5.1 Differentiation

Question Paper

Course	DP IB Maths
Section	5. Calculus
Topic	5.1 Differentiation
Difficulty	Medium

Time allowed: 110

Score: /86

Percentage: /100

Question 1a

The equation of a curve is $y = \frac{3}{2}x^2 - 15x + 2$.

(a) Find $\frac{dy}{dx}$.

[2 marks]

Question 1b

The gradient of the tangent to the curve at point A is -3.

(b) Find

(i) the coordinates of A

(ii) the equation of the tangent to the curve at point A.
Give your answer in the form $y = mx + c$.

[4 marks]

Question 2a

Consider the function $f(x) = 3x^7 - 12x$.

(a) Find $f'(x)$.

[1 mark]

Question 2b

(b) Find the gradient of the graph of f at $x = 0$.

[2 marks]

Question 2c

(c) Find the coordinates of the points at which the normal to the graph of f has a gradient of 4.

[3 marks]

Question 3a

The equation of a curve is $y = 4 - \frac{4}{x}$.

(a) Find the equation of the tangent to the curve at $x = 2$.
Give your answer in the form $y = mx + c$.

[3 marks]

Question 3b

(b) Find the coordinates of the points on the curve where the gradient is 16.

[3 marks]

Question 4a

Consider the function $f(x) = \frac{4}{x} + \frac{2x^4}{5} - \frac{2}{5},\ x \neq 0.$

(a) Calculate

(i) $f(2)$

(ii) $f'(2)$.

[3 marks]

Question 4b

A line, l, is tangent to the graph of $y = f(x)$ at the point $x = 2$.

(b) Find the equation of l. Give your answer in the form $y = mx + c$.

[3 marks]

Question 4c

The graph of $y = f(x)$ and l have a second intersection at point A.

(c) Use your graphic display calculator to find the coordinates of A.

[2 marks]

Question 5a

Consider the function $f(x) = x^2 - bx + c$.

(a) Find $f'(x)$.

[1 mark]

Question 5b

The equation of the tangent line to the graph $y = f(x)$ at $x = 2$ is $y = x - 1$.

(b) Calculate the value of b.

[2 marks]

Question 5c

(c) Calculate the value of c and write down the function $f(x)$.

[3 marks]

Question 6a

The curve with equation $y = ax^2 + bx + c$ has a gradient of -7 at the point $(-1, 13)$, and a gradient of -3 at the point $(1, 3)$.

(a) By considering $\frac{dy}{dx}$ show that $2a + b = -3$ and $-2a + b = -7$.

[2 marks]

Question 6b

(b) Hence find the values of a and b.

[1 mark]

Question 6c

(c) By considering a point that you know to be on the curve, find the value of c.

[2 marks]

Question 7a

The curve C has equation $y = 3x^2 - 6 + \frac{4}{x}$. The point $P(1, 1)$ lies on C.

(a) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

[2 marks]

Question 7b

(b) Show that an equation of the normal to C at point P is $x + 2y = 3$.

[3 marks]

Question 7c

This normal cut the x-axis at the point Q.

(c) Find the length of PQ, giving your answer as an exact value.

[2 marks]

Question 8

Find the values of x for which $f(x) = -9x^2 + 5x - 3$ is an increasing function.

[3 marks]

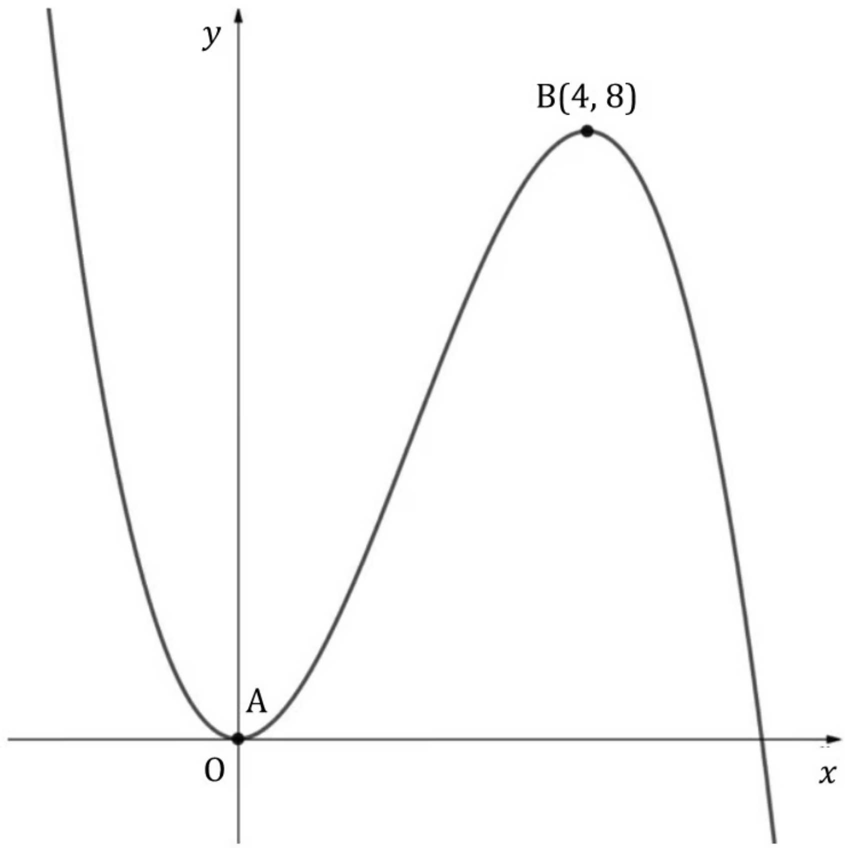
Question 9

Show that the function $f(x) = x^3 - 3x^2 + 6x - 7$ is increasing for all $x \in \mathbb{R}$.

[3 marks]

Question 10a

The graph of the cubic function $y = f(x)$ is shown below. Point A, a local minimum, is located at the origin and point B, a local maximum, sits at the point $(4,8)$.



(a) State the equations of the horizontal tangent to the curve.

[2 marks]

Question 10b

(b) Write down the value of x where the point of inflection is located.

[1 mark]

Question 10c

(c) Find the intervals where f is decreasing.

[2 marks]

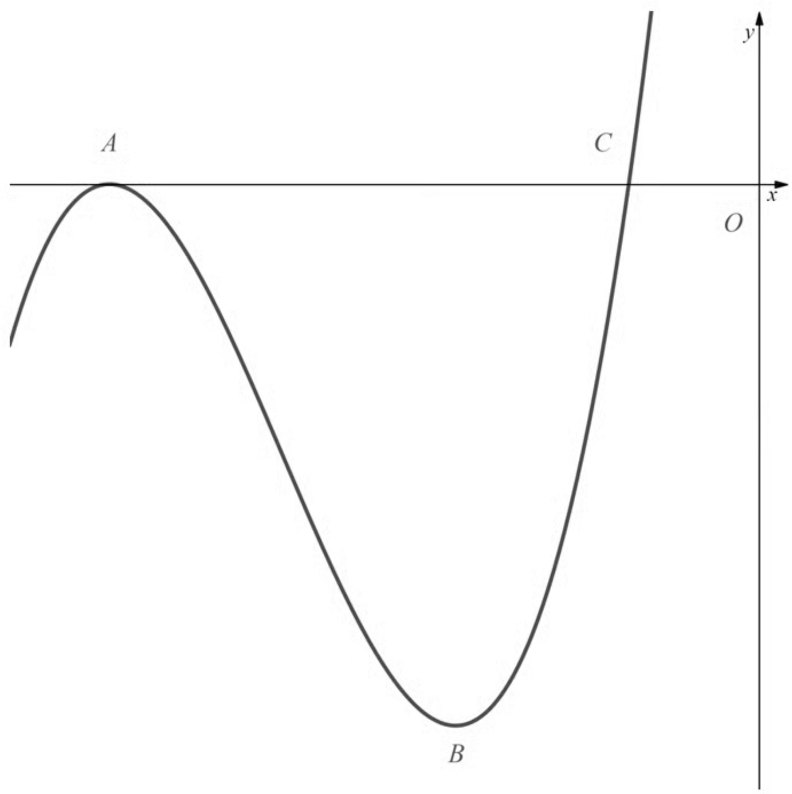
Question 10d

(d) Sketch the graph of $f'(x)$, labelling clearly any intercepts and axis of symmetry.

[3 marks]

Question 11a

The diagram below shows part of the curve with equation $y = x^3 + 11x^2 + 35x + 25$. The curve touches the x-axis at A and cuts the x-axis at C. The points A and B are stationary points on the curve.



(a) Using calculus, and showing all your working, find the coordinates of A and B.

[5 marks]

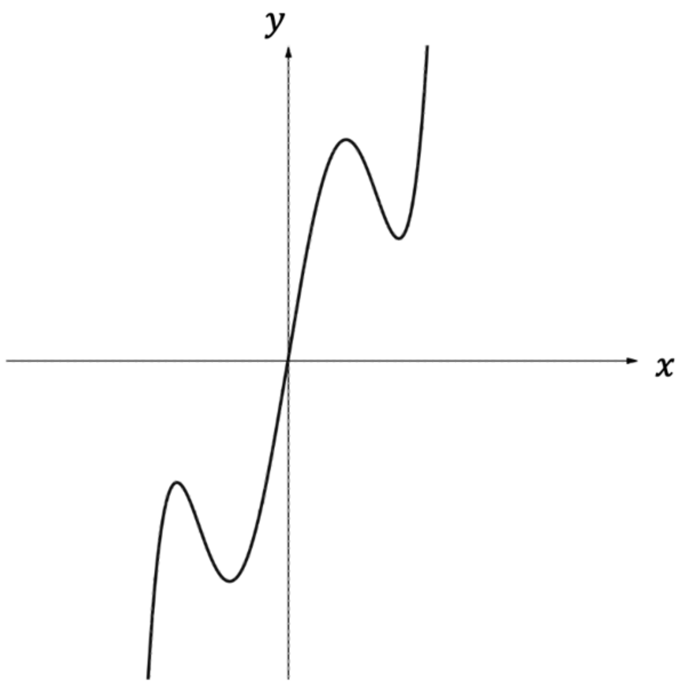
Question 11b

(b) Show that $(-1, 0)$ is a point on the curve and explain why those must be the coordinates of point C.

[2 marks]

Question 12a

The equation of the curve C is $y = \frac{1}{35}x^5 - \frac{3}{4}x^3 + 6x$. A section of the curve C is shown on the diagram below.



(a) Find $\frac{dy}{dx}$.

[2 marks]

Question 12b

There are two points, R and S, along the curve C at which the gradient of the tangent to the curve C is equal to 10.

(b) Calculate the x-coordinates of points R and S.

[4 marks]

Question 13a

(a) Find the x-coordinates of the stationary points on the graph with equation
$y = x^3 - 6x^2 + 9x - 1.$

[4 marks]

Question 13b

(b) Find the nature of the stationary points found in part (a).

[3 marks]

Question 13c

(c) Determine the x-coordinate of the point of inflection on the graph with equation $y = x^3 - 6x^2 + 9x - 1$.

[3 marks]

Question 13d

(d) Explain why, in this case, the point of inflection is not a stationary point.

[1 mark]

Question 14

The graph of a continuous function has the following properties:

The function is concave in the interval $(-\infty, a)$.

The function is convex in the interval (a, ∞).

The graph of the function intercepts the x-axis at the points $(b, 0)$, $(c, 0)$ and $(d, 0)$, where b, c and d are such that $d > c > b > 0$.

The x-coordinates of the turning points of the function are e and f, which are such that $f > e$.

The graph of the function intercepts the y-axis at $(0, g)$

Given that the value of the function is positive when $x = a$, sketch a graph of the function. Be sure to label the x-axis with the x-coordinates of the stationary points and the point of inflection, and also to label the points where the graph crosses the coordinate axes.

[4 marks]