

6.1 Extended Questions

Question Paper

Course	DPIB Maths
Section	6. Extended Questions
Topic	6.1 Extended Questions
Difficulty	Medium

Time allowed: 120
Score: /93
Percentage: /100

Question 1a

The number of seats a row has at a comedy festival follows a regular pattern where the first row has u_1 seats and the number of seats in each successive row increases by d seats. In the fourth row there are 25 seats and in the sixteenth row there are 49 seats.

- (a) Write down an equation, in terms of u_1 and d , for the number of seats
- (i) in the fourth row
 - (ii) in the sixteenth row.

[2 marks]

Question 1b

- (b) Find the value of u_1 and the value of d .

[2 marks]

Question 1c

The festival has 18 rows of seats in total.

- (c) Calculate the total number of seats.

[3 marks]

Question 1d

The price for a seat in the first row is \$22 and the price decreases by 5% each successive row.

- (d) (i) Find the row in which the price of a seat first falls below \$10.
- (ii) Find the total revenue the comedy festival generates if 22 tickets are sold for every row. Give your answer rounded to the nearest dollar.

[8 marks]

Question 2a

A study was conducted on 6 participants, measuring their body fat percentage (%) and their resting heart rate in beats per minute (BPM). The results are shown in the table below.

Body fat percentage (x)	22.0	14.2	15.5	12.6	29.8	10.1
Resting heart rate (y)	65	59	54	68	74	51

(a) Use your graphic display calculator to find

- (i) \bar{x} , the mean body fat percentage
- (ii) \bar{y} , the mean resting heart rate
- (iii) r , the Pearson's product-moment correlation coefficient.

[3 marks]

Question 2b

- (b) (i) Write down the equation of the regression line of y on x for this data, giving your answer in the form $y = mx + c$ where m and c are constants to be found.
- (ii) Show that the point $A(\bar{x}, \bar{y})$ lies on the regression line of y on x .

[4 marks]

Question 2c

A seventh participant, John, has a resting heart rate of 60 BPM.

- (c) (i) Use the regression line equation to estimate John's body fat percentage.
- (ii) Justify whether it is valid to use the regression line of y on x to estimate John's body fat percentage.

[4 marks]

Question 2d

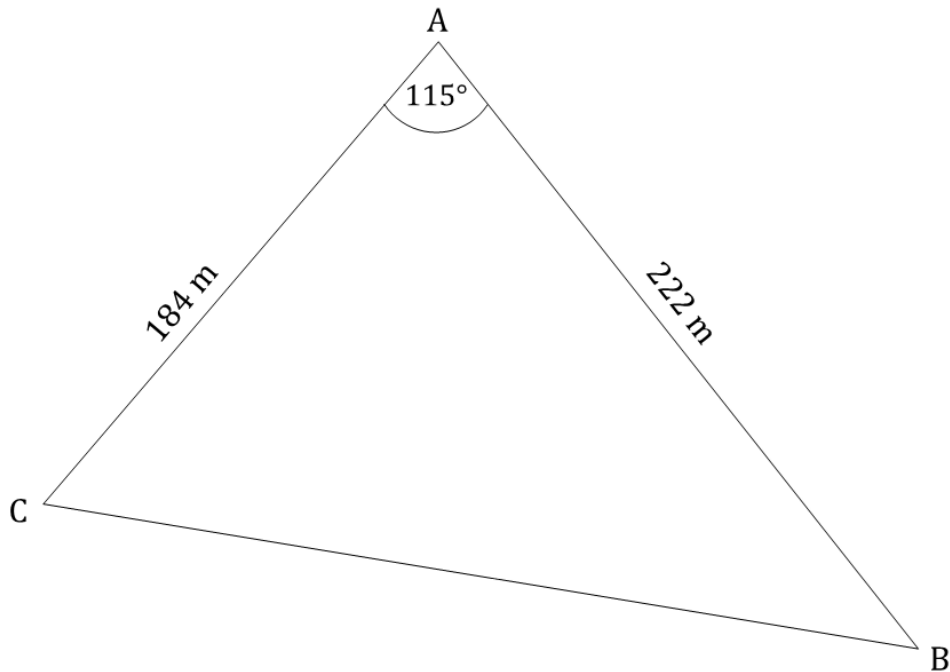
John's body fat percentage is 13.5%.

- (d) Calculate the percentage error in John's estimated body fat percentage from part (c).

[2 marks]

Question 3a

A farm is shown in the diagram below. A motorway runs in a straight line along the edge of the farm from point B to point C, and the farmhouse is located at point A. AB and AC form the other two sides of the farm, and the distances from the farmhouse to points B and C are 222 m and 184 m respectively. Angle \widehat{CAB} is 115° , and points A, B and C lie in a horizontal plane.



(a) Calculate the distance along the motorway from B to C.

[2 marks]

Question 3b

The cost of fencing in US dollars (USD) is \$89.99 per metre.

- (b) Calculate the total cost of fencing the whole perimeter of the farm. Give your answer to 2 decimal places.

[2 marks]

Question 3c

- (c) Calculate the area of the farm.

[2 marks]

Question 3d

- (d) Find the sizes of angles \widehat{ABC} and \widehat{ACB} .

[2 marks]

Question 3e

- (e) Calculate the shortest distance from the farmhouse to the motorway.

[3 marks]

Question 3f

A vertical signpost is located at point C, and the top of the signpost is designated as point D. The angle of elevation to the top of the signpost from point B is measured to be 1.4° .

(f) Calculate the distance CD, the vertical height of the signpost.

[2 marks]

Question 3g

(g) Calculate the distance between the top of the signpost, D, and point A.

[2 marks]

Question 3h

The rate of growth of the grass on the farm, G , in inches per month, can be modelled by the function

$$G(T) = -0.015(T - 40)(T - 80)$$

where T is the temperature in degrees Fahrenheit.

(h) Find the maximum rate of grass growth on the farm and the temperature required.

[3 marks]

Question 4a

The table below shows the distribution of the number of baskets scored by 150 netball players during a weekly game.

Number of baskets	0	1	2	3	4	5	6
Frequency	41	17	34	31	10	15	2

(a) Calculate

- (i) the mean number of baskets scored by a player
- (ii) the standard deviation.

[2 marks]

Question 4b

(b) Find the median number of baskets scored.

[1 mark]

Question 4c

(c) Find the interquartile range.

[2 marks]

Question 4d

(d) Determine if a player who scored 8 baskets would be considered an outlier.

[2 marks]

Question 4e

Two players are randomly chosen.

(e) Given that the first player scored 2 or less baskets, find the probability that both players scored exactly 1 basket.

[4 marks]

Question 4f

The number of hours each player trains each week is normally distributed with a mean of 5 hours and standard deviation of 0.8 hours.

- (f) (i) Calculate the probability that a player trains less than 6 hours a week.
- (ii) Calculate the probability that a player trains less than 4 hours a week.
- (iii) Calculate the expected number of players that train between 4 and 6 hours a week.

[3 marks]

Question 5a

Best Beans is a New Zealand-based company that sells baked beans packaged in cylindrical cans.

(a) Given that their cans have a height of 15 *cm* and a diameter of 8 *cm*, calculate

- (i) the volume of the can
- (ii) the surface area of the can.

[4 marks]

Question 5b

Every month, Best Beans expects to sell x thousand cans of baked beans. It is known that

$$\frac{dP}{dx} = -2x + 472, \quad x \geq 0$$

where P is the monthly profit, in New Zealand dollars (*NZD*), from the sale of x thousand cans of baked beans. It is also known that Best Beans makes a profit of 2450 *NZD* in a month where it sells 8000 cans of baked beans.

(b) Find $P(x)$.

[5 marks]

Question 5c

(c) Find the least number of cans which must be sold each month in order to make a profit.

[3 marks]

Question 5d

(d) Find the monthly sales level that will maximise profit, and the expected profit at this level.

[3 marks]

Question 5e

Best Beans wants to buy a new factory at a cost of 800 000 *NZD*. The CEO decides to invest 60% of the company's monthly profit into a savings account paying a nominal annual interest rate of 5.5%, **compounded monthly**.

(e) Under the assumption that the company's monthly profit will attain its maximum value every month throughout the period, determine whether Best Beans will have saved enough to buy the factory by the end of two years.

[4 marks]**Question 6a**

85 people are asked if they like juice (*J*), tea (*T*) and/or coffee (*C*) for breakfast.

- 12 like all three
- 16 like coffee and tea
- 14 like coffee and juice
- 5 like juice only
- 27 like coffee only
- 14 like tea only

(a)

- (i) Draw a Venn diagram to represent the information provided.
- (ii) Write down the number of people who like coffee but not tea.

[5 marks]

Question 6b

There are 31 people in total who like tea.

- (b) (i) Calculate the number of people who like tea or juice.
- (ii) Find the number of people who like none of the drinks for breakfast.

[4 marks]

Question 6c

A person is chosen at random from the 85 people.

(c) Find the probability that this person

- (i) likes coffee
- (ii) likes coffee and tea but not juice
- (iii) does not like either tea or juice
- (iv) does not like coffee given that the person does not like tea.

[5 marks]