5.7 Basic Limits & Continuity

Question Paper

Course	DP IB Maths
Section	5. Calculus
Topic	5.7 Basic Limits & Continuity
Difficulty	Medium

Time allowed: 60

Score: /43

Percentage: /100

Question 1a

For each of the following, either show that the limit converges and find its value, or else explain why the limit diverges:

a)

$$\lim_{x \to 4} \frac{1}{x^2 - 9}$$

[2 marks]

Question 1b

b)

$$\lim_{x \to 3} \frac{1}{x^2 - 9}$$

[2 marks]

Question 1c

c)

$$\lim_{x \to 3} \frac{x - 3}{x^2 - 9}$$

[3 marks]

Question 2a

a)
Evaluate the limit

$$\lim_{x \to -\infty} \left(13 - \frac{619}{x^2}\right)$$

justifying your answer by clear mathematical reasoning.

[2 marks]

Question 2b

b)
Show that the limit

$$\lim_{x \to +\infty} \frac{3x^2 - 5x + 7}{x^2}$$

converges, and find its value. Be sure to show clear algebraic working.

[3 marks]

Question 3a

A student has attempted to evaluate the limit

$$\lim_{x \to +\infty} (x^3 - x)$$

as follows:

$$\lim_{x \to +\infty} (x^3 - x) = (+\infty)^3 - (+\infty) = (+\infty) - (+\infty) = 0$$

a)
Explain what is wrong with the student's work.

[2 marks]

Question 3b

b)
Determine the correct evaluation of the limit, justifying your answer by clear mathematical reasoning.

[2 marks]

Question 3c

c)
Use technology to help you sketch the graph of $y = x^3 - x$, and show that the graph confirms your answer to part (b).

[2 marks]

Question 4a

Consider the function defined by

$$f(x) = \frac{1}{x^2}$$

a)
Evaluate the limits

(i)
$\lim_{x \to 0^-} f(x)$

(ii)
$\lim_{x \to 0^+} f(x)$

[3 marks]

Question 4b

b)
Evaluate the limits

(i)
$\lim_{x \to -\infty} f(x)$

(ii)
$\lim_{x \to +\infty} f(x)$

[3 marks]

Question 4c

c)
Use your results from parts (a) and (b) to write down the equations of any asymptotes on the graph of $y = f(x)$.

[2 marks]

Question 4d

d)
Use technology to help you sketch the graph of $y = f(x)$, and show that this confirms your results from parts (a), (b) and (c).

[2 marks]

Question 5a

Consider the function g defined by

$$g(x) = \frac{1}{x-5}$$

a)
Evaluate the limits

(i)
$\lim_{x \to 5^-} g(x)$

(ii)
$\lim_{x \to 5^+} g(x)$

[3 marks]

Question 5b

b)
Evaluate the limits

(i)
$\lim_{x \to -\infty} g(x)$

(ii)
$\lim_{x \to +\infty} g(x)$

[3 marks]

Question 5c

c)
Use your results from parts (a) and (b) to write down the equations of any asymptotes on the graph of $y = g(x)$.

[2 marks]

Question 5d

d)
Use technology to help you sketch the graph of $y = g(x)$, and show that this confirms your results from parts (a), (b) and (c).

[2 marks]

Question 6a

a)
The function f is a piecewise function defined by

$$f(x) = \begin{cases} x^2 \ , \ x \leq 2 \\ x+3, \ x > 2 \end{cases}$$

Explain why f is not continuous at $x = 2$.

[3 marks]

Question 6b

b)
A function g is defined for all $x \in \mathbb{R}$, and it is differentiable at all points $x \in \mathbb{R}$.

Explain why g is continuous at $x = 7$.

[2 marks]