2.2 Further Functions & Graphs
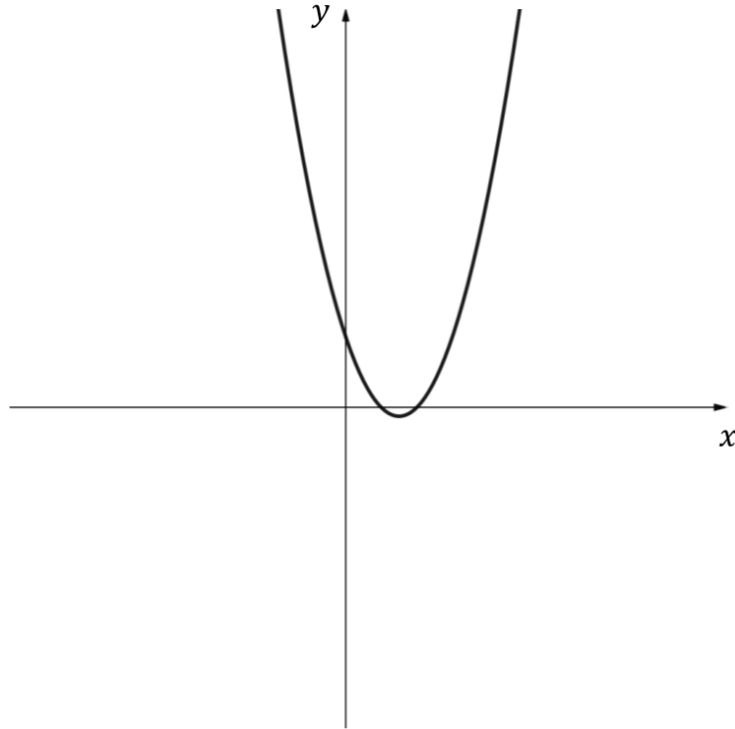
Question Paper

Course	DP IB Maths
Section	2. Functions
Topic	2.2 Further Functions & Graphs
Difficulty	Hard

Time allowed: 100

Score: /77

Percentage: /100

Question 1a

Let $f(x) = x^2 - 3x + 2$. The diagram below shows part of the graph of f.

Another function is defined by $g(x) = 2 - x$.

(a) Sketch the graph of g on the axes above.

[1 mark]

Question 1b

The graph of f and g intersect at points A and B.

(b) Find the coordinates of A and B and label them on the diagram above.

[3 marks]

Question 1c

(c) Find the length of the line AB.

[2 marks]

Question 2a

Let $f(x) = \frac{7}{2(x-7)} - 5$, for $x \neq 7$.

(a) For the graph of f, find the:

    (i) x-intercept

    (ii) y-intercept.

[2 marks]

Question 2b

(b) For the graph of f, write down the equation of any asymptotes.

[2 marks]

Question 2c

Let $g(x) = 2(1 - 2x)$, for $x \in \mathbb{R}$. The graphs of f and g intersect at points P and Q.

(c) Write down the coordinates of P and Q.

[2 marks]

Question 2d

(d) Find the distance of PQ.

[2 marks]

Question 3a

The perimeter, P, and area, A, of a given square can be expressed by $P = 4x$ and $A = x^2$ respectively, where x is the length of the side of the square.

(a) Write down an expression for:

    (i)   P in terms of A, P(A)

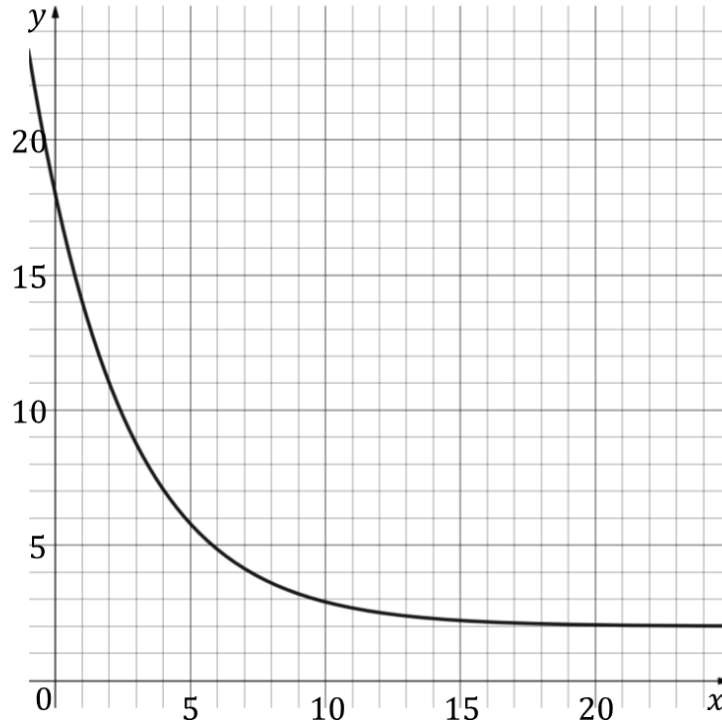   (ii)   A in terms of P, A(P).

[4 marks]

Question 3b

$\mathrm{P}^{-1}(40) = \mathrm{A}(k).$

(b) Find the value of k and $\mathrm{A}(k)$.

[2 marks]

Question 4a

Consider the function $f(x) = a(0.75)^x + b$ where a and b are constants. The graph of f passes through the points $(0, 18)$ and $(2, 11)$ and is shown below.

(a) Write down two equations relating a and b.

[2 marks]

Question 4b

(b) Find the value of a and b.

[2 marks]

Question 4c

(c) Write down the equation of the horizontal asymptote of the graph of f.

[2 marks]

Question 5a

The function $f(x) = ax^2 + bx + c$ intercepts the y-axis at -12 and has an x-intercept at $x = 3$. The function can be obtained by an appropriate shift of the graph $y = -4x^2$.

(a) Find the values of a, b and c.

[4 marks]

Question 5b

(b) Find the other x-intercept of $f(x)$.

[1 mark]

Question 5c

(c) Determine the coordinates of the maximum value of $f(x)$.

[2 marks]

Question 6a

A function is defined by $f(x) = \dfrac{1}{(x-3)^2} + 2$, $x \neq p$.

(a) Find the value of p.

[1 mark]

Question 6b

(b) For the graph of f write down the equation of any asymptotes.

[2 marks]

Question 6c

(c) Find the range of f.

[1 mark]

Question 6d

The line l intersects the graph of f when $x = 1$ and when $x = 4$.

(d) Find the equation of l. Give your answer in the form $ax + by + d = 0$, where a, b and d are integers.

[4 marks]

Question 7a

A function is defined by $f(x) = 4 - \dfrac{12}{5x+9}, \; x \neq a$.

(a) Find the value of a. Give your answer as a fraction.

[2 marks]

Question 7b
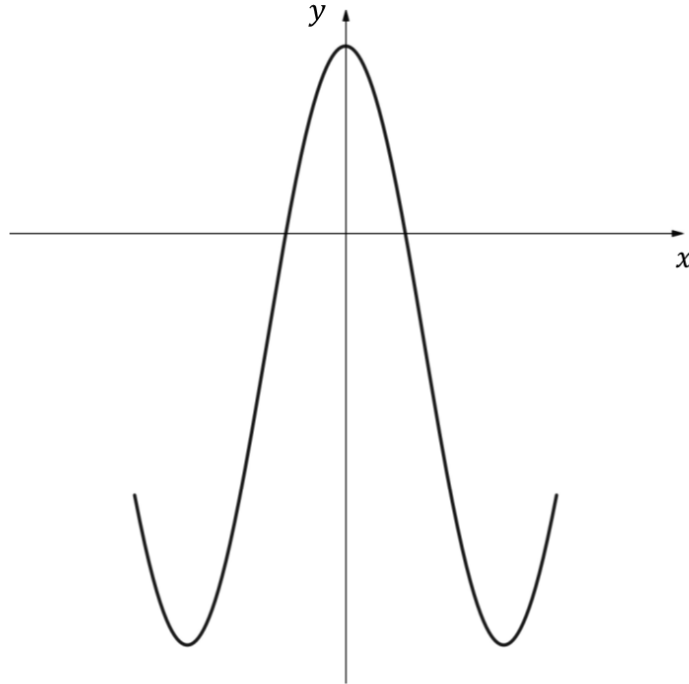
(b) Find the range of f.

[3 marks]

Question 7c

(c) Find the value of $f^{-1}(2)$. Give your answer as a fraction.

[2 marks]

Question 8a

The diagram below shows the graph of the function $f(x) = 8\cos(48x) - 3$, for $-5 \leq x \leq 5$.

(a) For the graph of f, find the:

    (i) x-intercepts

    (ii) y-intercept.

[2 marks]

Question 8b

(b) Write down the range of f.

[1 mark]

Question 8c

(c) Three lines are drawn connecting the two local minimum points and the local maximum point forming a triangle. Calculate the area of the triangle.

[4 marks]

Question 9a

Let $f(x) = 2 - 5\cos(30(3x - 1))$, for $x > 0$.

The nth maximum point on the graph of f has x coordinate x_n, where $n \in \mathbb{Z}^+$.

(a) Given that $x_n = x_1 + (n - 1)d$, find x_1 and d.

[4 marks]

Question 9b

(b) (i) Using sigma notation, write down an expression for $x_1 + x_2 + x_3 + \cdots x_8$.

(ii) Find the value of the sum from part (b) (i). Give your answer to 2 decimal places.

[4 marks]

Question 10a

The average fat-free mass, M, in kg, of footballers as a function of their age, a, in years, can be given by the logarithmic function:

$$M(a) = 10 \log(a - 15) + 50, \qquad 16 \leq a \leq 25.$$

(a) Calculate the average fat free mass of players aged:

(i) 16 years

(ii) 25 years.

[2 marks]

Question 10b

(b) Find an expression for a linear model using your answers to part (a) (i) and (ii).
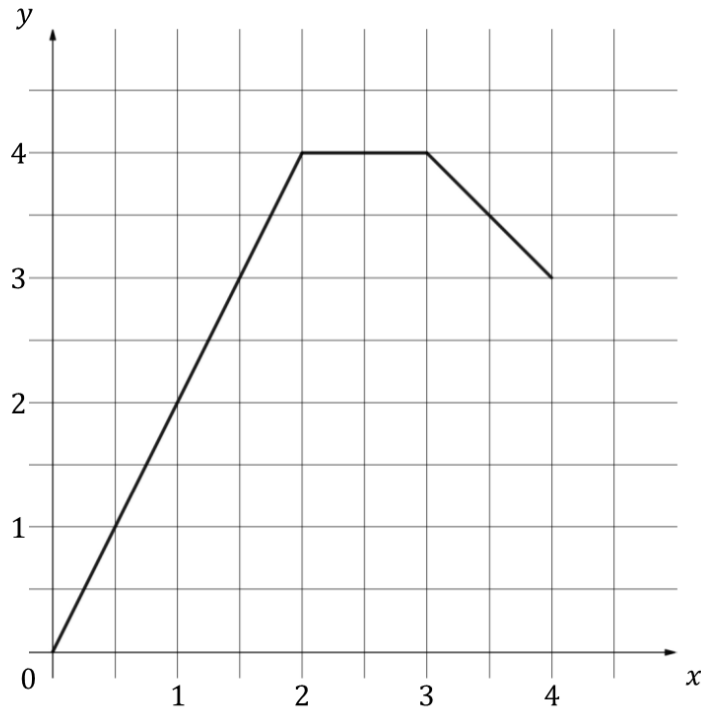
[3 marks]

Question 10c

(c) Calculate the percentage error from using the linear model found in part (b) to approximate the average fat free mass of a player aged 20 years old.

[3 marks]

Question 11a

The axes below shows the graph of the piecewise function, f

The gradient of the graph of f is 0 for $a \leq x < b$.

(a) Find the values of a and b.

[2 marks]

Question 11b

(b) Complete the following piecewise function for f

$$f(x) = \begin{cases} & 0 \leq x < 2 \\ & 2 \leq x < 3 \\ & 3 \leq x \leq 4 \end{cases}$$

[4 marks]