4.2 Correlation & Regression

Question Paper

Course	DP IB Maths
Section	4. Statistics & Probability
Topic	4.2 Correlation & Regression
Difficulty	Very Hard

Time allowed: 100

Score: /77

Percentage: /100

Question 1a

What to Watch (WTW) and Bingeable are two organisations that review and rank television series. Based on different sets of criteria, scores out of 5 are assigned to 6 recent television series (labelled A to F). The data is shown in the table below.

TV series	A	B	C	D	E	F
WTW'S score (x)	4.6	4.5	3.9	4.8	1.2	1.5
Bingeable's score (y)	4.9	2.5	1.5	3.2	1.1	1.4
WTW's rank						
Bingeable's rank						

(a) (i) Find the Pearson's product-moment correlation coefficient, r, for this data.

(ii) Write down the equation of the regression line y on x.

[4 marks]

Question 1b

(b) Fill in the two empty rank rows in the table above.

[2 marks]

Question 1c

(c) Find the value of the Spearman's rank correlation coefficient, r_s.

[2 marks]

Question 1d

(d) Comment on the difference in the values of r and r_s.

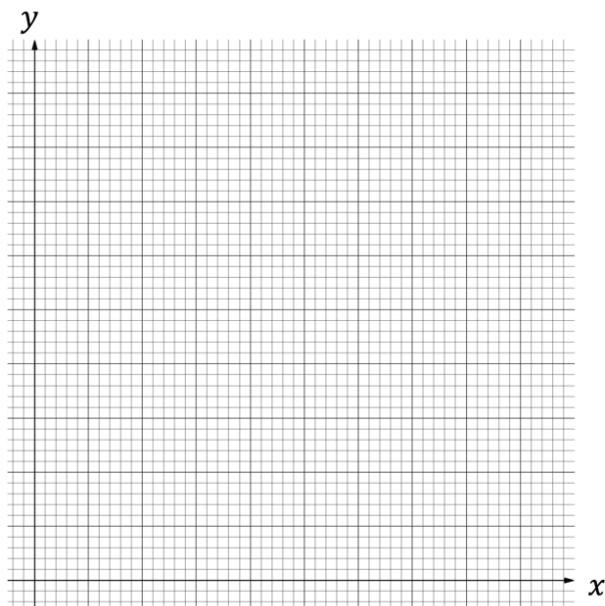**[2 marks]**

Question 2a

The table below shows the lengths, in km, of 5 taxi rides in Melbourne, Australia and the corresponding prices, in AUD.

Length, in km (x)	12.1	4.2	9.1	3.7	6.2
Price, in AUD (y)	26.75	5.75	8.50	5.50	6.95

(a) Draw a scatter diagram for the above data on the axes below.

[4 marks]

Question 2b

(b) Calculate

    (i) $\bar{x}$, the mean taxi ride length

    (ii) $\bar{y}$, the mean price

    (iii) Plot the point M$(\bar{x}, \bar{y})$ on your scatter diagram.

[3 marks]

Question 2c

(c)   (i)   Write down the equation of the regression line y on x.

      (ii)   Draw the regression line y on x on your scatter diagram.

[3 marks]

Question 2d

(d) Show that the point M$(\bar{x}, \bar{y})$ lies on the regression line y on x.

[1 mark]

Question 3a

A health study was conducted on 5 male and 5 female participants, measuring their average daily caffeine intake, in milligrams (mg), and their resting heart rate, in beats per minute (BPM). The following table shows the results of the study.

Average daily caffeine intake, in mg – male (x_m)	222	312	211	190	120
Resting heart rate, in BPM – male (y_m)	57	72	60	48	50
Average daily caffeine intake, in mg – female (x_f)	202	411	254	81	52
Resting heart rate, in BPM – female (y_f)	57	81	71	45	49

(a) Calculate the Pearson's product-moment correlation coefficient for,

   (i)   the male participants, r_m,

   (ii)   the female participants, r_f.

[4 marks]

Question 3b

(b) Write down the equation of the regression line

(i) y_m on x_m

(ii) y_f on x_f.

[4 marks]

Question 3c

(c) Find the intersection of the two regression lines found in part (b) and interpret its meaning.

[3 marks]

Question 4a

The following table shows the distance, in km, to 5 different ferry destinations from Rostock, Germany and the corresponding price of the cruise, in €.

Destination	Copenhagen	Oslo	Stockholm	Helsinki	Riga
Distance, in km (D)	174	620	730	933	810
Price, in € (P)	30.50	65.00	45.75	85.50	125.00

The regression line P on D can be written in the form $P = a + bD$.

(a) Calculate the values of a and b and interpret their meanings

[3 marks]

Question 4b

The distance to Aberdeen from Rostock is 1093 km.

(b) Estimate the cost of the ferry to Aberdeen.

[2 marks]

Question 4c

(c) Comment on the reliability of your estimate found in part (b).

[1 mark]

Question 4d

The ferry company decides to charge €135 to travel to Aberdeen.

(d) Calculate the percentage error between your estimate found in part (b) and the actual price.

[3 marks]

Question 5a

The following table shows the total revenue, R, in £, obtained weekly during the first 7 weeks of 2021 by Larry, an independent financial consultant, and the number of clients, x, served.

Week	1	2	3	4	5	6	7
Revenue, in £ (R)	2452	5751	6429	1203	4587	9786	6911
Clients, x	7	11	14	4	5	8	9

(a) Write down the equation of the regression line R on x.

[2 marks]

Question 5b

Larry's weekly operating costs are £2250 and the cost of serving each client is £35.

(b) Find an expression for the profit Larry makes when serving x clients in a week.

[3 marks]

Question 5c

(c) Estimate the least number of clients required to generate a minimum of £1000 profit.

[3 marks]

Question 6a

Sandy Café is located on a beach and is open all year. The manager wants to see whether the daily average temperature, in °C, is correlated with the average tip they receive, as a percentage of the customer's total bill. He records this data over 9 days and details it in the table below.

Daily average temperature, in °C (x)	22.4	27.8	15.4	12.2	8.8	2.1	33.4	14.7	19.4
Average tip as a percentage of the total bill (y)	20.1	16.3	12.4	12.8	10.1	9.4	18.8	13.1	15.9

(a) (i)   Find the Pearson's product-moment correlation coefficient, r, for this data.

(ii)   Write down the equation of the regression line y on x.

[2 marks]

Question 6b

On the 10th day, the average temperature is 25 °C and a customer tips their waiter $20.

(b) Use the regression line to estimate the customer's total bill. Give your answer to 2 decimal places.

[4 marks]

Question 6c

The customer's total bill was $98.50.

(c)   (i)   Calculate the percentage error between your estimate found in part (b) and the actual total bill.

(ii)   Calculate the tip as a percentage of the actual total bill. Give your answer to the nearest whole number.

[4 marks]

Question 7a

The table below shows the petrol prices, in New Zealand dollars (NZD) per litre, for 6 different petrol stations (labelled A to F) along with their distance **south** of Auckland's city centre.

Petrol station	A	B	C	D	E	F
Distance south of Auckland, in km (x)	122	314	456	231	178	392
Petrol price, in NZD per litre (y)	1.94	1.88	1.78	1.84	1.99	1.81

(a) Calculate the mean petrol price, $\bar{y}$.

[1 mark]

Question 7b

The equation of the regression line y on x can be written in the form $y = a + bx$.

(b) (i)   Calculate the value of a.

(ii) Calculate the value of b, giving your answer in the form $k \times 10^n$,
      where $1 \leq |k| < 10, n \in \mathbb{Z}$.

[3 marks]

Question 7c

The distance between Auckland's city centre and a new petrol station, G, is 200 km and the bearing of G from Auckland's city centre is 166°.

(c) Estimate the petrol price at G.

[2 marks]

Question 7d

Petrol station G decides to set their price at 0.44 NZD per litre below the mean petrol price calculated in part (a).

(d) Calculate the percentage error between the estimated petrol price for G calculated in part (c) and the actual petrol price at G.

[3 marks]

Question 8a

Jezebel is testing the accuracy and consistency of her new thermometer when it measures the temperature of an object. To test her thermometer's readings, Jezebel uses specialist scientific equipment to heat an object to different precise temperatures and uses the thermometer to read the temperature. She repeats this process twice using the same conditions to measure the reliability of her thermometer's reading. The data is shown in the table below.

Actual Temperature (°C)	5.0	15.0	25.0	35.0	45.0	55.0	65.0
1st Reading (°C)	4.6	14.3	24.4	34.3	44.3	54.5	64.6
2nd Reading (°C)	4.6	14.4	24.5	34.3	44.2	54.5	64.3

(a) State the name of this test for reliability.

[1 mark]

Question 8b

(b) Determine whether the thermometer's reading is reliable by calculating the Pearson's product-moment correlation coefficient between relevant sets of data.

[4 marks]

Question 8c

(c) Explain why using only Pearson's product-moment correlation coefficient is not necessarily a good measure of the criterion-related validity of the thermometer's readings.

[2 marks]

Question 8d

(d) Explain whether or not Jezebel's experiment is likely to have content validity for assessing the thermometer's ability of measuring the temperature accurately.

[2 marks]