4.7 Binomial Distribution

Question Paper

Course	DP IB Maths
Section	4. Statistics & Probability
Topic	4.7 Binomial Distribution
Difficulty	Medium

Time allowed: 80

Score: /63

Percentage: /100

Question 1a

(a) State the conditions that must be satisfied to be able to model a random variable X with a binomial distribution $B(n, p)$.

[4 marks]

Question 1b

(b) A fair spinner has 8 sectors labelled with the numbers 1 through 8. For each of the following cases, state with a reason whether or not a binomial distribution would be appropriate for modelling the specified random variable.

   (i)   The random variable S is the number of the sector that the spinner lands on when it is spun.

   (ii)   The random variable W is the number of times the spinner is spun until it lands on '7' for the first time.

   (iii) The random variable Y is the number of times the spinner lands on a prime number when it is spun twelve times.

   (iv) On the first spin, it is a 'win' if the spinner lands on an even number. On subsequent spins it is a 'win' if the spinner lands *either* on the same number as the previous spin *or* on a factor of the number from the previous spin. The random variable L is the number of wins when the spinner is spun ten times.

[4 marks]

Question 2a

A fair coin is tossed 20 times and the number of times it lands heads up is recorded.

(a) Find the expected number of times that the coin will land heads up.

[1 mark]

Question 2b

(b) Find the probability that the coin lands heads up 15 times.

[2 marks]

Question 3a

On any given day during a normal five-day working week, there is a 60% chance that Yussuf catches a taxi to work.

(a) Find $E(X)$, the expected number of times Yussuf will catch a taxi to work during a normal five-day working week.

[1 mark]

Question 3b

(b) Find the probability that, during a normal five-day working week, Yussuf never catches a taxi.

[2 marks]

Question 3c

(c) Find the probability that, during a normal five-day working week, Yussuf catches a taxi once at the most.

[2 marks]

Question 4a

A difficult operation to remove a rare form of cancer is found to have a success rate of 78%. Twelve patients are on the waiting list to undergo the operation.

(a) Find the probability that all twelve patients' operations are successful.

[2 marks]

Question 4b

(b) Find the probability that all but two patients undergo successful operations.

[2 marks]

Question 5

For a jellyfish population in a certain area of the ocean, there is a 95% chance that any given jellyfish contains microplastic particles in its body.

For a sample size of 40 jellyfish from this population, find the probability of:

    (i)   exactly 38 jellyfish

    (ii)   all the jellyfish

having microplastic particles in their bodies.

[4 marks]

Question 6

Giovanni is rolling a biased dice, for which the probability of landing on a two is 0.25. He rolls the dice 10 times and records the number of times that it lands on a two. Find the probability that

(i) the dice lands on a two 4 times.

(ii) the dice lands on a two 4 times by landing on a two 3 times in the first 9 rolls, and then landing on a two on the tenth roll.

[5 marks]

Question 7

For cans of a particular brand of soft drink labelled as containing 330 ml, the actual volume of soft drink in a can varies. Although the company's quality control assures that the mean volume of soft drink in the cans remains at 330 ml, it is known from experience that the probability of any particular can of the soft drink containing less than 320 ml is 0.0296.

Tilly buys a pack of 24 cans of this soft drink. It may be assumed that those 24 cans represent a random sample. Let L represent the number of cans in the pack that contain less than 320 ml of soft drink.

Find the probability that

     (i)   none of the cans

     (ii)   exactly two of the cans

     (iii) at least two of the cans

contain less than 320 ml of soft drink.

[4 marks]

Question 8a

The random variable $X \sim \mathrm{B}(40, 0.15)$.

(a) Find:

      (i) $\mathrm{P}(3 \leq X < 14)$

      (ii) $\mathrm{P}(5 < X < 12)$

[3 marks]

Question 8b

(b) Find $\mathrm{Var}(X)$.

[1 mark]

Question 8c

(c) Find $\mathrm{P}(X \leq 3 \mid X \leq 9)$.

[3 marks]

Question 9a

Zara is a gymnast. It is known that she has a 20% chance of making a mistake in any given routine.

Zara performs ten routines in a competition.

(a)   (i)   Find the expected number of routines in which Zara will make a mistake.

      (ii)   Find the standard deviation of the number of routines in which Zara makes a mistake.

[3 marks]

Question 9b

(b) Find the probability that Zara makes a mistake in:

      (i)   none of her routines,

      (ii)   exactly two of her routines,

      (iii)   no more than two of her routines.

[6 marks]

Question 9c

(c) Given that Zara makes a mistake in at least 2 of her routines, find the probability that she makes a mistake in exactly 3 of her routines.

[3 marks]

Question 9d

(d) Find the probability that the number of routines in which Zara makes a mistake is less than one standard deviation away from the mean.

[3 marks]

Question 10a

In the town of Wooster, Ohio, it is known that 90% of the residents prefer the locally produced Woostershire brand sauce when preparing a Caesar salad. The other 10% of residents prefer another well-known brand.

30 residents are chosen at random by a pollster. Let the random variable X represent the number of those 30 residents that prefer Woostershire brand sauce.

(a) Find

    (i) $\mathrm{E}(X)$

    (ii) $\mathrm{Var}(X)$.

[2 marks]

Question 10b

  (b) Find the probability that

    (i)   90% or more of the residents chosen prefer Woostershire brand sauce,

    (ii)   none of the residents chosen prefer the other well-known brand.

[6 marks]