5.4 Further Integration

Question Paper

Course	DP IB Maths
Section	5. Calculus
Topic	5.4 Further Integration
Difficulty	Hard

Time allowed: 100

Score: /82

Percentage: /100

Question 1a

Consider the function f defined by $f(x) = (x^2 - x - 2)(x - 5),\ -2 \leq x \leq 4$.

(a) Find the coordinates of the points where the graph of $y = f(x)$ intercepts the x-axis.

[3 marks]

Question 1b

(b) Hence calculate the area of the region enclosed by the graph of $y = f(x)$ and the x-axis.

[4 marks]

Question 2a

(a) Find the indefinite integral for

$$\int \cos\left(\frac{x}{2}\right)\,\mathrm{d}x$$

[2 marks]

Question 2b

(b) Find the indefinite integral for

$$\int 5e^{3x}\,dx$$

[2 marks]

Question 2c

(c) Find an expression for y given that

$$\frac{dy}{dx} = \sin\left(x - \frac{\pi}{3}\right)$$

[2 marks]

Question 3a

(a) Find the exact value of

$$\int_1^5 \frac{3}{2x}\,dx$$

[3 marks]

Question 3b

(b) Find the definite integral

$$\int_0^{\frac{\pi}{8}} 3\sin 4x \ \mathrm{d}x$$

[3 marks]

Question 3c

(c) Find an expression for y given that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^{2x+3} + 2$$

and also that $y = 5$ when $x = -\frac{3}{2}$.

[3 marks]

Question 4a

(a) Consider the function $f(x) = \ln(3x^2 - 12x + 1)$.

(i) Find $f'(x)$.

(ii) Hence, find

$$\int \frac{16 - 8x}{3x^2 - 12x + 1} \, \mathrm{d}x$$

[6 marks]

Question 4b

(b) Let $g'(x) = (x^2 - 5x + 6)\sin\left(2x^3 - 15x^2 + 36x - \frac{\pi}{3}\right)$

Find $g(x)$ given that $g(0) = 1$.

[5 marks]

Question 5

Use a suitable substitution to show that

$$\int_2^5 \frac{x}{2x-3}\,\mathrm{d}x = \frac{3}{2} + \frac{3}{4}\ln 7$$

[7 marks]

Question 6

Using a suitable trigonometric identity, find the exact value of

$$\int_{\pi}^{3\pi} \sin^2\left(\frac{\theta}{3}\right) \, \mathrm{d}\theta$$

[7 marks]

Question 7

Work out the value of the following definite integral

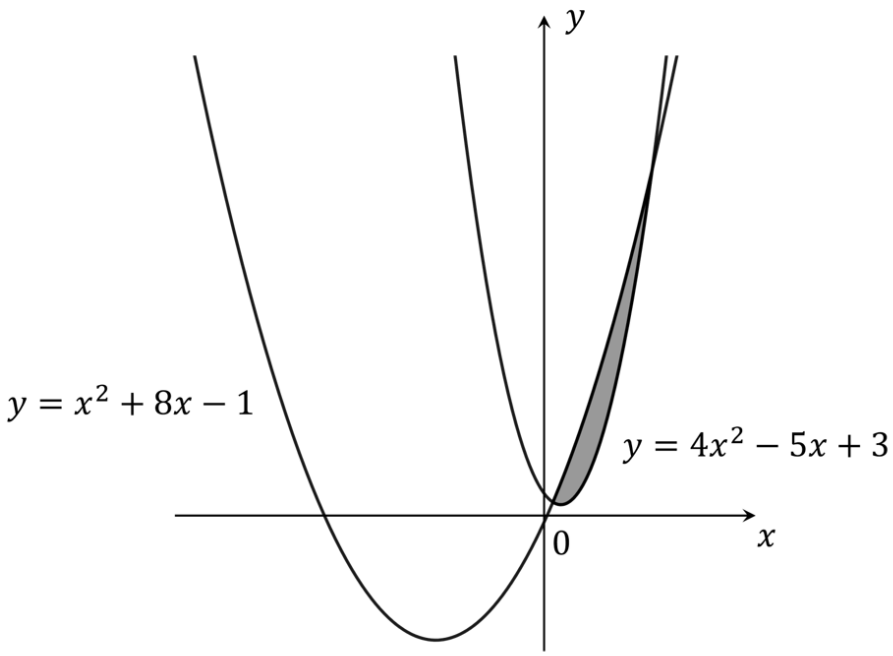
$$\int_2^5 \frac{x+1}{x^2+2x-5}\,\mathrm{d}x$$

giving your answer as an exact value.

[6 marks]

Question 8a

The diagram below shows a sketch of part of the curves with equations $y = x^2 + 8x - 1$ and $y = 4x^2 - 5x + 3$.



The shaded region in the diagram is the area bounded by the two curves.

(a) Show that the area of the shaded region is given by

$$\int_{\frac{1}{3}}^{4} (13x - 3x^2 - 4)\,\mathrm{d}x$$

[4 marks]

Question 8b

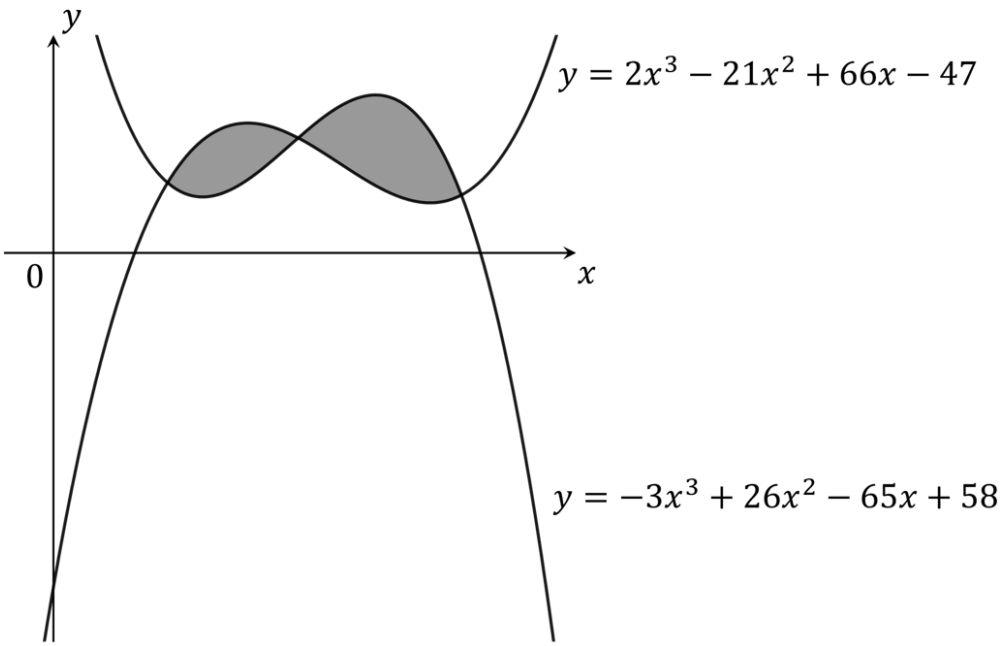
(b) Hence find the area of the shaded region.

[2 marks]

Question 9a

The diagram below shows a sketch of part of the curves with equations

$$y = 2x^3 - 21x^2 + 66x - 47 \quad \text{and} \quad y = -3x^3 + 26x^2 - 65x + 58$$



The shaded region in the diagram is the area bounded by the two curves.

(a) Work out the area of the region bounded by the positive x-axis, the negative y-axis and the graph of $y = 2x^3 - 21x^2 + 66x - 47$

[5 marks]

Question 9b

(b) Work out the area of the shaded region.

[7 marks]

Question 10a

Consider the function $h(x)$ such that

$$\int_0^7 h(x)\,\mathrm{d}x = 19 \quad \text{and} \quad \int_4^7 h(x)\,\mathrm{d}x = 12$$

(a) Find

(i) $\int_0^4 h(x)\,\mathrm{d}x$

(ii) $\int_7^4 h(x)\,\mathrm{d}x$

(iii) $\int_3^3 h(x)\,\mathrm{d}x$

[5 marks]

Question 10b

(b) Find

$$\int_4^7 \frac{4 - h(x)}{5} \, \mathrm{d}x$$

[3 marks]

Question 10c

(c) Find

$$\int_0^7 \left(2h(x) + \frac{3x^2}{7}\right) \mathrm{d}x$$

[3 marks]