1.3 Sequences & Series

Question Paper

Course	DP IB Maths
Section	1. Number & Algebra
Topic	1.3 Sequences & Series
Difficulty	Hard

Time allowed: 140

Score: /109

Percentage: /100

Question 1a

The sum of the first eight terms of a sequence is 200.

(a) Given that $u_1 = 5.75$, find:

     (i)   the common difference, d, in the case where the sequence is arithmetic

     (ii)   the common ratio, r, in the case where the sequence is geometric.

[4 marks]

Question 1b

(b) Find S_{12}, the sum of the first 12 terms for the arithmetic and geometric sequences found in part (a).

[4 marks]

Question 2a

The third term of a geometric sequence is 270 and the sixth term is -911.25.

(a) Find the 10th term of the sequence.

[4 marks]

Question 2b

(b) Find the sum of the first 21 terms of the sequence.
Give your answer in the form $a \times 10^k$, where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

[4 marks]

Question 3a

The table below shows information about the terms of three different sequences, a_n, b_n and c_n.

	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$
a_n	0.1			2.7		24.3
b_n	24.6		-19		-62.6	
c_n	880		220			-27.5

(a) Determine whether a_n is an arithmetic or geometric sequence and fill in the table accordingly.

[4 marks]

Question 3b

(b) Determine whether b_n is an arithmetic or geometric sequence and fill in the table accordingly.

[4 marks]

Question 3c

(c) Determine whether c_n is an arithmetic or geometric sequence and fill in the table accordingly.

[4 marks]

Question 4a

The 18th term of an arithmetic sequence is 54 and the common difference, d, is 2.2.

(a) Find S_{18}, the sum of the first 18 terms of the arithmetic sequence.

[4 marks]

Question 4b

The first and 18th terms of the arithmetic sequence are the first and second terms respectively of a geometric sequence.

(b) Find the smallest value of n such that $S_n > 10\,000$ for the geometric sequence.

[4 marks]

Question 5a

The fifth term of a geometric sequence is 1 and the common ratio, r, is $\frac{1}{3}$.

(a) Find S_5, the sum of the first five terms of the geometric sequence.

[4 marks]

Question 5b

(b) Find the exact value of the infinite sequence.

[2 marks]

Question 5c

The first and fifth terms of the geometric sequence are the 20th and 10th terms respectively of an arithmetic sequence.

(c) Find the largest value of n such that $S_n < 1000$ for the arithmetic sequence.

[4 marks]

Question 6a

Ashley and Emma are attempting to swim a total of 2000 m each by completing laps of a 25 m pool. Ashley swims her first lap in 17 s and takes 0.2 s longer each lap after that. Emma swims her first lap in 16.5 s and takes 1.01 s times longer each lap after that.

(a) (i)   Find the time Ashley takes to swim her final lap.

     (ii) Find the time Emma takes to swim her final lap.

[4 marks]

Question 6b

(b) (i)   State who swims the 2000 m the fastest.

     (ii) Find the mean lap time for both Ashley and Emma.

[4 marks]

Question 7a

The table below shows information about the terms of two different sequences, a_n and b_n.

	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$
a_n	1		$\dfrac{9}{4}$			$\dfrac{243}{32}$
b_n	18.6			38.1		51.1

(a) State which sequence is arithmetic, and which is geometric.

[1 mark]

Question 7b

(b) Fill in the missing values in the table.

[4 marks]

Question 7c

(c) Find the largest value of n such that $a_n < b_n$.

[3 marks]

Question 8a

A basketball is dropped from a height of 1 m and bounces on the ground n times. The height that the basketball reaches after each bounce forms a geometric sequence. The height of the basketball after the first bounce is 80 cm and the height after the third bounce is 51.2 cm.

(a) Find the common ratio, r, of the geometric sequence.

[2 marks]

Question 8b

(b) Find the height that the ball reaches after the second bounce.

[2 marks]

Question 8c

(c) Find the total vertical distance, in metres, travelled by the basketball after the first four bounces.

[4 marks]

Question 8d

(d) Find the total distance travelled by the ball.

[2 marks]

Question 9a

Since the start of 2020, Malcolm has been on a diet and fitness plan aiming to decrease his waist size. To measure his progress, he has been noting when he goes down a size in trousers. In January he wore a size 46, in April he wore a size 44, in July he wore a size 42 and now, in October, he wears a size 40.

(a) Show that the decrease in Malcolm's size in trousers forms an arithmetic sequence and find how much his size in trousers decreases each month.

[4 marks]

Question 9b

(b) Find the month and year when Malcolm's size in trousers will be 34.

[4 marks]

Question 9c

(c) State a more accurate way Malcolm could measure the reduction in his waist size.

[1 mark]

Question 10a

Grace is a photographer and joins Instagram to advertise her photos. She made one post in the first week and four posts in the fifth week.

(a) (i) Given that the number of posts that Grace makes each week forms an arithmetic sequence, calculate the common difference, d.

    (ii) Comment on the validity of the common difference, d, found in part (a).

[2 marks]

Question 10b

(b) Find the week in which Grace will make her 1000$^{\text{th}}$ post.

[2 marks]

Question 10c

After 11 weeks Grace has 100 followers and after 21 weeks she has 200 followers.

(c) Assuming the increase in Grace's followers forms a geometric sequence, calculate:

    (i) r

    (ii) u_1

[2 marks]

Question 10d

Grace believes that once she reaches 10 000 followers, companies will start paying her to take photographs of their products.

(d) Find the week in which Grace will reach 10 000 followers.

[2 marks]

Question 11a

Georgia buys a new computer for herself that costs $1099. At the same time, she buys her son, Duncan, a new gaming computer that costs $2749.

It is anticipated that Georgia's computer will depreciate at a rate of 11% per year, whereas Duncan's gaming computer will depreciate at 18% per year.

(a) Estimate the value of Georgia's computer after 6 years.

[2 marks]

Question 11b

Georgia and Duncan's computers will have the same estimated value k years after they were purchased.

(b) Find:

  (i)   the value of k

  (ii)   the estimated value after k years.

[4 marks]

Question 12

The sixth term of an arithmetic sequence is equal to 3 and the sum of the first 12 terms is 12.

Find the common difference and the first term.

[6 marks]

Question 13a

A geometric sequence has $u_1 = 135$ and $u_4 = 5$.

(a) Find the common ratio, r.

[2 marks]

Question 13b

(b) Find u_3.

[2 marks]

Question 13c

(c) Find u_7. Give your answer as a fraction.

[2 marks]

Question 13d

(d) Find S_∞.

[2 marks]