

5.12 Further Limits (inc l'Hôpital's Rule)

Question Paper

Course	DPIB Maths
Section	5. Calculus
Topic	5.12 Further Limits (inc l'Hôpital's Rule)
Difficulty	Medium

Time allowed: 60
Score: /49
Percentage: /100

Question 1a

For each of the following limits,

(i) determine whether or not l'Hôpital's rule may be used to evaluate the limit, giving a reason for your answer; and

(ii) if l'Hôpital's rule may be used, then use the rule to evaluate the limit.

a)

$$\lim_{x \rightarrow 0} \frac{\sin x}{x^2 + 2x}$$

[4 marks]

Question 1b

b)

$$\lim_{x \rightarrow 0} \frac{\cos x}{x^2 + 2x}$$

[2 marks]

Question 1c

c)

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\sec x}{1 - \tan x}$$

[5 marks]

Question 2a

Consider the following limit:

$$\lim_{x \rightarrow 0} \frac{-1 + \cos 2x}{x^2}$$

a)

Explain why it is appropriate to use l'Hôpital's rule to attempt to evaluate this limit.

[2 marks]

Question 2b

b)

Show that employing l'Hôpital's rule once leads to an indeterminate form when you attempt to evaluate the limit.

[2 marks]

Question 2c

c)

By employing l'Hôpital's rule a second time, show that the limit exists and find its value.

[2 marks]**Question 3a**Consider the function f defined by

$$f(x) = \frac{7-3x}{12x+5}$$

a)

Use l'Hôpital's rule to evaluate $\lim_{x \rightarrow \infty} f(x)$.**[3 marks]****Question 3b**

b)

Hence write down the equation(s) of any horizontal asymptotes on the graph of $y = f(x)$, giving a reason for your answer.**[2 marks]**

Question 3c

c)

(i)

Show that $f(x)$ may be rewritten in the form

$$f(x) = \frac{\frac{7}{x} - 3}{12 + \frac{5}{x}}$$

(ii)

Hence show that $\lim_{x \rightarrow \infty} f(x)$ may also be evaluated without the use of l'Hôpital's rule.**[4 marks]****Question 4a**

a)

By substituting $-x$ into the Maclaurin series for e^x , determine the Maclaurin series for e^{-x} .**[2 marks]**

Question 4b

Consider the limit

$$\lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{2x}$$

b)

Use Maclaurin series to evaluate the limit.

[5 marks]**Question 4c**

c)

(i)

Show that it would also be appropriate to use l'Hôpital's rule to attempt to evaluate the limit.

(ii)

Evaluate the limit using l'Hôpital's rule, and confirm that this matches your answer in part (b).

[4 marks]

Question 5a

a)

Find the Maclaurin series for $\cos 2x$.

[3 marks]

Question 5b

b)

Hence evaluate the limit

$$\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2}$$

[4 marks]

Question 6

Use an appropriate method to evaluate the limit

$$\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3}$$

[5 marks]