4.12 Further Hypothesis Testing

Question Paper

Course	DP IB Maths
Section	4. Statistics & Probability
Topic	4.12 Further Hypothesis Testing
Difficulty	Hard

Time allowed: 100

Score: /82

Percentage: /100

Question 1

In order to test the hypotheses $H_0: \mu = 330,\ H_1: \mu > 330$, a sample of size 12 is taken from a normally distributed population.

The mean of the sample is found to be 332.1, and the standard deviation of the sample is found to be 2.7.

Using the *p*-value from a *t*-test, test at the 5% significance level whether the sample is from a population with a mean of 330, or from one with a larger mean.

[5 marks]

Question 2a

George owns an ice cream store and his previous records suggest that only 15% of customers like his famous chilli chocolate flavour ice cream. To increase the proportion of customers who like the chilli chocolate flavour ice cream, George increases the amount of chilli in the ice cream. George wants to test, using a 5% level of significance, whether the extra chilli has increased the proportion of customers who like his chilli chocolate flavour ice cream.

a)
Write down null and alternative hypotheses for George's test.

[2 marks]

Question 2b

George takes a random sample of 120 customers and gives them a sample of his new chilli chocolate ice cream.

b)
Find the probability that George will make a Type I error in his test conclusion.

[4 marks]

Question 2c

George finds that 25 out of 120 customers liked the new chilli chocolate flavoured ice cream.

c)
State George's conclusion to the test. Justify your answer.

[2 marks]

Question 2d

d)
State, with a reason, which error, Type I or Type II, George can be certain that he did not make in his conclusion.

[2 marks]

Question 3a

Midge is playing an online game about dragons where the player scores bonus points if a purple dragon appears. One day Midge notices that the game has been updated and she suspects that the purple dragons now appear less frequently. Before the update, it was known that the number of purple dragons appearing in a 10–minute period could be modelled by a Poisson distribution with mean 3.2. To test her suspicion, Midge uses a 10% significance level and plays the game for 60 minutes to see how many purple dragons appear.

a)
(i)
Write down null and alternative hypotheses to test Midge's suspicion.
(ii)
Find the critical value for Midge's test. Justify your answer.

[5 marks]

Question 3b

The game designers did reduce the mean rate at which purple dragons appeared to a mean of 2.5 times in a 10–minute period.

b)
Find the probability that Midge will make a Type II error in her test conclusion.

[2 marks]

Question 3c

After Midge plays the game for 60 minutes, she finds that 13 purple dragons appeared.

c)
State Midge's conclusion to her test. Justify your answer.

[2 marks]

Question 3d

d)
Midge repeats this test 100 times in total. Find the expected number of times that Midge makes a Type II error in her test conclusion.

[1 mark]

Question 4a

Rachel, Nathan and Hope are three siblings who raise female cows together on a dairy farm. The masses of female cows follow a normal distribution with standard deviation 29.4 kg. It is claimed that the average mass of a female cow is 723 kg. The siblings have 20 cows on their farm.

The three siblings have different beliefs about their cows so they each use a hypothesis test with the same null hypothesis, $H_0 : \mu = 723$. Rachel believes that their cows weigh less than the average and she tests her belief using a 5% significance level. Nathan believes that their cows weigh more than the average and will reject the null hypothesis if the average mass of their cows is more than 732 kg. Hope believes that the average mass of their cows is different to the claimed average and she tests her belief using a 5% significance level.

a)
(i)
Find the critical region for Rachel's test.
(ii)
Find the probability that Nathan's test will lead to a Type I error.

[4 marks]

Question 4b

The three siblings find that the total mass of their cows is 14221 kg.

b)
Show, giving reasons, that only one of the siblings reject the null hypothesis.

[5 marks]

Question 5a

Lucinda is investigating the petrol prices at stations in a town in North Carolina. Lucinda visits 9 random petrol stations and records the price of petrol per gallon, p USD, and the distance between that station and the next closest station, d miles. The results are shown in the table below.

Price (p USD)	3.92	3.98	4.03	3.89	4.23	3.86	3.99	4.11	3.74
Distance (d miles)	2.4	2.9	5.1	3.0	7.3	2.5	3.1	5.0	1.6

Lucinda believes there is a linear relationship between the two variables. Lucinda uses a hypothesis test with a 5% significance level to test for linear correlation.

a)
(i) State why the hypothesis test should be two-tailed.
(ii)
State the null and alternative hypotheses for this test.
(iii)
Determine whether there is significant evidence of a linear correlation between the price of petrol per gallon and the distance between the station and the next closest station.

[5 marks]

Question 5b

b)
Describe one way in which Lucinda could improve the reliability of her test.

[1 mark]

Question 5c

Lucinda calculates the equation for the least squares regression line of p on d. She uses the equation to predict the price of a gallon of petrol at a station given that the next closest station is 6.5 miles away.

c)
State, with a reason, whether Lucinda's prediction is valid.

[2 marks]

Question 6a

Brock sells bags of sand which he claims weigh on average 10 kg. It is known that the mass, in kg, of a bag of sand can be modelled using a normal distribution with variance 0.215. An inspector takes a sample of 8 bags of sand to test whether the average mass differs significantly from the average mass claimed by Brock. The masses, in kg, of the 8 bags are shown below.

9.89	10.01	9.74	9.63	10.11	9.61	9.25	10.04

The inspector uses a 5% significance level for his test.

a)
(i) State, with a reason, the name of the test the inspector should use.
(ii)
Write down the null and alternative hypotheses for the inspector's test.

[3 marks]

Question 6b

b)
State the conclusion of the inspector's test. Justify your answer.

[3 marks]

Question 6c

The inspector repeats the test a week later using another sample of 8 bags. Brock is considering reducing the significance level to 1%.

c)
Explain how reducing the significance level would affect:
(i)
the probability of the inspector making a Type I error in his conclusion.
(ii)
the probability of the inspector making a Type II error in his conclusion.

[3 marks]

Question 7a

Gina, a member of the US gymnastic committee, is interested in analysing the gymnasts' results to support future development of the team. In order to analyse the scoring of two gymnastic disciplines, the vault and the horizontal bar, she collated the scores from 10 competitors in both events. The results are shown in the table below.

Competitor	Vault	Horizontal Bar
A	13.932	14.008
B	12.648	13.565
C	13.200	13.738
D	14.102	14.252
E	13.883	13.821
F	12.100	12.278
G	13.214	13.677
H	12.722	13.459
I	13.040	13.610
J	13.175	13.668

Gina carries out a hypothesis test, using a 5% significance level, on the correlation coefficient to investigate whether an increase in the vault score is associated with an increase in the horizontal bar.

a)
(i) Explain why the hypothesis test should be one-tailed.
(ii)
Write down the null and alternative hypotheses for this test.
(iii)
Calculate the value of r, Pearson's product-moment correlation coefficient, for these scores.

[5 marks]

Question 7b

b)
The critical value for this test is 0.5494. Determine whether there is significant evidence of a positive linear correlation between scores in the vault and the horizontal bar. Justify your answer.

[2 marks]

Question 8a

Chloe, a gym instructor, has two training programs for her clients: *The Sweat Inducer* and *The Calorie Destroyer*. At the start of a week, Chloe picks one of the programs and uses that program for all seven sessions that week, however she does not tell her clients which one she picked. It is known that the times taken to complete each program can be modelled using normal distributions with standard deviation 8 minutes. The average time for *The Sweat Inducer* is 60 minutes and the average time for *The Calorie Destroyer* is 70 minutes.

Lara, a keen statistician, attends Chloe's sessions once each day for a week. Lara knows Chloe will use the same program each day and wants to determine which one it is. Lara uses the seven sessions in the week as a random sample to test the null hypothesis $H_0 : \mu = 60$ against the alternative hypothesis $H_1 : \mu = 70$. Lara will calculate the mean time for the seven sessions and will reject the null hypothesis if it is greater than 63 minutes.

a)
(i) Explain what a Type I error is in the context of the question.
(ii)
Find the probability that Lara makes a Type I error in her test conclusion.

[4 marks]

Question 8b

b)
(i)
Explain what a Type II error is in the context of the question.
(ii)
Find the probability that Lara makes a Type II error in her test conclusion.

[4 marks]

Question 9a

Jars are filled with a large number of jelly bears, the colour of each jelly bear is random. Simon's favourite colour is red and through extensive research he has found that the probability of picking a red jelly bear from a full jar is 0.13. A new company produces the jelly bears and Simon suspects that the proportion of red jelly bears has decreased. Simon uses a hypothesis test with a 5% significance level to test his suspicion.

a)
State the null and alternative hypotheses for Simon's test.

[2 marks]

Question 9b

Simon takes a random sample of 20 jelly bears.

b)
Show that this test will never produce a Type I error.

[3 marks]

Question 9c

Simon decides his sample was too small, so he repeats his test using a bigger sample with the same significance level. Out of a total of 200 jelly bears, Simon finds that 17 of them are red.

c)
By finding the *p*-value for the test, determine the conclusion of Simon's test.

[3 marks]

Question 10a

Pizza Prince is a fast-food restaurant which is known for its Crown pizza. The weights of Crown pizzas are normally distributed with standard deviation 42 g. It is claimed that the mean weight, μ, is 350 g.

The restaurant receives a complaint from a customer who claims that the average weight of a Crown pizza is less than 350 g. Imogen, an independent inspector, investigates the customer's claim by visiting the restaurant over a week and randomly sampling five Crown pizzas. Imogen uses the data to test the null hypothesis $H_0 : \mu = 350$ against the alternative hypothesis $H_1 : \mu < 350$ using a 5% significance level.

When Imogen writes up her report, she can only find the values for four of the weights, these are shown below.

325.2 356.1 319.7 300.5

a)
Imogen remembers that there was insufficient evidence to reject the null hypothesis using a 5% significance level. Calculate the minimum possible value for the missing weight, w. Give your answer correct to 1 decimal place.

[4 marks]

Question 10b

b)
Imogen also remembers that the result of the hypothesis test would have been different if she had used a 10% significance level. Using this information, write down an inequality for w. Give the endpoints correct to 1 decimal place.

[4 marks]