5.12 Further Limits (inc l'Hôpital's Rule)

Question Paper

Course	DP IB Maths
Section	5. Calculus
Topic	5.12 Further Limits (inc l'Hôpital's Rule)
Difficulty	Very Hard

Time allowed: 70

Score: /57

Percentage: /100

Question 1a

For each of the following limits,

(i)
determine whether or not l'Hôpital's rule may be used to evaluate the limit, giving a reason for your answer; and
(ii)
if l'Hôpital's rule may be used, then use the rule to evaluate the limit.
(a)

$$\lim_{x \to 0} \frac{x(x+1)^{17}}{(x+1)^{17}-1}$$

[4 marks]

Question 1b

(b)

$$\lim_{x \to 0} \frac{\cos x}{\arctan x}$$

[2 marks]

Question 1c

(c)

$$\lim_{x \to +\infty} \frac{\ln x}{\sqrt{x}}$$

[4 marks]

Question 2a

Consider the following limit:

$$\lim_{x \to 0} \frac{e^{x^2} - 1}{1 - \cos 3x}$$

(a)
Explain why it is appropriate to use l'Hôpital's rule to attempt to evaluate this limit.

[2 marks]

Question 2b

(b)
Use l'Hôpital's rule to evaluate the limit.

[5 marks]

Question 3a

Consider the function f defined by

$$f(x) = \frac{9 - 4e^x}{3e^x + 7}$$

(a)
(i)
Use l'Hôpital's rule to evaluate $\lim_{x \to +\infty} f(x)$.
(ii)
Find $\lim_{x \to -\infty} f(x)$.

[5 marks]

Question 3b

(b)
Hence write down the equation(s) of any horizontal asymptotes on the graph of $y = f(x)$, giving a reason for your answer.

[3 marks]

Question 3c

(c)

Show that $\lim_{x \to +\infty} f(x)$ may also be evaluated without the use of l'Hôpital's rule, and confirm that the limit found matches the answer from part (a)(i).

[3 marks]

Question 4a

(a)

Find the Maclaurin series for $\arctan(\sin x)$, giving the terms and coefficients explicitly in ascending powers of x up through the term in x^5.

[5 marks]

Question 4b

Consider the limit

$$\lim_{x \to 0} \frac{\arctan(\sin x)}{x}$$

(b)
Use Maclaurin series to evaluate the limit.

[3 marks]

Question 4c

(c)
(i)
Show that it would also be appropriate to use l'Hôpital's rule to attempt to evaluate the limit.

(ii)
Evaluate the limit using l'Hôpital's rule, and confirm that this matches your answer in part (b).

[4 marks]

Question 5

Use Maclaurin series to evaluate the limit

$$\lim_{x \to 0} \frac{\sin px + x \cos^2 qx}{x}$$

where $p, q \in \mathbb{R}$ are non-zero constants. Give your answer in terms of p and q as appropriate.

[6 marks]

Question 6

Use an appropriate method to show that

$$\lim_{x \to +\infty} \frac{x^n}{e^x} = 0$$

for any $n \in \mathbb{N}$.

[6 marks]

Question 7

Use the fact that $x = \frac{1}{1/x}$ to evaluate the following limit:

$$\lim_{x \to 0^+} x \ln x$$

Be sure to justify the validity of any method you use to determine the limit.

[5 marks]