3.7 Vector Properties

Question Paper

Course	DP IB Maths
Section	3. Geometry & Trigonometry
Topic	3.7 Vector Properties
Difficulty	Very Hard

Time allowed: 110

Score: /88

Percentage: /100

Question 1a

a)

Given $|\boldsymbol{a}| = 6,\ |\boldsymbol{b}| = 2$ and $\boldsymbol{a} \times \boldsymbol{b} = \begin{pmatrix} -5 \\ 2 \\ 7 \end{pmatrix}$ find the possible values of $\boldsymbol{a} \cdot \boldsymbol{b}$.

[4 marks]

Question 1b

Consider a third vector $\mathbf{c}$, where $|c| = 5$.

b)

Given that the angle between a and c is $\frac{\pi}{3}$,

(i)

find $|a \cdot c|$ and $|a \times c|$,

(ii)

find the component of vector c acting perpendicular to vector a.

[4 marks]

Question 2a

The points A and B have position vectors a and b respectively.

$|a| = \sqrt{35}$, $b = 2\mathrm{i} + 2\mathrm{j} + \mathrm{k}$, $a \times b = \begin{pmatrix} 7 \\ -11 \\ 8 \end{pmatrix}$ and θ is the angle between a and b.

a)
Find $\cos\theta$.

[4 marks]

Question 2b

A third point C is located such that its position vector $c = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

b)
Given that $c = n(a \times b)$ find the unit vector in the direction of c.

[2 marks]

Question 3

$ABCDE$ is a pentagon, where $A(0,4,-1)$, $D(-1,7,3)$, $\overrightarrow{AB} = \begin{pmatrix} -4 \\ 2 \\ 5 \end{pmatrix}$ and $\overrightarrow{CD} = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$.

Given that $\overrightarrow{BD} = \overrightarrow{AE}$ find the area of triangle $B\widehat{C}D$ as a percentage of the total area of the pentagon.

[8 marks]

Question 4a

Three points, $A(-3,-1)$, $B(2,2)$ and C, are located on a straight line where $\overrightarrow{AC} = \lambda\overrightarrow{AB}$. A fourth point D, is located such that $\overrightarrow{DB}$ is perpendicular to $\overrightarrow{OB}$ and $|DB| = 2\sqrt{22}$.

(a)

Find $D\widehat{B}C$.

[3 marks]

Question 4b

(b)
Given that the area of the triangle $\mathrm{BCD} = 53.1$ units^2 correct to 3 significant figures, find λ.

[4 marks]

Question 5

Consider a regular hexagon ABCDEF with sides of length $9\sqrt{2}$ units. The position vectors of A and E are $a = -11\mathrm{i} - 4\mathrm{j} + 5\mathrm{k}$ and $e = 3\mathrm{i} - 3\mathrm{j} - 12\mathrm{k}$ respectively.

Given that the coordinates of F are (r, t, t), where $r, t \in \mathbb{Z}$, $r \neq 0$. find the value of r and t.

[9 marks]

Question 6a

ABCD is a parallelogram defined by the vectors $\overrightarrow{AB} = p$ and $\overrightarrow{AD} = q$, where $p = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$ and $q = \begin{pmatrix} 2t+2 \\ t \end{pmatrix}$.

(a)
Given that the angle $B\widehat{A}D$ is acute, find the range of values for t.

[3 marks]

Question 6b

$\overrightarrow{AB}$ is enlarged by a factor of k.

(b)
Show that $kp \cdot q = k(p \cdot q)$.

[3 marks]

Question 6c

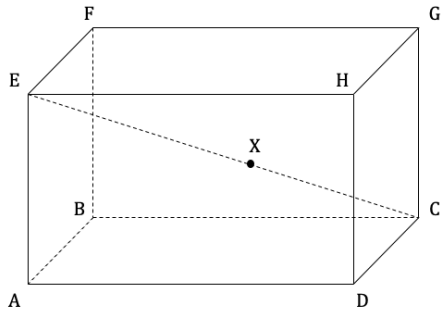
(c)
Given that $k = 7$, find the range of possible values for the area of the enlarged parallelogram.

[4 marks]

Question 7

Consider the cuboid $\mathrm{ABCDEFGH}$ as shown in the diagram below. The position vectors of A, C, D and E are $a = -5\mathrm{i} + 2\mathrm{j} - 4\mathrm{k}$, $c = 2\mathrm{i} + 3\mathrm{j} + \mathrm{k}$, $d = -\mathrm{i} + \mathrm{j}$ and $e = -3\mathrm{i} + \mathrm{j} + 2\mathrm{k}$ respectively.

X is a point located on the line $[\mathrm{EC}]$ such that $\overrightarrow{\mathrm{EX}} = \lambda\overrightarrow{\mathrm{EC}}$.

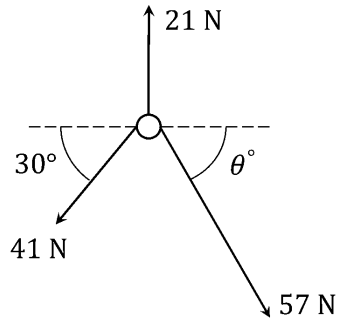


Find the shortest length $|\mathrm{HX}|$.

[7 marks]

Question 8

The following force diagram shows three forces acting on a particle:



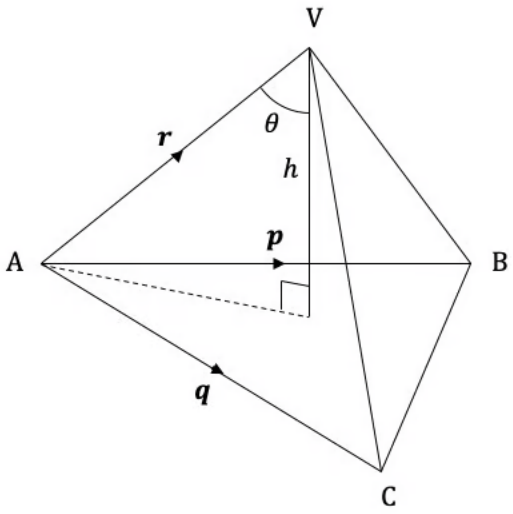
Given that the resultant force on the particle in the vertical direction is 14.195 N downwards, find the size of the angle θ.

[5 marks]

Question 9a

Consider a tetrahedron ABCV where $\overrightarrow{\mathrm{AB}} = p$, $\overrightarrow{\mathrm{AC}} = q$ and $\overrightarrow{\mathrm{AV}} = r$. The perpendicular height, h, of the tetrahedron from the base ABC makes an angle of θ with r.

This information is shown in the diagram below.



(a)
Find an expression for the volume of the tetrahedron in terms of p, q *and* r.

[4 marks]

Question 9b

(b)

Find the volume of the tetrahedron when $p = \begin{pmatrix} 4 \\ 5 \\ -2 \end{pmatrix}, q = \begin{pmatrix} 2 \\ 2 \\ -5 \end{pmatrix}, r = \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix}$

[3 marks]

Question 9c

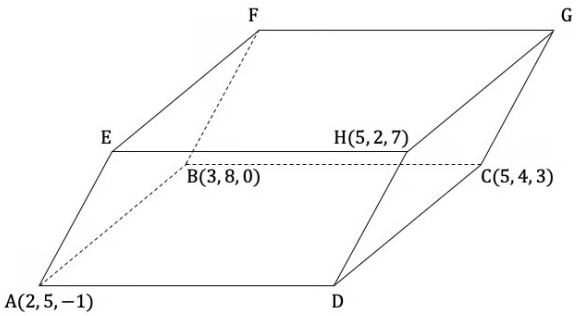
(c)

Hence find the shortest distance between vertex A and its opposite face.

[4 marks]

Question 10

Consider a parallelepiped $ABCDEFGH$ with vertices $A(2,5,-1)$, $B(3,8,0)$, $C(5,4,3)$ and $H(5,2,7)$ as seen in the diagram below.



By first finding an expression for the perpendicular height of the object, find the volume of the parallelepiped.

[8 marks]

Question 11a

An explorer is trying to find some treasure that they believe is hidden in a wild region near an underground river. They set off from a point O and travel at a velocity of $4\ \text{kmh}^{-1}$ along a bearing of $229°$. The river flows in the direction given by the vector $\begin{pmatrix} 1 \\ -5 \end{pmatrix}$.

The base vectors $\mathbf{i}$ and $\mathbf{j}$ represent due east and due north respectively.

b)
Write down the velocity of the explorer as a column vector.

[2 marks]

Question 11b

b)
Find the component of the explorer's velocity in the direction of the underground river.

[4 marks]

Question 11c

After they have travelled for 2.5 hours, the explorer finds the treasure.

c)
Find the perpendicular distance between the location of the treasure and the underground river.

[3 marks]