

# 1.5 Complex Numbers

## Question Paper

Course	DPIB Maths
Section	1. Number & Algebra
Topic	1.5 Complex Numbers
Difficulty	Medium

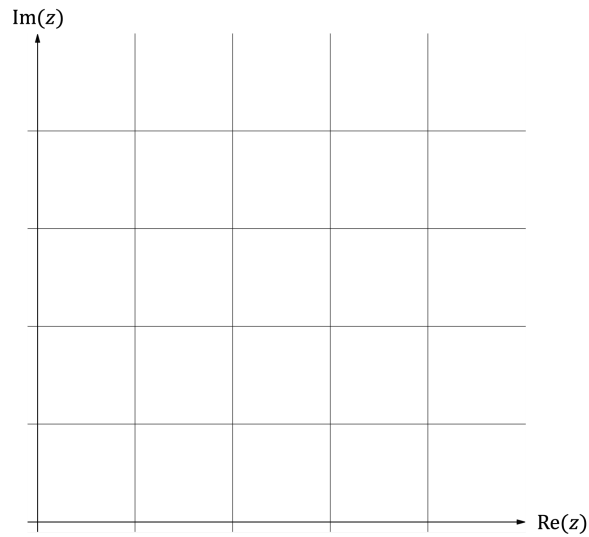
**Time allowed:** 120  
**Score:** /95  
**Percentage:** /100

### Question 1a

Consider the complex numbers  $z_1 = 2 + 2i$  and  $z_2 = 2 + 2\sqrt{3}i$ .

a)

Sketch  $z_1$  and  $z_2$  on the Argand diagram below, be sure to include an appropriate scale.



[2 marks]

### Question 1b

b)

Find the modulus of  $z_1$  and  $z_2$ .

[3 marks]

**Question 1c**

c)

Find the argument of  $z_1$  and  $z_2$ .**[3 marks]****Question 2**Solve the following equations for  $x$ 

(i)

$$x^2 + 4x + 5 = 0$$

(ii)

$$x^2 = -625$$

(iii)

$$x^4 = 24 - 2x^2.$$

**[7 marks]**

**Question 3a**

Let  $w_1 = z_1 z_2$ , where  $z_1 = 5 + i$  and  $z_2 = 1 + 2i$ .

a)

Express  $w$  in the form  $w = a + bi$ .

[2 marks]

**Question 3b**

b)

Find the modulus and argument for  $w$

[4 marks]

**Question 4a**

Let  $z = \frac{w_1}{w_2}$ , where  $w_1 = 4 - i$  and  $w_2 = 1 - 2i$ .

a)

Express  $z$  in the form  $z = a + bi$ .

[3 marks]

**Question 4b**

b)

Find the modulus and argument for  $z$ .

[4 marks]

**Question 5a**Consider the complex numbers  $z = 3 - 4i$  and  $w = 7 - 2i$ .

a)

Find

(i)

 $z + w$ 

(ii)

 $w - z$ .

[2 marks]

**Question 5b**

Let  $z^*$  and  $w^*$  represent the complex conjugates of  $z$  and  $w$ , respectively.

b)

Write down  $z^*$  and  $w^*$ , giving your answers in the form  $a + bi$ .

[2 marks]

**Question 5c**

c)

Find

(i)

$$z^* w$$

(ii)

$$\frac{w^*}{z}$$

[4 marks]

**Question 6**

Find all possible real values for  $a$  and  $b$  such that

(i)  
 $a + bi = 8i$

(ii)  
 $(2 + 3i)(a + bi) = 13$

(iii)  
 $(a + i)(2 + bi) = -6 + 22i.$

[7 marks]

**Question 7**

Consider the complex numbers  $w = iz$  and  $w + 2z = 7 + 6i$ .

Find

(i)  
 $\operatorname{Re}(w)$

(ii)  
 $\operatorname{Im}(w)$

(iii)  
 $\operatorname{Re}(z)$

(iv)  
 $\operatorname{Im}(z)$ .

[7 marks]



**Question 8**

It is given that  $z_1 = 3 + 4i$  and  $z_2 = -2 + 2i$ .

Find

(i)

$$iz_1 + z_2$$

(ii)

$$\frac{z_1}{iz_2}$$

(iii)

$$i(z_1 z_2).$$

[7 marks]

**Question 9**Find the complex numbers  $z$  and  $w$  such that

$$2z - iw^* = 5 + 7i$$

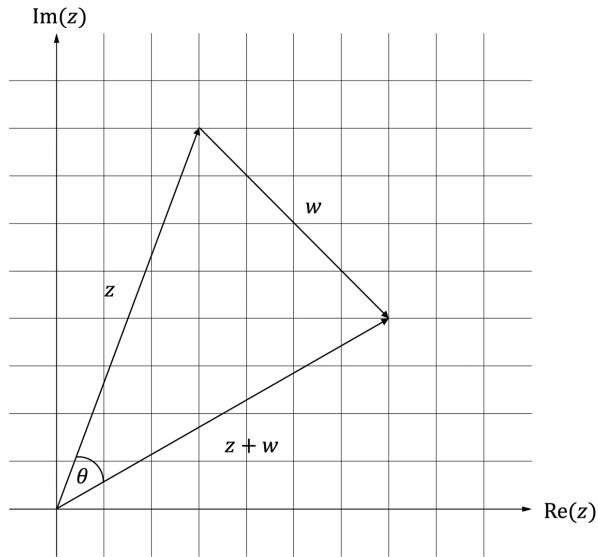
$$w + iz^* = 5 + 16i$$

**[8 marks]**

**Question 10a**

Let  $z = 3 + 8i$  and  $w = 4 - 4i$ .

a)  
Find  $\theta$ , the angle shown on the diagram below.



[5 marks]

**Question 10b**

b)  
Find the area of the triangle formed in the diagram above.

[3 marks]

**Question 11a**

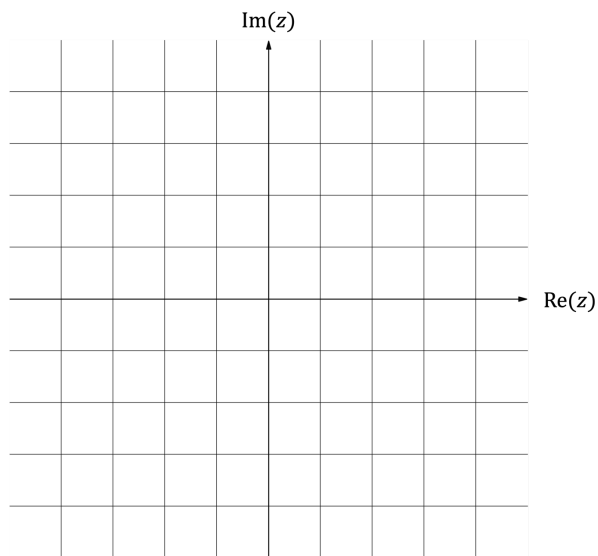
Let  $z = -1 - 3i$  and  $w = 1 + i$ .

a)  
Find  $zw$ .

[2 marks]

**Question 11b**

b)  
Sketch  $z$ ,  $w$  and  $zw$  on the Argand diagram below.



[3 marks]

**Question 11c**

Let  $\theta$  be the angle between  $z$  and  $zw$  and  $\phi$  be the angle between  $w$  and  $zw$ .

c)

Find the angles  $\theta$  and  $\phi$ , giving your answers in degrees.

[4 marks]

**Question 12a**

Let  $w = \frac{z+1}{z^*+1}$ , where  $z = a + bi$ ,  $a, b \in \mathbb{R}$ .

a)

Write  $w$  in the form  $x + yi$ ,  $x, y \in \mathbb{R}$ .

[4 marks]

**Question 12b**

b)

Determine the conditions under which  $w$  is purely imaginary.**[3 marks]****Question 13a**Consider the equation  $x^2 + bx + c = 0$ .(a) Write down an inequality, in terms of  $b$  and  $c$ , that shows the equation has no real solutions.**[1 mark]****Question 13b** $5 - 3i$  is one solution to the equation  $x^2 + bx + c = 0$ .(b) Find the values of  $b$  and  $c$ .**[4 marks]**

**Question 13c**

Let  $z = c + bi$ .

(c) Find  $z^5$  using technology.

[1 mark]