

4.9 Further Normal Distribution (inc Central Limit Theorem)

Question Paper

Course	DPIB Maths
Section	4. Statistics & Probability
Topic	4.9 Further Normal Distribution (inc Central Limit Theorem)
Difficulty	Medium

Time allowed: 80
Score: /65
Percentage: /100

Question 1a

Chiara records the length of time in minutes that it takes for members of a population of cats to find their ways from the edge of a maze to its centre. The time taken, T , follows a normal distribution where $T \sim N(\mu, \sigma^2)$.

Chiara selects a random sample of 7 of the results, these results are displayed below.

4.3, 2.9, 3.5, 4.1, 3.6, 3.1, 3.7

(a)

Determine

(i)

the mean of the sample

(ii)

the variance of the sample.

[3 marks]

Question 1b

(b)

Hence find

(i)

an unbiased estimate of the mean of the population

(ii)

an unbiased estimate of the variance, s_{n-1}^2 of the population.

[3 marks]

Question 1c

It is subsequently discovered that the true standard deviation, σ , for the population is 0.23 minutes.

(c)

Find a 95% confidence interval for the mean of the population.

[2 marks]

Question 2a

Let D be a normally distributed random variable that represents the distance travelled in metres by a slug in one day. The distance covered by a random sample of 21 slugs on a randomly selected day can be summarized as follows

$$\Sigma d = 341, \Sigma d^2 = 5881.$$

(a)

Find an unbiased estimate of the mean, μ , of D .

[1 mark]

Question 2b

(b) Use the formula $s_{n-1}^2 = \frac{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}{n-1}$ to find an unbiased estimate of the variance of D .

[1 mark]

Question 2c

(c)

Find a 95% confidence interval for μ .

[2 marks]

Question 2d

Justin believes that the average slug travels 15 m per day.

(d) State whether or not Justin's statement is valid. Give a reason for your answer.

[2 marks]

Question 3a

A farm grows pumpkins and transports them in crates of 24. The mass of the pumpkins follows a normal distribution with mean 7.9 kg and standard deviation 0.4 kg.

(a)

Find the mean mass of a crate of pumpkins.

[2 marks]

Question 3b

(b)

Find the standard deviation of the mass of a crate of pumpkins.

[2 marks]

Question 3c

(c)

Find the probability that a crate selected at random has a mass of between 170 kg and 190 kg.

[2 marks]

Question 4a

The time taken for a customer services advisor to complete a phone call follows a normal distribution with mean 4.2 minutes and standard deviation 1.3 minutes.

A customer service advisor deals with 5 phone calls one after the other. It is assumed that the phone calls are independent events.

(a)

Find the expected total time to complete the 5 phone calls.

[2 marks]

Question 4b

(b)

Find the variance of the total time to complete the 5 phone calls.

[2 marks]

Question 4c

(c)

Find the probability that the total time taken to complete the 5 phone calls will be more than 25 minutes.

[2 marks]

Question 5a

A fisherman catches 16 fish from a local population of mackerel. He measures the fish that he catches and finds that they have mean length of 30.5 cm with standard deviation 5 cm.

(a)

Find s_{n-1}^2 .

[2 marks]

Question 5b

(b)

Find a 95 % confidence interval for the population mean.

[2 marks]

Question 5c

The fisherman advertises the population from which he fishes as having an average length of 34 cm.

(c)

Comment on the fisherman's claim using your answer from part (b).

[2 marks]

Question 6a

A gardener is laying a pathway of pebbles from a large sack of pebbles. The mass of the pebbles is normally distributed with mean 564 g and standard deviation 57g.

(a)

Find the probability that a pebble that the gardener picks at random from the sack has a mass of less than 500 g.

[2 marks]

Question 6b

The gardener decides that any pebbles that have a mass greater than 620 g are “oversized” and should not be used to create the pathway.

(b)

Find the probability that a pebble selected at random from the sack will be considered “oversized”.

[2 marks]

Question 6c

The gardener decides to select 8 of the pebbles at random.

(c)

Find the probability that the mean mass of the 8 pebbles selected would fall in the “oversized” range.

[3 marks]

Question 7a

In a busy office all workers are able to send jobs to the printer to be printed. It is assumed that each print job is an independent event, and that more than one print job does not arrive in the print queue at the same time. The number of jobs arriving in the print queue in 1 hour follows a Poisson distribution given by $X \sim Po(17)$.

(a)

Find the probability that the number of print jobs sent to the printer in 1 hour is less than 15.

[2 marks]

Question 7b

Helen wants to investigate the mean number of print jobs sent to the printer in an hour over the course of a working week of 35 hours.

(b)

Using the central limit theorem, define a probability distribution that may be used to approximate the distribution of the random variable \bar{X} .

[2 marks]

Question 7c

(c)

Using the answer to part (b), find the probability that in a working week of 35 hours the mean number of print jobs in a single hour is less than 15.

[2 marks]

Question 8a

A population of leatherback turtles has a mean swimming speed of 31 km/h with standard deviation 2.3 km/h.

(a)

Using the central limit theorem, find an estimate for the probability that a sample of 32 leatherback turtles have a mean swimming speed greater than 31.8 km/h.

[2 marks]

Question 8b

(b)

Also using the central limit theorem, find an estimate for the probability that a sample of 50 leatherback turtles have a mean swimming speed greater than 31.8 km/h.

[2 marks]**Question 8c**

(c)

Explain why there is a difference between your answers to part (a) and part (b).

[2 marks]**Question 9a**

On a flower farm the height of a tulip, in centimetres, is normally distributed with mean μ and standard deviation σ . A random sample of 60 flowers is taken from the farm and can be summarised as follows

$$\Sigma h = 1950, \Sigma h^2 = 66075$$

(a)

Find an unbiased estimate for μ .

[1 mark]**Question 9b**

(b)

Given that $s_n^2 = 45$, find an unbiased estimate for the variance of the height of the tulips.

[2 marks]

Question 9c

It is subsequently discovered that the actual standard deviation, σ , of the tulip population is 6.47 cm.

A second sample of 40 tulips is picked.

\bar{H} denotes the mean height of the new sample.

(c)

State a distribution that may reasonably be used to model \bar{H} .

[2 marks]

Question 9d

(d)

Using the answer to part (c), find an estimate for the probability that the mean height of the flowers in the new sample is between 20 and 30 cm.

[2 marks]

Question 10a

Jessamy is interested in the quality of the soil in her local area and decides to test 100 soil samples for levels of nitrogen. From her past research Jessamy knows that the level of nitrogen in an individual sample, N_i , has a mean of 41 ppm and a standard deviation of 7 ppm.

Let $X = \sum_{i=1}^{100} N_i$ be the total of the levels of nitrogen in Jessamy's batch of samples.

(a)

Find

(i)

$E(X)$

(ii)

$\text{Var}(X)$

[3 marks]

Question 10b

(b)

Explain why a normal distribution can be used to give an approximate model for X .

[2 marks]

Question 10c

(c)

Use the model to find the an estimate for the value of such that $P(X < a) = 0.1$.

[2 marks]