5.5 Kinematics

Question Paper

Course	DP IB Maths
Section	5. Calculus
Topic	5.5 Kinematics
Difficulty	Medium

Time allowed: 100

Score: /81

Percentage: /100

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Question 1a

A skydiver jumps from a moving aircraft at a point directly above a fixed point, O, on the ground. The trajectory of the skydiver is then modelled by the function

$$h(x) = 3200 - 0.5x^2$$

where h m is the height of the skydiver above the ground and x m is the horizontal distance along the ground from point O.

(a) (i) Explain the significance of the value 3200 in the model.

(ii) Calculate the horizontal distance the skydiver covered upon landing.

[2 marks]

Question 1b

(b) Sketch a graph of h against x.

[2 marks]

Question 1c

(c) Explain why the model is not suitable for values of x larger than 80 m.

[1 mark]

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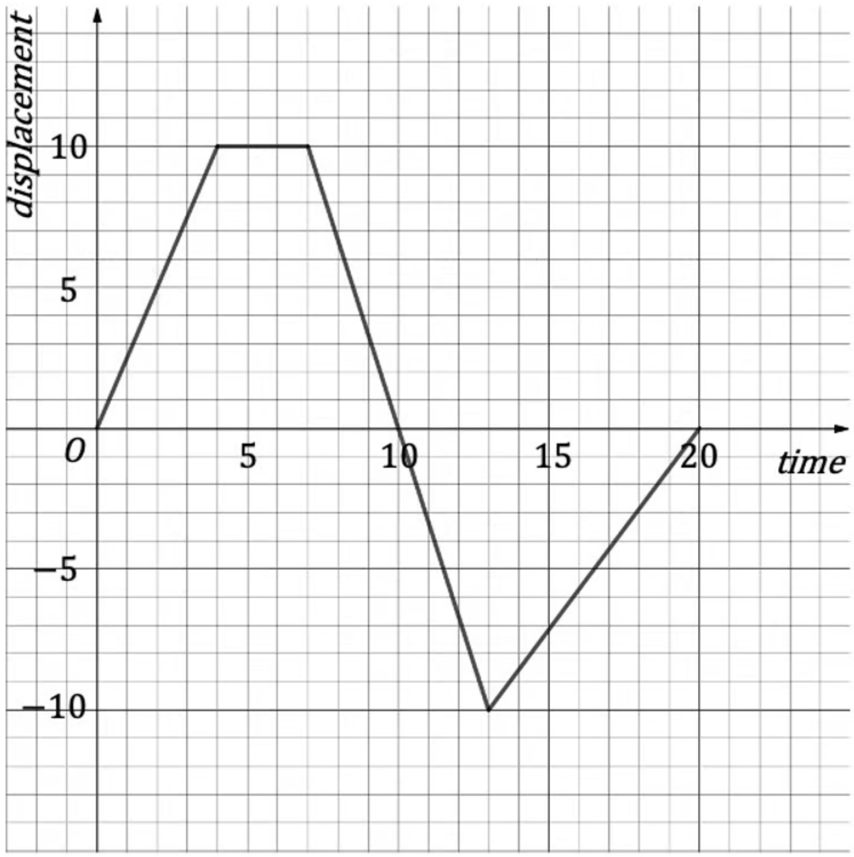
Question 2a

A particle moves along a horizontal line starting at the point O. The displacement-time graph for the first 20 seconds of its motion is shown below. Displacement is measured in metres.



(a) (i) Write down the displacement of the particle after 2 seconds.

(ii) Write down the displacement of the particle after 4 seconds.

[2 marks]

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Question 2b

(b) Find the velocity of the particle between 13 and 20 seconds.

[1 mark]

Question 2c

(c) Find the speed of the particle between 7 and 10 seconds.

[1 mark]

Question 2d

(d) Find the total distance travelled by the particle after 20 seconds.

[2 marks]

Question 3a

A cricket ball is projected directly upwards from ground level. The motion of the cricket ball is modelled by the function

$$h(t) = 13t - 4.9t^2 \qquad\qquad t > 0$$

where h metres is the height of the cricket ball above ground level after t seconds.

(a) Find the times at which the cricket ball is exactly 3 m above the ground.

[2 marks]

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Question 3b

(b) For how long is the cricket ball at least 3 m above the ground?

[1 mark]

Question 3c

A player catches the cricket ball (on its way down) at a height of 0.8 m above the ground.

(c) Find the length of time the ball was in the air.

[2 marks]

Question 3d

(d) Find the total distance travelled by the ball.

[2 marks]

Question 3e

(e) Find the velocity of the cricket ball at $t = 1$ second.

[2 marks]

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Question 4a

A soft ball is thrown upwards from the top of a 10 m tall building.
The height, h m of the ball above the ground after t seconds is modelled by the function

$$h(t) = H + 7.8t - 4.9t^2 \qquad t > 0$$

(a) Write down the value of H.

[1 mark]

Question 4b

(b) Find the height of the ball after 2 seconds.

[2 marks]

Question 4c

(c) Find the time at which the ball is at the same height as it was when thrown.

[2 marks]

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Question 4d

(d) Find the time the ball first hits the ground.

[2 marks]

Question 4e

(e) Find $h''(t)$ and hence show that the acceleration at any time is -9.8 m/s^2.

[3 marks]

Question 5a

A particle moves along a straight line with a velocity, v ms^{-1}, given by $v = 2^t - 2$ where t is measured in seconds such that $0 \leq t \leq 4$.

(a) Find the acceleration of the particle at time $t = 2$.

[2 marks]

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Question 5b

(b) State the time when the particle comes to rest.

[1 mark]

Question 5c

(c) Find the total distance travelled by the particle.

[3 marks]

Question 6a

A particle is found to have an acceleration, a ms^{-2}, according to the function

$$a = \frac{1}{t^2} + \sin t, \text{ where } t \geq 1$$

(a) Find an expression for the velocity, v, of the particle given that $v(1) = 1$.

[4 marks]

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Question 6b

(b) Find the velocity of the particle at $t = 2$.

[2 marks]

Question 7a

A particle, moving in a straight line, is found to have a velocity $v = \sin t + \cos 2t$ where v is measured in ms^{-1} and time t is measured in seconds such that $0 \leq t \leq 5$.

(a) Find the time(s) when the particle is instantaneously at rest.

[2 marks]

Question 7b

(b) Find the time(s) when the particle changes direction.

[1 mark]

Question 7c

(c) Find the distance travelled in the first second of motion.

[3 marks]

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Question 7d

(d) Find the acceleration of the particle at the instant it first changes direction.

[3 marks]

Question 7e

(e) Find the displacement of the particle from its starting point to the point when $t = 5$.

[4 marks]

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Question 8a

A particle is moving along a straight line. The position of the particle at time t seconds, measured in metres relative to a fixed origin point, is denoted by $x(t)$.

The particle starts at the origin at time $t = 0$, and its motion over the next eight seconds is described by the equation

$$\dot{x}(t) = \frac{1}{\cos^2\left(\frac{\pi}{20}t\right)} - 3, \quad 0 \le t \le 8$$

(a) Find an expression for $x(t)$.

[4 marks]

Question 8b

(b) Hence determine the maximum distance of the particle from the origin during the first eight seconds of its movement.

[3 marks]

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Question 8c

(c) Find the change in displacement of the particle during the first eight seconds of its movement.

[2 marks]

Question 8d

(d) Find the total distance travelled by the particle during the first eight seconds of its movement.

[2 marks]

Question 8e

(e) Find an expression for the particle's acceleration $\ddot{x}(t)$.

[3 marks]

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Question 9a

A particle is moving along a straight line. The position of the particle at any given time, measured in metres relative to a fixed origin point, is denoted by x.

It is known that the velocity, v ms^{-1}, of the particle is dependent on the particle's position, and that the velocity may be described by the equation

$$v(x) = \sqrt{1 - x^2}, \quad -1 \leq x \leq 1$$

(a) Use the chain rule to explain why the acceleration, a ms^{-2}, of the particle may be expressed in the form

$$a = v\frac{\mathrm{d}v}{\mathrm{d}x}$$

[3 marks]

Question 9b

(b) Show that the derivative of $\sqrt{1 - x^2}$ is $-\frac{x}{\sqrt{1-x^2}}$.

[4 marks]

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Question 9c

(c) Hence find an expression for the acceleration of the particle in terms of x, being sure to indicate the domain of x values for which the expression is valid.

[2 marks]

Question 9d

(d) Identify the minimum and maximum values of

(i) the speed of the particle

(ii) the magnitude of the particle's acceleration

along with the values of x for which those occur.

[3 marks]

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