1.1 Number Toolkit

Question Paper

Course	DP IB Maths
Section	1. Number & Algebra
Topic	1.1 Number Toolkit
Difficulty	Very Hard

Time allowed: 70

Score: /55

Percentage: /100

Question 1a

Consider the numbers $a = 11\sqrt{2}, b = (5 + 6\pi), c = \sqrt{2}, d = 6(\pi - 1)$.

(a) Giving your answer to 1 decimal place, calculate the value of

(i) a.

(ii) b.

(iii) c.

(iv) d.

[2 marks]

Question 1b

Points P and Q have coordinates (a, b) and (c, d) respectively.

The formula for the distance, d, between two points with coordinates (x_1, y_1) and (x_2, y_2) is given in your formula booklet.

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

(b) Using your answers from part (a), calculate the distance, d, between points P and Q. Give your answer correct to 1 decimal place.

[2 marks]

Question 1c

(c) Find the percentage error between the distance, correct to 1 decimal place, found in part (b) and the exact distance between points P and Q.

[4 marks]

Question 2a

Let $Y = (pq)^{-1}r$ and $T = pqr^{-1}$, where $p = \sin 60°$, $q = \sqrt{3}$, $r = 2$

(a) Giving your answer to 1 decimal place, calculate the value of

(i) Y.

(ii) T.

[2 marks]

Question 2b

(b) Using your answers to part (a), estimate the value of YT. Give your answer as a fraction.

[1 mark]

Question 2c

(c) Calculate the percentage error between your estimated value of YT found in part (b) and the exact value of YT.

[4 marks]

Question 3a

A cuboid has length, l = 0.102 m, width, w = 9.4 cm and height, h = 0.25 m.

(a) Calculate the exact volume of the cuboid

(i) in cm^3.

(ii) in m^3.

[2 marks]

Question 3b

(b) Write your answers to part (a) (i) and (ii) in the form $a \times 10^k$, where $1 \leq a < 10$, $k \in \mathbb{Z}$.

[2 marks]

Question 3c

William estimates the volume of the cuboid as being Q cm^3 and the percentage error in his estimate is 5%.

(c) Calculate the exact possible values of Q.

[4 marks]

Question 4a

Let $S = (a \sin^2 4b)(c^2 \tan^2 12d)^{-1}\left(\sqrt{a} + c - \cos 48b\right)$, where $a = 16, b = 7.5°, c = 3$ and $d = 5°$.

Note: $\sin^2\theta = (\sin\theta)^2$

(a) Find the value of S, giving your answer as a fraction.

[2 marks]

Question 4b

Let $X = \frac{\sqrt{a}+c^2-2\sin 54d}{\sqrt{a^3}-a-c}$

(b) Find the value of X, giving your answer as a fraction.

[2 marks]

Question 4c

(c) Calculate the value of SX, giving your answer as a fraction.

[2 marks]

Question 4d

John estimates the value of SX to be 0.3.

(d) Calculate the percentage error in John's estimate.

[2 marks]

Question 5a

Consider the numbers $p = 2.41 \times 10^4$ and $q = 4.12 \times 10^5$.

(a) Giving your answers in the form $a \times 10^k$, where $1 \leq a < 10, k \in \mathbb{Z}$, calculate

(i) $p + q$

(ii) $p - q$

(iii) $q - p$

(iv) $\frac{p}{q}$

[4 marks]

Question 5b

The formula for the distance, d, between two points with coordinates (x_1, y_1) and (x_2, y_2) is given in your formula booklet.

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

(b) Using your answers to part (a), estimate the distance between points A$(p + q, p - q)$ and B$\left(q - p, \frac{p}{q}\right)$.

[2 marks]

Question 5c

(c) Calculate the percentage error between the estimate of the distance between points P and Q found in part (b) and the exact distance.

[4 marks]

Question 6a

A shop sells bags of potatoes labelled as "5 kg". The shop owner weighs five bags, in kilograms, at random and recorded the following:

4.96, 4.89, 5.07, 5.11, 5.02

(a) (i) Find the mean of the shop owner's recorded weights.

(ii) Calculate the percentage error between the mean and the stated weight of 5 kg.

[2 marks]

Question 6b

The shop owner shares his findings with his potato supplier, who weighs another five bags and recorded the following:

5.05, 5.01, 4.97, 5.09, X

The supplier allows a maximum percentage error of 1%.

(b) Find the interval for the values of X such that the percentage error from the five bags that the supplier weighed is less than 1%.

[3 marks]

Question 6c

(c) Find the interval for the values of X such that the percentage error from all the ten bags weighed is less than 1%.

[3 marks]

Question 7

Solve the following systems of linear equations using technology.

(i)

$$2x - 5y - 7z = -21$$
$$3z + x - 4y = 44$$
$$x + z - y = 12$$

(ii)

$$z - x - y = -11$$
$$5x + 11z - 2y = -28$$
$$3y - 4z + x = 30$$

[6 marks]