

# 3.4 Further Trigonometry

## Question Paper

|            |                            |
|------------|----------------------------|
| Course     | DPIB Maths                 |
| Section    | 3. Geometry & Trigonometry |
| Topic      | 3.4 Further Trigonometry   |
| Difficulty | Hard                       |

**Time allowed:** 100  
**Score:** /82  
**Percentage:** /100

**Question 1**

Complete the following table. In all cases the values for the angle should be given between  $0$  and  $360^\circ$  or  $0$  and  $2\pi$  radians, as appropriate.

| Degrees     | Radians          | sin                  | cos           | tan |
|-------------|------------------|----------------------|---------------|-----|
| $45^\circ$  |                  |                      |               | 1   |
|             |                  | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ |     |
| $150^\circ$ |                  | $\frac{1}{2}$        |               |     |
|             |                  | $-1$                 |               |     |
|             | $\frac{7\pi}{4}$ |                      |               |     |

[5 marks]

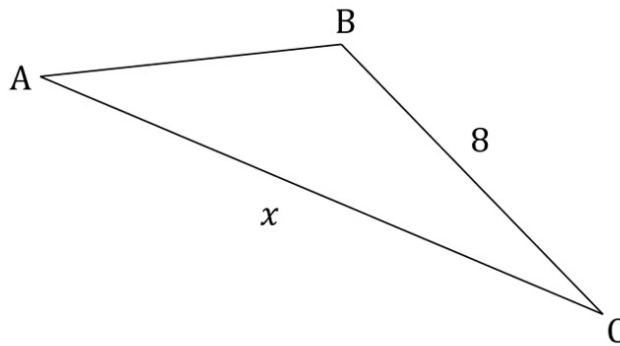
**Question 2**

Given that  $\sin \theta = -\frac{12}{13}$ , find the possible values of  $\cos \theta$  and the corresponding values of  $\tan \theta$ .

[4 marks]

**Question 3**

The following diagram shows triangle ABC, with  $BC = 8$  and  $AC = x$ .

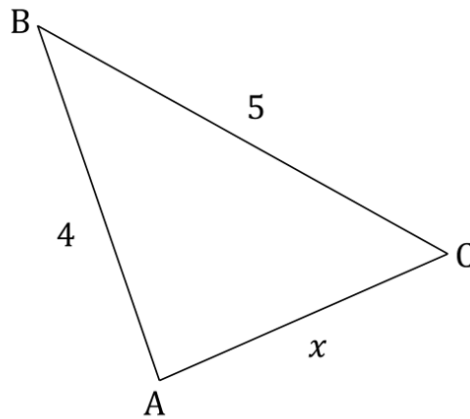


Given that  $\tan \hat{ACB} = \frac{5}{12}$  and that the area of triangle ABC is equal to 20 units<sup>2</sup>, find the value of  $x$ .

[6 marks]

**Question 4a**

The following diagram shows triangle ABC, with  $AB = 4$ ,  $BC = 5$ , and  $AC = x$ .



(a) Given that  $\cos \widehat{ABC} = \frac{3}{4}$ , find the exact area of triangle ABC.

[6 marks]

**Question 4b**

(b) Find the exact perimeter of triangle ABC.

[4 marks]

**Question 5**

A sector of a circle, OPQ, is such that the angle at its centre, O, is  $\frac{5\pi}{6}$  radians.

The area of sector OPQ in  $\text{cm}^2$  is one-fifth of the length of the arc PQ in cm.

- (i) Show that the radius of the sector is equal to 0.4 cm, and hence
- (ii) find the area of sector OPQ and the length of arc PQ.

Give your answers in part (ii) correct to 3 significant figures.

[6 marks]

**Question 6**

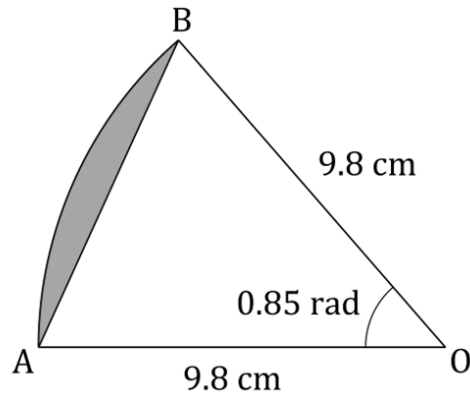
The lengths of two sides in a right-angled triangle are 9 cm and 12 cm.

Find the possible values of  $\sin \theta$ , and the corresponding values of  $\cos \theta$  and  $\tan \theta$ , where  $\theta$  is the smallest angle in the triangle. All your answers should be given as exact values.

[7 marks]

**Question 7a**

The diagram below shows the sector of a circle  $OAB$ , with centre  $O$  and radius  $9.8$  cm. The angle at the centre of the sector,  $\widehat{AOB}$ , is  $0.85$  radians.



- (a) Find the area of the shaded segment, bounded by arc  $AB$  and chord  $AB$ . Give your answer correct to 3 significant figures.

[5 marks]

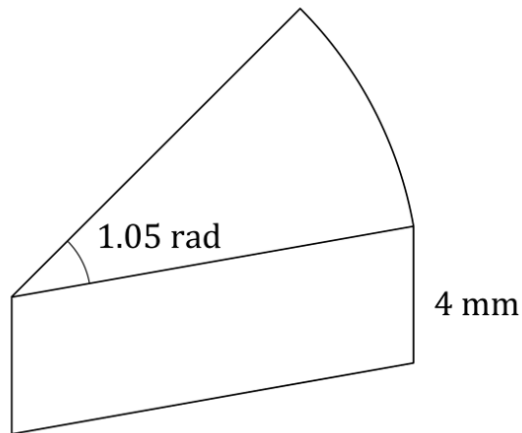
**Question 7b**

- (b) Find the perimeter of sector  $OAB$ .

[3 marks]

**Question 8**

A games design company produces a popular game called ‘Inconsequential Endeavour’. Each game set includes solid plastic game pieces which are in the form of a right prism with a cross-section that is the sector of a circle, as shown in the diagram below. The angle at the centre of the sector is 1.05 radians, and the height of the game piece is 4 mm.



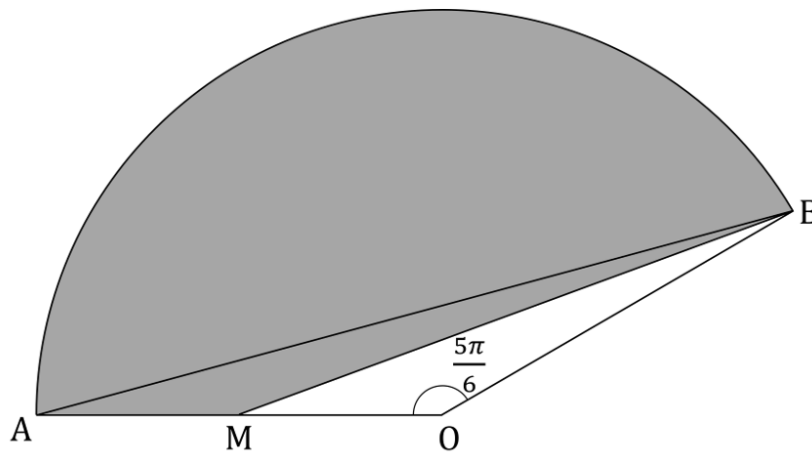
Given that the volume of the game piece is  $0.412 \text{ cm}^3$ , work out the radius of the sector. Give your answer correct to 3 significant figures.

[5 marks]



**Question 9a**

The diagram below shows the sector of a circle  $OAB$  with centre  $O$ . The angle at the centre of the sector,  $\widehat{AOB}$ , is  $\frac{5\pi}{6}$  radians. Point  $M$  is the midpoint of line segment  $OA$ , and the shaded region is the combination of triangle  $ABM$  with the region enclosed by the arc  $AB$  and the chord  $AB$ .



- (a) Show that the ratio of the area of triangle  $OMB$  to the area of the shaded region may be expressed as

$$1 : \left( \frac{10\pi}{3} - 1 \right)$$

[6 marks]

**Question 9b**

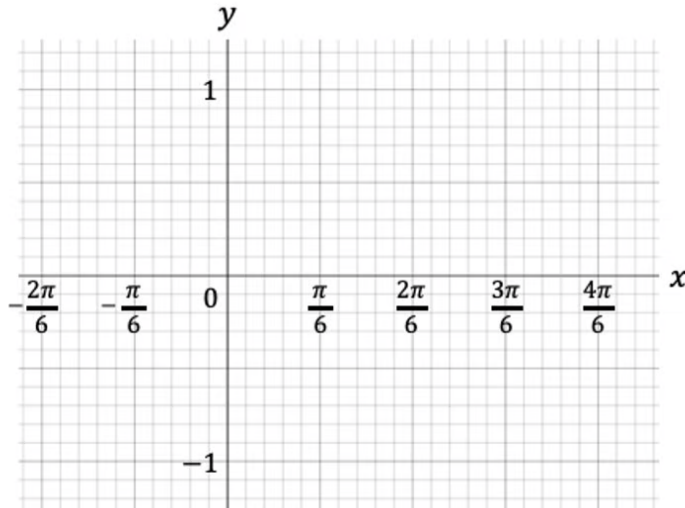
(b) Given that the area of the shaded region is equal to  $30\pi - 9$  units<sup>2</sup>, find the exact area of triangle OAB.

[3 marks]

**Question 10a**

A graph has the equation  $y = \cos 3x$  for the interval  $-\frac{\pi}{3} \leq x \leq \frac{2}{3}\pi$ .

(a) Sketch the graph on the axes below.



[3 marks]

**Question 10b**

A straight line with equation  $y = \frac{1}{2}$  intersects the graph of  $y = \cos 3x$ .

(b) (i) Sketch the line  $y = \frac{1}{2}$  on to the same set of axes.

(ii) Find the coordinates of the points of intersection between  $y = \cos 3x$  and  $y = \frac{1}{2}$ .

[5 marks]

**Question 11**

- (i) Sketch the graph of  $y = \sin(2\theta - 60)$  in the interval  $-180^\circ \leq \theta \leq 180^\circ$ .
- (ii) Write down all the values for  $\theta$ , where  $y = \sin(2\theta - 60)^\circ = 0$  in the given interval.

**[5 marks]**

**Question 12a**

(a) Use the fact that

$$16x^3 - 12x^2 - 4x + 3 = (4x - 3)(4x^2 - 1)$$

to fully factorise  $16x^3 - 12x^2 - 4x + 3$ .

[2 marks]

**Question 12b**

(b) Use your result from part (a) to solve the equation

$$16 \sin^3 3\theta - 12 \sin^2 3\theta - 4 \sin 3\theta + 3 = 0$$

in the interval  $0 \leq \theta \leq \frac{\pi}{2}$ . You should give your answers as exact values where possible.

[7 marks]



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