2.9 Further Functions & Graphs

Question Paper

Course	DP IB Maths
Section	2. Functions
Topic	2.9 Further Functions & Graphs
Difficulty	Hard

Time allowed: 80

Score: /65

Percentage: /100

Question 1a

Consider the function $f(x) = x^2 - 4|x| - 5,\ x \in R.$

(a)
Solve $f(x) = 0.$

[4 marks]

Question 1b

(b)
Sketch the graph of f. Clearly indicate the intersections with the coordinate axes and any turning points.

[4 marks]

Question 2

Given that

$$f(x) = e^x, x \in R,$$

sketch on separate axes the graphs of

(i)
$y = f(|x - 1|)$
(ii)
$y = |f(x) - 1|$
(iii)
$y = f(-|x|).$

Show any intercepts with the axes, label any local maximum and minimum points and give the equations of any asymptotes. Leave numbers in terms of e where appropriate.

[9 marks]

Question 3a

Consider the function f defined by $f(x) = 4x^2 - 8x - 5$.

(a)
Sketch the graph of $y = |f(x)|$. Clearly indicate any intercepts with the axes and any turning points.

[6 marks]

Question 3b

(b)
Sketch the graph of $y = [f(x)]^2$. Clearly indicate any intercepts with the axes and any turning points.

[3 marks]

Question 4a

(a)

Sketch the curve $y = \dfrac{4}{x+3}$ and the line $y = 3 - x$ on the same diagram, clearly indicating any x- and y- axes intercepts as well as any asymptotes.

[3 marks]

Question 4b

(b)

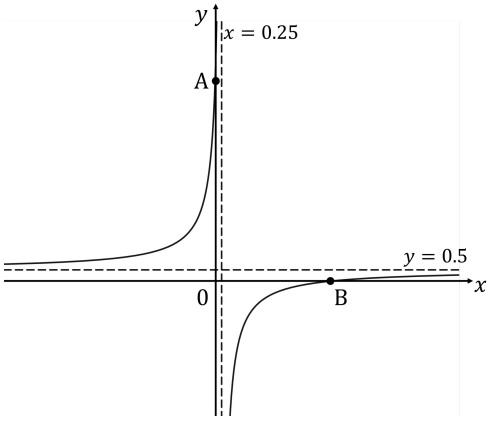
Hence find the exact solutions to the equation

$$3 - x = \left| \frac{4}{x+3} \right|.$$

[5 marks]

Question 5

The graph of f has two asymptotes with equation $x = 0.25$ and $y = 0.5$ as shown below. The graph passes through the points $\mathrm{A}(0,6)$ and $\mathrm{B}(3, 0)$.



Sketch the graph of $y = \dfrac{1}{f(x)}$. Clearly indicate the points where the graph intersects the axes or has a discontinuity and state the equations of any asymptotes.

[6 marks]

Question 6a

Consider the function defined by $f(x) = \ln(k - x)$ where f has the largest possible valid domain and k is a positive constant such that $k > 1$.

(a)
Sketch the graph of $y = f(x)$. Give the equations of any asymptotes and any intercepts with the axes in terms of k. Clearly state the domain and range of f.

[4 marks]

Question 6b

The function g is defined by $g(x) = f(|x|)$. The range of g is $g(x) \leq 1$.

(b)
(i)
Find the exact value of k.
(ii)
State the domain of g.
(iii)
Sketch the graph of g.

[4 marks]

Question 6c

(c)

Given that $|g(x)| = p$ has exactly two distinct real solutions find the range of values of p.

[3 marks]

Question 7

Find the set of values of x which satisfy the inequality

$$|2x^2 - 13x + 15| > 3x - 15$$

[6 marks]

Question 8a

(a)

By considering the inverse of an appropriate function, sketch the graph $y = \sqrt{x}$.

[2 marks]

Question 8b

The function f is defined by $f(x) = \sqrt{16 - 2x}$ and its domain is the largest possible set of real values.

(b) (i) State the domain and range of f.

(ii) Sketch the graph of $y = f(x)$. Clearly label the points where the graph intersects the axes.

[3 marks]

Question 8c

(c)

On separate sets of axes, sketch the graphs of:

(i)

$y = f(|x|)$

(ii)

$y = [f(x)]^2$.

For each graph, define the domain and range and clearly label the points where the graph intersects the axes.

[3 marks]