

# 4.10 Poisson Distribution

## Question Paper

Course	DPIB Maths
Section	4. Statistics & Probability
Topic	4.10 Poisson Distribution
Difficulty	Hard

**Time allowed:** 100  
**Score:** /79  
**Percentage:** /100

**Question 1a**

State which distribution – normal, binomial or Poisson – is likely to be appropriate for calculating the final value of each of the following probabilities. In each case specify any assumptions that would need to be made, and any parameters of which you would need to know the value in order to carry out the calculation.

- a)  
The probability that at least 7 college students in a class of 30 receive a text message in the next hour.

**[3 marks]****Question 1b**

- b)  
The probability that a randomly selected college student in a class receives at least 7 text messages in the next hour.

**[3 marks]****Question 1c**

- c)  
The probability that a randomly selected college student takes less than 5 minutes to write three random text messages.

**[3 marks]**

**Question 2a**

The random variable  $X$  follows a Poisson distribution which has a standard deviation of 2.25.

a)

Write down the value for  $E(X)$ .

[1 mark]

**Question 2b**

b)

Find  $P(X = 4)$ .

[2 marks]

**Question 2c**

c)

Find  $P(1.5 \leq X < 5)$ .

[3 marks]

**Question 2d**

d)

Find  $P(X = 4 | X > 0)$ .

[3 marks]

### Question 3a

Blue, a dog, barks an average of 25 times every hour. His owner, Hayley, uses a Poisson distribution to model the number of times that Blue barks for any interval of time.

- a)  
Write down two assumptions Hayley has made about Blue's barks in order to use a Poisson distribution.

[2 marks]

### Question 3b

- b)  
Find the probability that Blue barks:  
(i)  
exactly 5 times in a 10-minute period,  
(ii)  
at most 4 times in a 15-minute period,  
(iii)  
more than 47 times but no more than 51 times in a two-hour period.

[7 marks]

### Question 4a

Grace, a grumpy toddler, attends nursery five days a week. The number of tantrums that Grace has in a day follows a Poisson distribution with variance 3.14.

a)

(i)

Write down the distribution for the total number of tantrums that Grace has during a week at nursery. State any assumptions that are needed.

(ii)

Hence, find the probability that Grace has exactly 17 tantrums during a week at nursery.

[2 marks]

### Question 4b

b)

Find the probability that Grace has fewer than four tantrums in a two-day period at nursery.

[3 marks]

**Question 4c**

c)

Given that Grace has fewer than four tantrums at nursery one day, find the probability that she had no tantrums at nursery that day.

**[4 marks]****Question 5a**

The table below shows the number of detentions per school day that Ms Ottway, a teacher, issues to students over a period of 150 days.

Number of detentions	Number of days
0	51
1	54
2	36
3	6
4	3
5 or more	0

a)

By calculating unbiased estimates for the mean and the variance, show that a Poisson distribution is an appropriate model for the number of detentions Ms Ottway issues.

**[4 marks]**

**Question 5b**

b)

Using a Poisson distribution with the unbiased estimate of the mean, find the probability that Ms Ottway issues at least 5 detentions in a day.

**[3 marks]****Question 5c**

Students at Ms Ottway's school attend school 5 days a week for 40 weeks a year.

c)

Estimate the number of weeks in a school year that Ms Ottway issues fewer than 3 detentions.

**[4 marks]**

**Question 6**

$X$ ,  $Y$ , and  $Z$  are random variables with  $W \sim \text{Po}(75)$ ,  $X \sim \text{Po}(x)$  and  $Y \sim \text{Po}(y)$ .

(i)

Find the largest integer value of  $a$  such that  $P(W \leq a) < 0.5$ .

(ii)

Given that  $P(X = 0) = 0.253$ , find the value of  $x$ .

(iii)

Given that  $5 \text{Var}(Y) = (\text{E}(Y))^2 - 6$ , find the value of  $y$ .

[7 marks]

**Question 7a**

In a game, enemies appear independently and randomly at an average rate of 2.5 enemies every minute.

a)

Find the probability that exactly 10 enemies will appear in a five-minute period.

[2 marks]



### Question 7b

b)

Find the probability that at least 3 enemies will appear in a 90-second period.

[3 marks]

### Question 7c

c)

The probability that at least one enemy appears in  $k$  seconds is 0.999. Find the value of  $k$  correct to 3 significant figures.

[3 marks]

### Question 8a

Matt has three best friends: Danny, Jessica and Luke. Matt receives messages from Danny at an average rate of 1.7 messages per hour, from Jessica at an average rate of 1.5 messages per hour, and from Luke at an average rate of 1.1 messages per hour.

a)

Stating any necessary assumptions, find the probability that Matt receives fewer than 4 messages from Danny in a two-hour period.

[3 marks]

**Question 8b**

b)

Stating any additional assumptions needed, find the probability that Matt receives exactly 6 messages in total within a one-hour period.

**[2 marks]****Question 8c**

c)

Find the probability that Matt will receive at least one message from any of the three friends within a one-hour period.

**[2 marks]****Question 8d**

d)

Find the probability that Matt will receive at least one message from any of the three friends within a one-hour period.

**[3 marks]**

**Question 9a**

Mick runs a business printing designs onto t-shirts. He has five identical machines which he uses at the same time. The number of times that a given machine malfunctions within any given time interval can be modelled using a Poisson distribution. Whether or not a given machine malfunctions is independent of whether or not the other machines malfunction. On average, each machine malfunctions at a rate of 0.8 times per hour.

a)

Find the probability that the combined total number of times that the five machines malfunction in a 10-hour period is fewer than 30.

**[3 marks]****Question 9b**

Each day the five machines operate for 10 hours. At the end of each day, Mick asks an engineer to check a machine if it has malfunctioned more than 12 times throughout the day.

b)

Find the probability that at the end of a random day Mick will ask the engineer to check at least two of the machines.

**[4 marks]**