

5.4 Further Integration

Question Paper

Course	DPIB Maths
Section	5. Calculus
Topic	5.4 Further Integration
Difficulty	Hard

Time allowed: 100
Score: /82
Percentage: /100

Question 1a

Consider the function f defined by $f(x) = (x^2 - x - 2)(x - 5)$, $-2 \leq x \leq 4$.

a)

Find the coordinates of the points where the graph of $y = f(x)$ intercepts the x -axis.

[2 marks]**Question 1b**

b)

Find the indefinite integral

$$\int (x^2 - x - 2)(x - 5) dx$$

[3 marks]**Question 1c**

c)

Use your answer to part (b) to calculate the area of the region enclosed by the graph of $y = f(x)$ and the x -axis.

[2 marks]

Question 2a

a)

Find the indefinite integral for

$$\int \cos\left(\frac{x}{2}\right) dx$$

[2 marks]**Question 2b**

c)

Find an expression for y given that

$$\frac{dy}{dx} = \sin\left(x - \frac{\pi}{3}\right)$$

[2 marks]**Question 3a**

a)

Find the indefinite integral

$$\int \frac{3}{2x} dx$$

[2 marks]

Question 3b

b)

Find an expression for given that

$$\frac{dy}{dx} = e^{2x+3} + 2$$

and also that $y = 5$ when $x = -\frac{3}{2}$.**[3 marks]****Question 4a**

a)

Find the indefinite integral for

$$\int \left(\frac{1}{4} \sqrt{x} - \frac{2}{\sqrt[3]{x}} \right) dx$$

[3 marks]**Question 4b**

b)

Find the indefinite integral for

$$\int \frac{x^{\frac{5}{6}} - 5x^{\frac{7}{3}}}{x\sqrt{x}} dx$$

[3 marks]

Question 5a

a)

Consider the function $f(x) = \ln(3x^2 - 12x + 1)$.

i)

Find $f'(x)$.

ii)

Hence, find

$$\int \frac{16 - 8x}{3x^2 - 12x + 1} dx$$

[6 marks]**Question 5b**

b)

Let $g'(x) = (x^2 - 5x + 6)\sin\left(2x^3 - 15x^2 + 36x - \frac{\pi}{3}\right)$ Find $g(x)$ given that $g(0) = 1$.

[5 marks]

Question 6a

Work out the following indefinite integrals:

a)

$$\int \frac{2}{3\cos^2 3x} dx$$

[2 marks]

Question 6b

b)

$$\int \frac{7\tan\left(\frac{x}{2}\right)}{6\sin\frac{x}{2}\cos\frac{x}{2}} dx$$

[3 marks]

Question 7

Use definite integration to find the exact value of

$$\int_2^5 \frac{x+1}{x^2+2x-5} dx$$

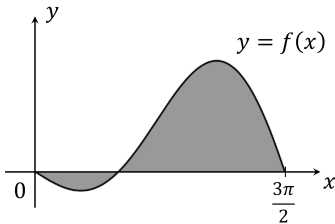
giving your final answer in as simple a form as possible.

[6 marks]

Question 8a

The diagram below shows the graph of the function f which is defined by

$$f(x) = -x \cos x, \quad 0 \leq x \leq \frac{3\pi}{2}$$



The shaded region in the diagram is the region enclosed by the x -axis and the graph of $y = f(x)$.

a)

Explain why the area of the shaded region is not equal to

$$\int_0^{\frac{3\pi}{2}} -x \cos x \, dx$$

[2 marks]

Question 8b

b)

Find the area of the entire shaded region.

[3 marks]

Question 8c

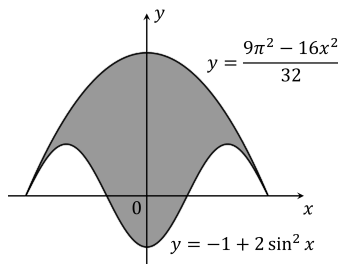
c)

Find the individual areas of the parts of the shaded region (i) above and (ii) below the x -axis.

[4 marks]

Question 9a

The shaded region in the diagram below depicts the design for a new company logo. The upper border of the logo is formed by a part of the curve with equation $y = \frac{9\pi^2 - 16x^2}{32}$, while the lower border of the logo is formed by a part of the curve with equation $y = -1 + 2 \sin^2 x$. The points where the two curves intersect lie on the x -axis, as shown.



a)

Find the exact coordinates of

- (i) the points of intersection of the two curves
- (ii) the other points where the lower border of the logo intersects the x -axis.

[4 marks]

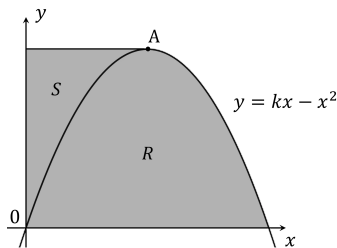
Question 9b

b) Hence find the area of the company logo.

[5 marks]

Question 10a

The following diagram shows a part of the graph of the curve $y = kx - x^2$, where $k > 0$ is a constant. The point marked A is the vertex of the curve. Region R is the region enclosed by the curve and the x-axis. Region S is the region enclosed by the curve, the positive y-axis, and the line through point A with gradient zero.



a) Show that the part of the curve bordering the region can also be represented by the curve with equation $x = \frac{1}{2}(k - \sqrt{k^2 - 4y})$.

[3 marks]

Question 10b

When region R is rotated 2π radians about the x -axis, the resultant solid of revolution has a volume equal to $\frac{1296\pi}{5}$ units³.

(b) Find the value of k .

[5 marks]

Question 10c

c)

Use the result from part (a) to find the area of region S .

[3 marks]

Question 10d

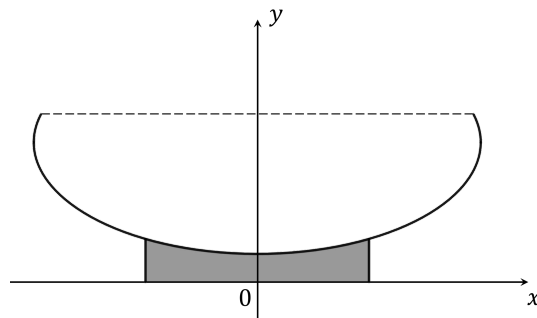
d)

Use your answer to part (c) to write down the area of region R.

[2 marks]

Question 11

The diagram below shows the cross-section of a miniature goldfish bowl produced by Some Things Fishy, a specialist company supplying products for miniature goldfish enthusiasts.



The glass part of the bowl sits on a solid base, indicated by the shaded region on the diagram. The cross-section of the glass part of the bowl is symmetrical about the y -axis, and may be described by the curve with equation

$$\frac{x^2}{64} + \frac{(y-5)^2}{16} = 1$$

The dashed horizontal line represents the diameter of the open top of the fishbowl. All coordinates are expressed in centimetres, and for purposes of answering this question the thickness of the glass sides of the bowl may be regarded as negligible.

Given that the diameter of the open top of the fishbowl is 15 cm, and that this is less than the diameter of the fishbowl at its widest point, find the capacity of the glass part of the fishbowl.

[7 marks]

