5.7 Basic Limits & Continuity

Question Paper

Course	DP IB Maths
Section	5. Calculus
Topic	5.7 Basic Limits & Continuity
Difficulty	Hard

Time allowed: 60

Score: /46

Percentage: /100

Question 1a

For each of the following, either show that the limit converges and find its value, or else explain why the limit diverges:

(a)

$$\lim_{x \to \frac{2}{5}} \frac{1}{4x^2 - 25}$$

[2 marks]

Question 1b

(b)

$$\lim_{x \to \frac{5}{2}} \frac{1}{4x^2 - 25}$$

[2 marks]

Question 1c

(c)

$$\lim_{x \to \frac{5}{2}} \frac{2x - 5}{4x^2 - 25}$$

[3 marks]

Question 2a

(a)

Evaluate the limit

$$\lim_{x \to -\infty} \left(\frac{5 + (2x - 3)^2}{2x^2} \right)$$

justifying your answer by clear mathematical reasoning.

[3 marks]

Question 2b

(b)

(i)

Show that the limit

$$\lim_{x \to +\infty} \frac{x^2 - x + 3}{x}$$

diverges. Be sure to show clear algebraic working.

(ii)

Determine any asymptotes on the graph of the curve with equation

$$y = \frac{x^2 - x + 3}{x}$$

[4 marks]

Question 3a

A student has attempted to evaluate the limit

$$\lim_{x \to +\infty}\left(\frac{x^2+x}{x^3}\right)$$

as follows:

$$\lim_{x \to +\infty}\left(\frac{x^2+x}{x^3}\right)=\frac{(+\infty)^2+(+\infty)}{(+\infty)^3}=\frac{+\infty}{+\infty}=1$$

(a)
Explain what is wrong with the student's work.

[2 marks]

Question 3b

(b)
Determine the correct evaluation of the limit, justifying your answer by clear mathematical reasoning.

[2 marks]

Question 3c

(c)

Use technology to help you sketch the graph of $y=\frac{x^2+x}{x^3}$, and show that the graph confirms your answer to part (b).

[2 marks]

Question 4a

Consider the function f defined by

$$f(x) = \frac{1}{(2x+6)^2}$$

(a)

Evaluate the limits

(i)

$$\lim_{x \to -3^-} f(x)$$

(ii)

$$\lim_{x \to -3^+} f(x)$$

[3 marks]

Question 4b

(b)

Evaluate the limits

(i)

$$\lim_{x \to -\infty} f(x)$$

(ii)

$$\lim_{x \to +\infty} f(x)$$

[3 marks]

Question 4c

(c)

Use your results from parts (a) and (b) to write down the equations of any asymptotes on the graph of $y = f(x)$.

[2 marks]

Question 4d

(d)

Use technology to help you sketch the graph of $y = f(x)$, and show that this confirms your results from parts (a), (b) and (c).

[2 marks]

Question 5a

Consider the function g defined by

$$g(x) = \frac{1}{x^3 - 8} + 1$$

(a)

Evaluate the limits

(i)

$$\lim_{x \to 2^-} g(x)$$

(ii)

$$\lim_{x \to 2^+} g(x)$$

[3 marks]

Question 5b

(b)
Evaluate the limits
(i)
$\lim_{x \to -\infty} g(x)$
(ii)
$\lim_{x \to +\infty} g(x)$

[3 marks]

Question 5c

(c)
Use your results from parts (a) and (b) to write down the equations of any asymptotes on the graph of $y = g(x)$.

[2 marks]

Question 5d

(d)
Use technology to help you sketch the graph of $y = g(x)$, and show that this confirms your results from parts (a), (b) and (c).

[2 marks]

Question 6a

(a)

The function f is a piecewise function defined by

$$f(x)=\begin{cases}3x-7, & x<3\\ 1, & x=3\\ x^2-8x+17, & x>3\end{cases}$$

Explain why f is not continuous at $x=3$.

[3 marks]

Question 6b

(b)

Give an example of a function g that is continuous for all values of $x \in \mathbb{R}$, but is not differentiable for all values of $x \in \mathbb{R}$.

Include a sketch of the graph of the function, identifying the point(s) where the function is not differentiable.

[3 marks]