

3.1 Geometry Toolkit

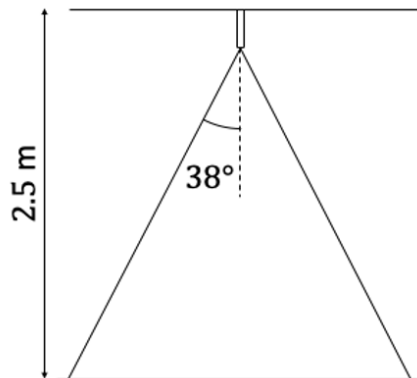
Question Paper

Course	DPIB Maths
Section	3. Geometry & Trigonometry
Topic	3.1 Geometry Toolkit
Difficulty	Very Hard

Time allowed: 130
Score: /103
Percentage: /100

Question 1a

A security lamp is situated at a height of 2.5 m and positioned so that the central axis of the light bulb is directed perpendicularly to the horizontal. When the lamp is switched on the light spreads out in all directions up to an angle of 38° from the central axis of the light bulb. This information is shown in the diagram below.

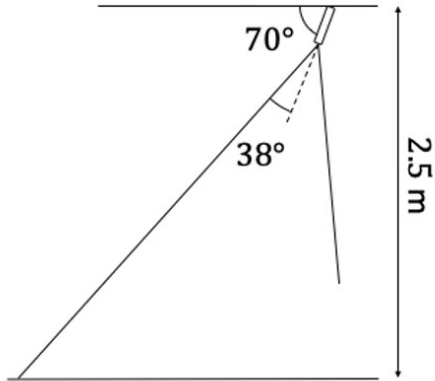


(a) Calculate the horizontal distance on the floor that is illuminated by the lamp.

[2 marks]

Question 1b

The area illuminated is not sufficient so the lamp is repositioned at the same height so that the central axis of the light bulb is now at an angle of 70° from the horizontal.

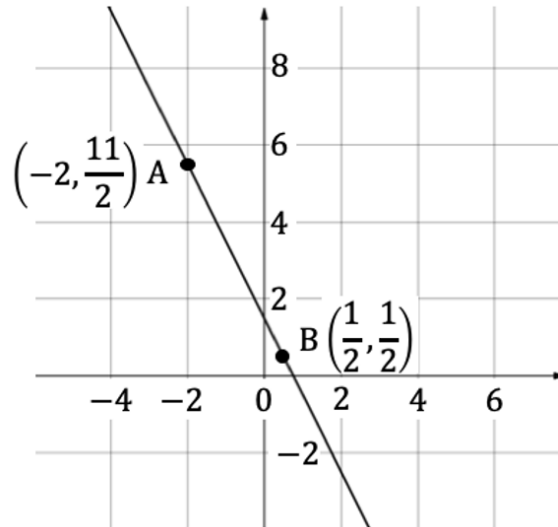


(b) Calculate the percentage increase in the horizontal distance that is now illuminated.

[4 marks]

Question 2a

On the diagram below, points A $\left(-2, \frac{11}{2}\right)$ and B $\left(\frac{1}{2}, \frac{1}{2}\right)$ indicate the position of metal spikes called pitons that aid mountaineers as they climb. Each unit on the graph is 16 cm in real life.



A third piton is required halfway between points A and B at point M.

(a) Calculate the distance AM.

[4 marks]

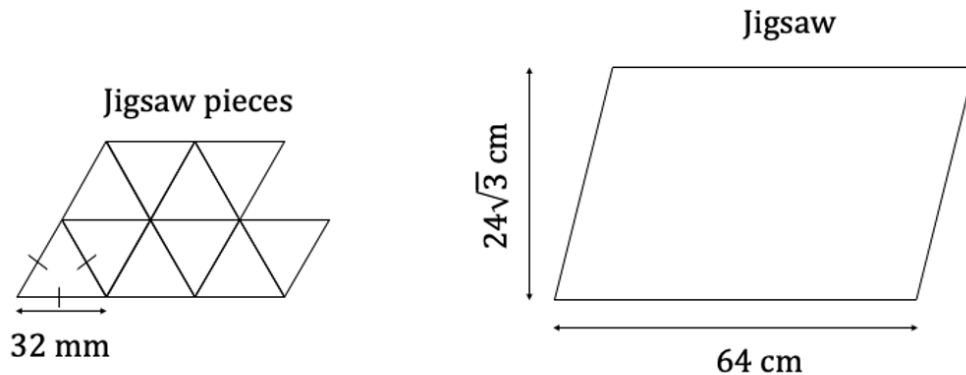
Question 2b

(b) Given that all pitons should be spaced at the distance calculated in part (a), work out the number of pitons required on a mountain with a slope length of $25\sqrt{5}$ m.

[2 marks]

Question 3a

An equilateral triangular jigsaw piece has an edge length of 32 mm. Several of these pieces are connected together with the vertices of the triangular pieces alternately pointing up and then down. The completed jigsaw puzzle is in the shape of a parallelogram with a side length of 64 cm and a perpendicular height of $24\sqrt{3}$ cm. A diagram illustrating this information can be seen below.



(a) Calculate the number of individual jigsaw pieces in the puzzle.

[4 marks]

Question 3b

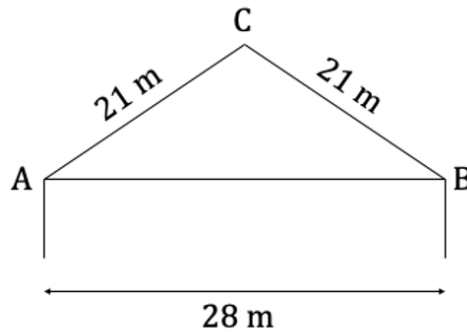
A second jigsaw is to be designed using 289 of the same type of individual pieces. The completed puzzle will this time be in the shape of an equilateral triangle.

(b) Work out the number of pieces required along each length.

[5 marks]

Question 4a

A roof with a symmetrical triangular cross-section, ABC, is being designed for the top of a building. The horizontal width that the roof must span is 28 m and the lengths of the timbers used for the angled part of the cross-section are 21 m, as shown in the diagram below.

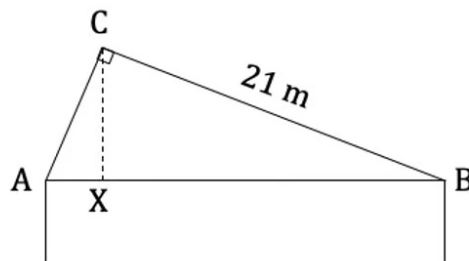


(a) Calculate the angle \widehat{CAB} .

[2 marks]

Question 4b

An alternative design idea for the roof is to shorten AC and to make the apex of the roof a right angle. BC remains the same length as it was originally. These changes can be seen in the diagram below. The point X is situated such that it is directly beneath point C.



(b) Calculate the new length of AC.

[2 marks]

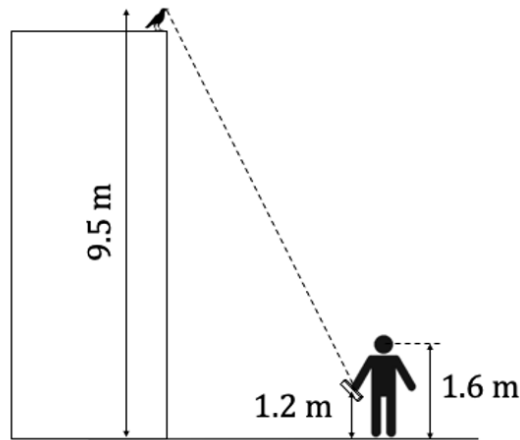
Question 4c

(c) Calculate the vertical height CX of this alternative design for the roof.

[3 marks]

Question 5a

A bird is perched on the edge of a building with its eye at a height of 9.5 m above ground level. A person is holding a sandwich at a height of 1.2 m from the ground and the distance between the ground and the person's eye level is 1.6 m. A diagram showing this is below.



The bird sees the sandwich at an angle of depression of 52° .

(a) Calculate the distance that the bird must fly to reach the food.

[3 marks]

Question 5b

The person's eyes are 0.3 m further away from the building than the sandwich.

(b) Find the angle of elevation at which the person sees the bird.

[4 marks]

Question 5c

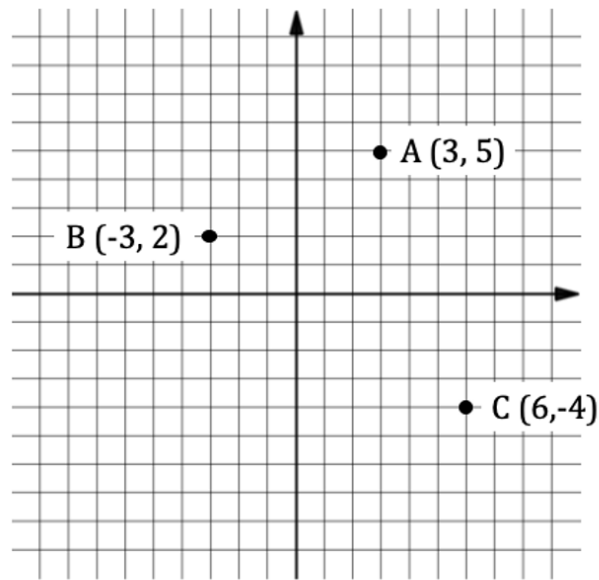
A second bird is perched on a lamp post on the other side of the person at a horizontal distance of 5 m. The person sees this bird at an angle of elevation of 37° .

(c) Find the vertical distance between the two birds.

[3 marks]

Question 6a

Three locations in a forest, A (3, 5), B (−3, 2) and C (6, −4) are marked out for an orienteering activity. These can be seen on the grid below. Each unit on the grid indicates a distance of 1 km.



(a) For AC, find:

- (i) The length AC
- (ii) The bearing of A from C.

[5 marks]

Question 6b

Point X is the midpoint between B and the point D. X is located on the line AC such that $AX : XC = 1 : 3$.

(b) Find the coordinates of the point D.

[4 marks]

Question 7a

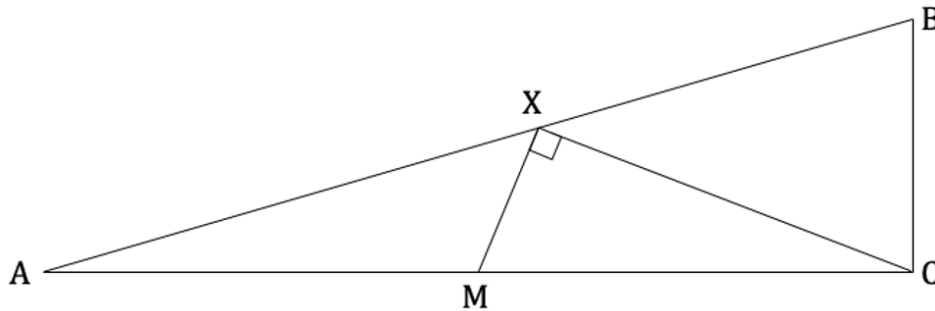
A wheelchair ramp is required to provide access to a building with a door that is located 22 cm above ground level. The maximum angle that a ramp must be from the horizontal is 4.8° .

(a) Calculate the minimum horizontal distance that the ramp must extend out.

[2 marks]

Question 7b

The wheelchair ramp is built using the minimum distance found in part (a), rounded to 3 significant figures. The ramp is supported by a steel frame, a cross section of which can be seen in the diagram below. A metal strut joins M, the midpoint of AC, to a point X on the line AB. XM is 11.1 cm in length and \widehat{MXC} forms a right angle.

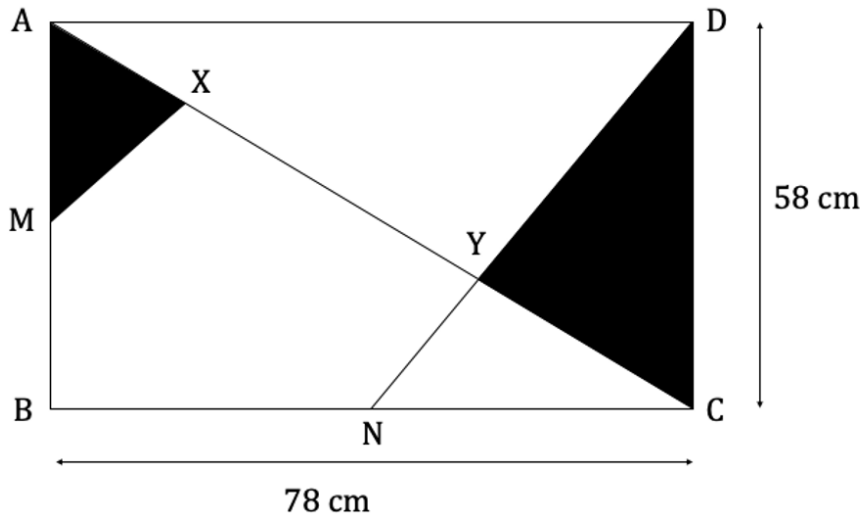


(b) Calculate the length XB.

[6 marks]

Question 8a

In the diagram below, ABCD shows a piece of geometric art on canvas measuring 58 cm by 78 cm. N is the midpoint of BC and M is the midpoint of AB. X is a point on AC such that $AX : XC = 1 : 5$. A straight line connects M to point X. Y is the point where AC intersects ND.



(a) Calculate the area of the artwork that is painted black.

[6 marks]

Question 8b

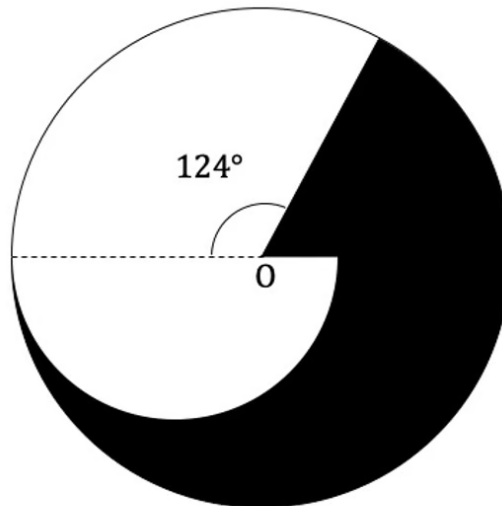
The piece of artwork is to be enlarged by a length scale factor of 6 and painted on an exterior wall of an art gallery. A 200 ml tin of paint costs \$8 and covers an area of 2.4 m^2 .

- (b) Calculate the cost of the paint that must be purchased to re-create the same black sections from part (a) on the wall.

[4 marks]

Question 9a

The logo of a new company comprises a circle of radius r cm and centre O , with part of the interior area shaded. The diameter of the unshaded interior semi-circle (the unshaded area below the dashed line in the diagram) is $\frac{2}{3}$ that of the larger circle. The remainder of the unshaded area is a sector of the main circle with a sector angle of 124° . This information is shown in the diagram below.



(a) Show that the area of the shaded section is equal to $\frac{13}{30}\pi r^2$ cm².

[4 marks]

Question 9b

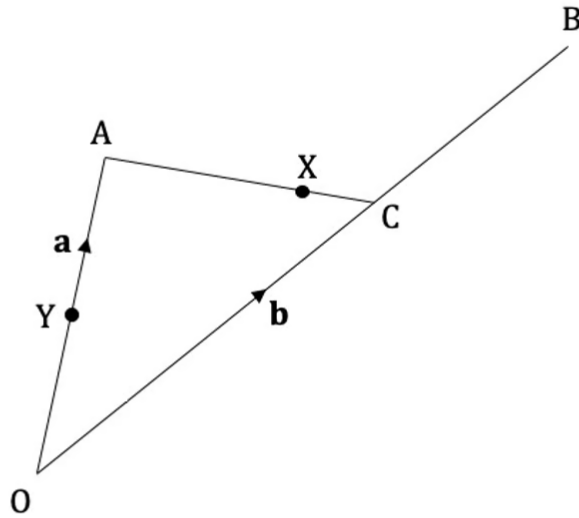
The sector angle of the unshaded sector is decreased.

- (b) Find the sector angle of the unshaded sector that is required to make the areas of the logo that are shaded and unshaded equal.

[3 marks]

Question 10a

In the diagram below $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. Y is the midpoint of [OA]. C is a point on [OB] such that $\overrightarrow{OC} : \overrightarrow{CB} = 3 : 2$.



(a) Find

- (i) \overrightarrow{OC} in terms of \mathbf{a} and \mathbf{b} .
- (ii) \overrightarrow{BY} in terms of \mathbf{a} and \mathbf{b} .

[3 marks]

Question 10b

X is a point on [AC]. It is given that $\overrightarrow{BX} = \alpha\overrightarrow{BY}$ and $\overrightarrow{CX} = \beta\overrightarrow{CA}$, where $\alpha, \beta \in \mathbb{R}$.

(b) Find an expression for \overrightarrow{OX}

(i) in terms of **a**, **b** and α .

(ii) in terms of **a**, **b** and β .

[4 marks]

Question 10c

(c) Hence find the values of α and β .

[4 marks]

Question 10d

(d) Write down an expression for \overrightarrow{XY} in terms of **a** and **b** only.

[3 marks]

Question 11a

A parallelogram is formed by the points $A(-1, 7)$, $B(2, 12)$, C and D , where $\overrightarrow{AC} = \begin{pmatrix} 12 \\ 7 \end{pmatrix}$.

(a) Find the coordinates of C and D .

[2 marks]

Question 11b

(b) Calculate the perpendicular height of the parallelogram.

[4 marks]

Question 11c

(c) Hence find the area of the parallelogram ABCD.

[2 marks]

Question 12a

Points A and B have coordinates $(4, -7, n)$ and $(-1, 3p, 2)$ respectively.

(a) Given that $\overrightarrow{AB} = \begin{pmatrix} -5 \\ 19 \\ 6 \end{pmatrix}$, find the value of n and p .

[2 marks]

Question 12b

(b) Consider a unit vector \mathbf{v} , such that $\mathbf{v} = \frac{4}{9}\mathbf{i} + q\mathbf{j} - \frac{7}{9}\mathbf{k}$, where $q > 0$.

Point C is such that $\overrightarrow{AC} = 18\mathbf{v}$.

Find the coordinates of point C.

[7 marks]



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