3.7 Inverse & Reciprocal Trig Functions

Question Paper

Course	DP IB Maths
Section	3. Geometry & Trigonometry
Topic	3.7 Inverse & Reciprocal Trig Functions
Difficulty	Hard

Time allowed: 80

Score: /62

Percentage: /100

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Question 1

If $x = \sin\left(\frac{2\pi}{3}\right)$ find

(i)
the exact value of $\cos(3 \arccos x)$.
(ii)
the exact value of $\cos(\arcsin x)$.

[6 marks]

Question 2a

(a)
Sketch the graph of $y = \operatorname{cosec} x$ for $-\pi \leq x \leq 2\pi$

[2 marks]

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Question 2b

(b)

Given that $2\ \text{cosec}\ \theta = -\frac{9}{4}$ and $-\pi < \theta < -\frac{\pi}{2}$, find the exact value of $\cot\ \theta$.

[4 marks]

Question 3a

(a)

Show that the equation $33 + \frac{6\cos x}{\cot^2 x} = \sqrt{2}(11\sec x + 2)$ can be rewritten in the form $a\cos^2 x + b\cos x + c = 0$.

[4 marks]

Question 3b

(b)

Hence solve $33 + \frac{6\cos x}{\cot^2 x} = \sqrt{2}(11\sec x + 2)$ for the interval $-\frac{3\pi}{2} \leq x \leq \pi$.

[3 marks]

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Question 4a

(a)

Show that the equation

$$\cot 2\theta \sin \theta + \frac{8}{\sec \theta} = \sin \theta$$

can be rewritten as

$$\tan^2 \theta + 2 \tan \theta - 17 = 0.$$

.

[5 marks]

Question 4b

(b)

Hence solve the equation $\cot 2\theta \sin \theta + \frac{8}{\sec \theta} = \sin \theta$ for all values of θ in the interval $-\pi \leq \theta \leq \frac{\pi}{2}$.

[3 marks]

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Question 5a

Consider the function $f(x) = \frac{1}{2}\arccos x + 1$, where $-1 \leq x \leq 1$.

(a)
Sketch the graph of f indicating clearly any intercepts with the coordinate axes and any maximum and minimum values.

[3 marks]

Question 5b

(b)
Write down the domain of $f^{-1}(x)$.

[1 mark]

Question 5c

(c)
Find an expression for $f^{-1}(x)$.

[2 marks]

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Question 6a

Consider the function $f(x) = \arccos x, \ -1 \leq x \leq 1$.

(a)
State whether the function is f even, odd or neither. Give a reason for your answer.

[2 marks]

Question 6b

A second function g is such that $g(x) = 3f(x) + a$, where $g(x)$ is an odd function.

(b)
Find the value of a.

[2 marks]

Question 6c

(c)
Sketch the graph of $g(x)$ and state the range of g.

[3 marks]

Question 6d

(d)
State whether $g^{-1}(x)$ will also be odd. Give a reason for your answer.

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[2 marks]

Question 7a

(a)

Show that $\frac{\cot\theta \sec^2\theta}{\text{cosec}\,\theta} \equiv \sec\theta$.

[2 marks]

Question 7b

(b)

Hence solve in the range $-\pi \leq \theta \leq 2\pi$, the equation $\frac{\sqrt{3}\cot\theta \sec^2\theta}{\text{cosec}\,\theta} = 2$.

[3 marks]

Question 8a

Consider the function defined by $f(x) = \sec(2\ \arcsin x)$.

(a)

Find the domain of f.

[2 marks]

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Question 8b

(b)

Show that $f(x)$ can be written as $\frac{1}{1-2x^2}$ for all x in its domain.

[6 marks]

Question 9a

(a)

Show that the equation

$$3\left(\frac{1}{\cot^2 x}-\frac{3}{\cos x}\right)=8\sec x+25$$

can be written in the form

$$(a\sec x+b)(\sec x+c)=0.$$

[4 marks]

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Question 9b

a)

Hence solve the equation $3\left(\frac{1}{\cot^2 x}+\frac{3}{\cos x}\right)=8\sec x+25$ in the interval $-270° \leq x \leq 90°$. Give your answers correct to 1 decimal place.

[3 marks]

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