2.9 Further Functions & Graphs

Question Paper

Course	DP IB Maths
Section	2. Functions
Topic	2.9 Further Functions & Graphs
Difficulty	Medium

Time allowed: 80

Score: /63

Percentage: /100

Question 1a

a)
Sketch the graph of $y = (x-1)^2 - 2|x-1| - 1$, for $-3 \leq x \leq 6$.

[3 marks]

Question 1b

b)
Hence, solve the equation $y = (x-1)^2 - 2|x-1| - 1 = 0$.

[2 marks]

Question 2

Given that

$$f(x) = \ln x, \qquad x > 0$$

sketch on separate axes the graphs of

(i)
$y = f(x)$

(ii)
$y = |f(x)|$

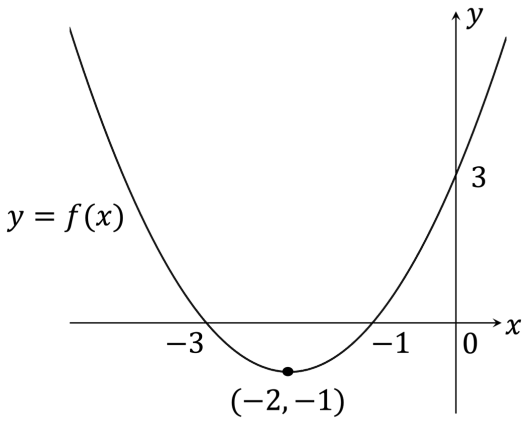
(iii)
$y = -f(x-3)$

On each diagram, show the x-intercepts along with any asymptotes, including their equations.

[7 marks]

Question 3a

The graph of $y = f(x)$ is given below.



On separate axes, draw the graphs of

a)
$|f(x)|$

[3 marks]

Question 3b

b)

$[f(x)]^2$

[3 marks]

Question 4a

a)

Sketch the curve $y = \frac{3}{x+4}$ and line $y = 4 - x$ on the same axes, clearly indicating any x- and y- intercepts and any asymptotes.

[3 marks]

Question 4b

b)
Consider the equation

$$4 - x = \left| \frac{3}{x+4} \right|$$

(i)
Explain why the cases $x < -4$, $x = -4$ and $x > -4$ must be considered separately in attempting to solve the equation.

(ii)
Hence find the exact solutions to the equation.

[5 marks]

Question 5a

Consider the function f defined by $f(x) = 3x^2 \arcsin x, -1 \leq x \leq 1.$

a)
Sketch the graph of $y = f(x).$

[3 marks]

Question 5b

b)
State the range of f.

[2 marks]

Question 5c

c)
Solve the inequality $|3x^2\arcsin x| > 1$.

[3 marks]

Question 6a

Consider the function f defined by $f(x) = \sqrt{9 - x}$, where f has the largest possible valid domain.

a)
(i)
Sketch the graph of $y = f(x)$, labelling the x- and y-intercepts.

(ii)
State the domain and range of f.

[4 marks]

Question 6b

b)
(i)
On the same set of axes, sketch the graph of the function $f(|x|)$, labelling the x- and y-intercepts.

(ii)
State the domain and range of the function $f(|x|)$.

[4 marks]

Question 7a

Let $f(x) = \frac{7-9x}{cx-12}$, $x \neq \frac{12}{c}$, where c is a non-zero constant.

The line $x = 4$ is a vertical asymptote to the graph of $y = f(x)$.

a)
(i)
Find the value of c.

(ii)
State the equation of the horizontal asymptote to the graph of $y = f(x)$.

[4 marks]

Question 7b

b)
The line $y = k$, where $k \in \mathbb{R}$, intersects the graph of $y = |f(x)|$ at exactly one point. Find the possible values of k.

[3 marks]

Question 8a

Let $f(x) = 2x^3 - 2x$, for $x \in \mathbb{R}$.

(a)
(i)
Sketch the graph of $y = |f(x)|$.

(ii)
State the transformation of the graph $y = f(x)$ to $y = |f(x)|$ for $f(x) < 0$.

[3 marks]

Question 8b

(b)

(i)

Sketch the graph of $y = f(|x|)$.

(ii) State the transformation of the graph $y = f(x)$ to $y = f(|x|)$ for $x < 0$.

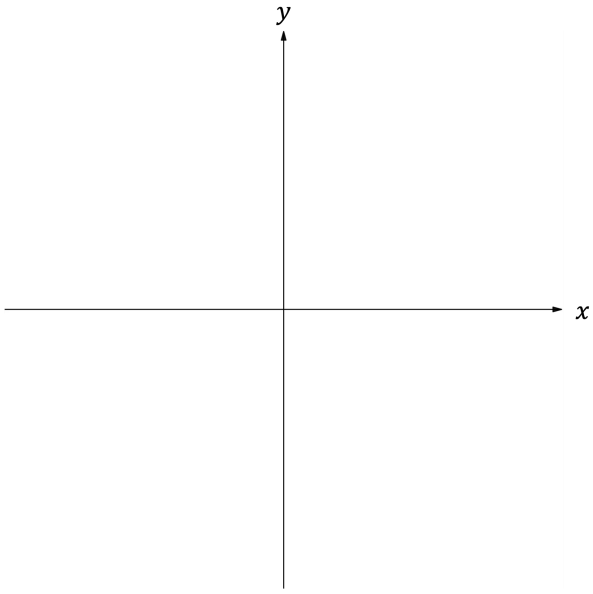
[3 marks]

Question 9a

Let $f(x) = x(x-2)$.

(a)

Sketch the graph of $y = f(x)$ on the coordinate axes below. Be sure to label anywhere the graph intersects the coordinate axes and any extrema.



[3 marks]

Question 9b

(b)

On the same axes, sketch the graph of the reciprocal $y = \frac{1}{f(x)}$. Be sure to label anywhere the graph intersects the coordinate axes and any extrema.

[3 marks]

Question 9c

(c)

Find the equation of the horizontal and vertical asymptotes of the graph of $y = f(x)$.

[2 marks]