4.10 Poisson Distribution

Question Paper

Course	DP IB Maths
Section	4. Statistics & Probability
Topic	4.10 Poisson Distribution
Difficulty	Medium

Time allowed: 100

Score: /83

Percentage: /100

Question 1a

State which distribution – normal, binomial or Poisson – is likely to be appropriate for calculating the final value of each of the following probabilities. In each case specify any assumptions that would need to be made, and any parameters of which you would need to know the value in order to carry out the calculation.

(a) The probability that the next person to walk through the door of a shop has a height of 1.7 metres or more.

[3 marks]

Question 1b

(b) The probability that exactly 3 people with a height of 1.7 metres or more walk through the door of a shop in the next 20 minutes.

[3 marks]

Question 1c

(c) The probability that of the next 12 people to walk through the door of a shop, exactly three of them have a height of 1.7 metres or more.

[3 marks]

Question 2a

Amira has a bad Internet connection at her house. Her internet disconnects on average 5 times each day.

(a) Define a suitable distribution to model the number of times the internet at Amira's house disconnects during a day. State any assumptions you make.

[2 marks]

Question 2b

(b) Find the probability that during a random day the internet at Amira's house disconnects:

(i) exactly four times

(ii) at most three times

(iii) no fewer than two times.

[4 marks]

Question 3a

Lucy loves the cinema and goes on average four times a week. The number of times she goes to the cinema in a week can be modelled as a Poisson distribution with a mean of four times.

(a) Find the probability that Lucy goes to the cinema exactly five times in a week.

[2 marks]

Question 3b

(b) Find the probability that Lucy goes to the cinema no more than four times in a fortnight.

[2 marks]

Question 3c

(c) Find the probability that Lucy goes to the cinema at least once in a day.

[3 marks]

Question 4a

Comic Stans is a comic book store in the city of Krakoa. Customers enter the store randomly and independently at an average rate of 8 people every 15 minutes.

(a) Find the probability that exactly three people enter the store in a 1-minute period.

[2 marks]

Question 4b

(b) Find the probability that someone enters the store in a 15-second period.

[3 marks]

Question 4c

(c) Find the probability that at most three people enter the store in a 10-minute period.

[3 marks]

Question 4d

(d) Find the variance of the number of people entering the store in a 1-hour period.

[2 marks]

Question 5a

Amber suggests that she can model the number of times that she hiccups using a Poisson distribution.

(a) Write down two conditions that must apply for this model to be applicable.

[2 marks]

Question 5b

The mean number of hiccups in a 30-second period is 2.9.

(b) Assuming a Poisson distribution is applicable, find the probability that

(i) Amber hiccups exactly three times in a 30-second period

(ii) Amber hiccups at least twice but no more than five times in a 15-second period

(iii) Amber hiccups during a one-minute period.

[6 marks]

Question 6a

(a) The table below shows the data from a sample of 50 observations of a variable x.

(i) Calculate unbiased estimates for the mean and the variance.

(ii) State, with a reason, whether a Poisson distribution could be used to model the population's data.

x	0	1	2	3	4
Frequency	3	5	17	15	10

[5 marks]

Question 6b

(b) The table below shows the data from a sample of 100 observations of a variable y.

(i) Calculate unbiased estimates for the mean and the variance.

(ii) State, with a reason, whether a Poisson distribution could be used to model the population's data.

y	0	1	2	3	4	5
Frequency	15	19	25	21	12	8

[5 marks]

Question 7a

Jim is a bird watcher and is trying to model the number of birds that fly past his window. During a 10-minute period he records the number of birds that fly past his window, and he repeats this a total of 120 times to form a sample.

Number of birds	Frequency
0	43
1	44
2	22
3	8
4	3
5 or more	0

(a) Calculate unbiased estimates for the mean and the variance for the number of birds that fly past Jim's window in a ten-minute period.

[4 marks]

Question 7b

(b) Explain why a Poisson distribution would be appropriate to model the number of birds that fly past Jim's window in a 10-minute period.

[1 mark]

Question 7c

Jim uses the distribution $\text{Po}(1)$ to model the number of birds that fly past his window in a 10-minute period.

(c) Use Jim's model to calculate the probability that:

(i) exactly two birds fly past Jim's window in a 30-minute period

(ii) fewer than two birds fly past Jim's window in a 1-minute period

(iii) at least four birds fly past Jim's window in a 1-hour period.

[7 marks]

Question 8a

Roberto orders a pizza from Pizza Palace and asks for two types of meat toppings: ham and salami. It is known that the number of pieces of ham that Pizza Palace put on their pizzas follows a Poisson distribution with a mean of 6.2 pieces per pizza. It is also known that the number of pieces of salami on their pizzas follows a Poisson distribution with a mean of 4.9 pieces per pizza. The ham and salami are put on the pizza independently.

(a) Write down the distribution that can be used to model the total number of pieces of meat on Roberto's pizza.

[1 mark]

Question 8b

(b) Find the probability that Roberto's pizza contains a total of exactly 10 pieces of meat.

[2 marks]

Question 8c

(c) Find the probability that Roberto's pizza contains more than 9 but fewer than 13 pieces of meat.

[3 marks]

Question 9a

André has is a keen amateur astronomer who spends his nights with a telescope trying to discover new comets. Based on his past record of success, the number of times per year that André makes a new discovery may be modelled as a Poisson distribution with mean 0.6.

(a) Use the model to find the probability that André makes exactly one new discovery in any given year.

[1 mark]

Question 9b

(b) Over the course of three consecutive years, find the probability that André

(i) makes exactly two new discoveries

(ii) makes new discoveries in the second and third years only

[5 marks]

Question 9c

André's partner Boglárka is a keen amateur entomologist who spends her spare time trying to discover new species of insects. Based on her past record of success, the number of times per year that Boglárka makes a new discovery may be modelled as a Poisson distribution with mean 1.3.

(c) Find the probability that, between them, André and Boglárka make at least one new discovery over a 3-month period, specifying any assumptions you make.

[5 marks]

Question 9d

(d) Find the probability that over a period of 12 years there will be exactly 4 years during which neither André nor Boglárka make a new discovery.

[4 marks]