

Question 7b

(b) Find the range of $f(x)$.

[3 marks]

Question 7c

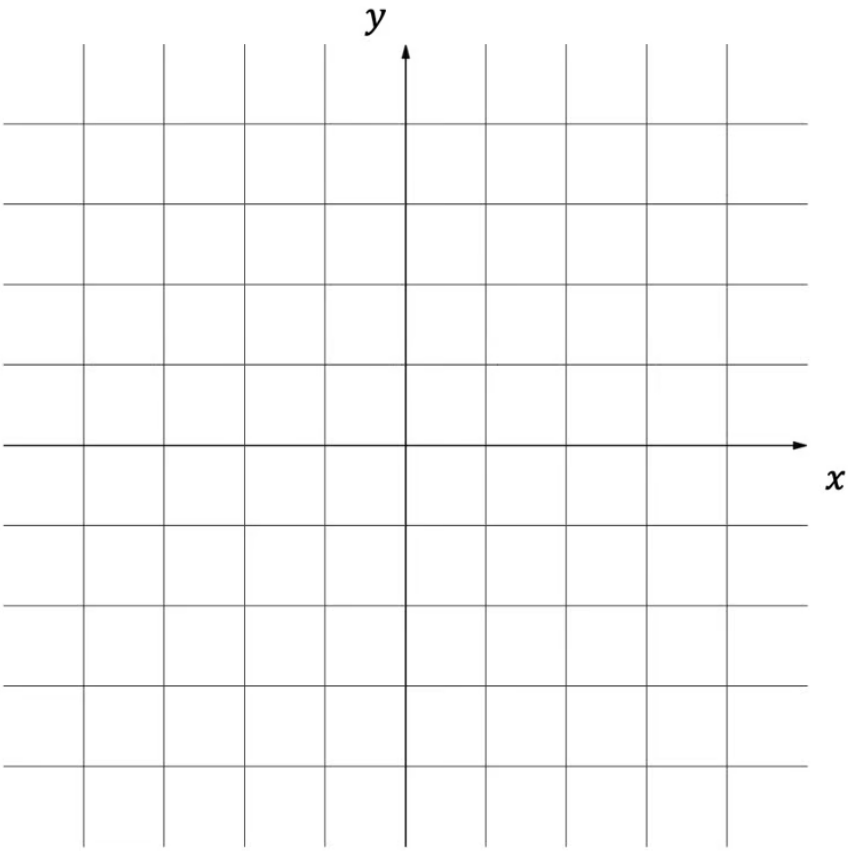
(c) Write down the domain of the inverse function.

[3 marks]

Question 8a

Consider the function $g(x) = \sqrt{4 - x}$.

(a) Sketch the graph of the function $g(x)$, labelling the x and y intercepts.



[3 marks]

Question 8b

(b) Find

(i) $g(-5)$

(ii) x when $g(x) = \frac{1}{2}$.

[2 marks]

Question 8c

(c) Find

(i) the maximum possible domain of the function $g(x)$

(ii) the range of the function $g(x)$ that corresponds to the domain found in part (c) (i).

[2 marks]

Question 9a

The functions f and g are defined for $x \in R$ by $f(x) = 3x^2 + 10x + 7$ and $g(x) = x + d$, where $d \in R$.

(a) Find the range of f.

[2 marks]

Question 9b

(b) Given that $(g \circ f)(x)$ is always positive for all x, determine the set of possible values for d.

[4 marks]

Question 10a

Let $f(x) = \frac{2x-5}{x+8}$, where $x \neq a, x \in \mathbb{R}$.

(a) Write down

(i) the value of a

(ii) the range of f.

[2 marks]

Question 10b

(b) For the graph of f, find the equations of all the asymptotes.

[1 mark]

Question 10c

(c) Find $f^{-1}(x)$.

[2 marks]

Question 10d

(d) For the graph of f^{-1}, find the equation of

(i) the horizontal asymptote

(ii) the vertical asymptote.

[2 marks]

Question 11

Determine, for each of the following functions, whether they are even, odd or neither:

(i) $f(x) = \frac{1}{x^2} + 2$

(ii) $g(x) = x^3 - 3x$

(iii) $h(x) = x^2 + 2x - 5.$

[5 marks]

Question 12

Prove that the sum of two odd functions is also an odd function.

[5 marks]

Question 13a

Let $f(x) = \frac{\pi^2}{x}$, where $x \neq 0, x \in \mathbb{R}$.

(a) Show that $f(x)$ is a self-inverse function.

[2 marks]

Question 13b

Let $g(x) = \frac{-x-2}{5x+1}$, where $x \neq p, x \in \mathbb{R}$.

(b) Find the value of p.

[1 mark]

Question 13c

(c) Show that $g(x)$ is a self-inverse function.

[3 marks]

Question 14a

Consider the function f defined by $f(x) = 2x^3 + 3x^2 - 36x + 7,\ x \in \mathbb{R}$.

(a) Sketch the graph of f. Clearly label the points where the graph intersects the axes, along with any points that are local maxima or minima.

[2 marks]

Question 14b

Let the function g be defined by $g(x) = 2x^3 + 3x^2 - 36x + 7,\ x \leq p$.

(b) Given that g has an inverse:

(i) Find the largest possible value of p

(ii) Find the domain of g^{-1} for the value of p identified in part (b)(i)

(iii) Find the value of $g^{-1}(0)$.

[3 marks]

Question 14c

Let the function h be defined by $h(x) = 2x^3 + 3x^2 - 36x + 7,\ x \geq q$.

(c) Given that h has an inverse:

(i) Find the smallest possible value of q

(ii) Find the domain of h^{-1} for the value of q identified in part (c)(i)

(iii) Find the value of $h^{-1}(0)$.

[3 marks]