3.8 Further Trigonometry

Question Paper

Course	DP IB Maths
Section	3. Geometry & Trigonometry
Topic	3.8 Further Trigonometry
Difficulty	Hard

Time allowed: 80

Score: /60

Percentage: /100

Question 1

Solve the following equations in the given intervals.

(i)

$\cos^2 x - \sin^2 x = \frac{1}{2} \qquad -\pi \leq x \leq \pi$

(ii)

$\frac{2\tan x}{1-\tan^2 x} = -\sqrt{3} \qquad 0 \leq x \leq \pi$

[7 marks]

Question 2a

(a)

Show that $\frac{5\sin(\pi + x)}{\tan x} \equiv -5\cos x \qquad x \neq \frac{n\pi}{2}$.

[3 marks]

Question 2b

(b)

Hence find the exact value of $\sin x$ when $\frac{5\sin(\pi + x)}{\tan x} = 4$ for $\frac{\pi}{2} \leq x \leq \pi$.

[3 marks]

Question 3a

(a)

Show that $\cos x - \sqrt{3}\sin x \equiv 2\cos\left(x + \frac{\pi}{3}\right)$.

[3 marks]

Question 3b

(b)

Hence solve the equation $2\cos\left(x + \frac{\pi}{3}\right) = \sqrt{3}\cos x \sin x - 1$ for the interval $-\pi \leq x \leq \pi$. Show each step of your working.

[4 marks]

Question 4

Show that $2\cos\theta - 5\sin\theta$ can be written in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give R in the form $\sqrt{k}$ where k is an integer.

[5 marks]

Question 5

Given that $\cos x = \frac{\sqrt{2}}{5}$ and $\pi \leq x \leq 2\pi$, find the exact value of $\tan(2x)$.

[6 marks]

Question 6

Prove that $2 \sin 2\theta(2 \cos^2 \theta - 1) \equiv \sin 4\theta$.

[4 marks]

Question 7

Solve the equation $\frac{1}{\cos^2 2x} = 1 + \tan 2x$ for the interval $-\pi < x < 0$.

[7 marks]

Question 8a

(a)

Prove the identity $\frac{1-\sin 2\theta}{2\cos 2\theta} \equiv \frac{1-\tan\theta}{2(1+\tan\theta)}$.

[4 marks]

Question 8b

(b)

Solve the equation $\frac{1-\sin 2\theta}{\cos 2\theta} = 2\sqrt{2}$ for $-\pi \leq \theta \leq \pi$.

[3 marks]

Question 9

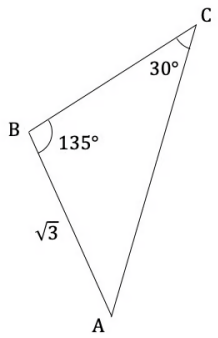
Determine the value of the following expression

$$\cos 55° \cos 5° - \cos 85° \cos 35°.$$

[5 marks]

Question 10

The following diagram shows the triangle ABC where $AB = \sqrt{3}$, $A\hat{B}C = 135°$ and $B\hat{C}A = 30°$.



Find the length AC giving your answer in its simplest form.

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[6 marks]