3.8 Vector Equations of Lines

Question Paper

Course	DP IB Maths
Section	3. Geometry & Trigonometry
Topic	3.8 Vector Equations of Lines
Difficulty	Medium

Time allowed: 80

Score: /62

Percentage: /100

Question 1a

The points A and B are given by $A(4, 2, -3)$ and $B(0, 5, 1)$.

(a) Find a vector equation of the line L that passes through points A and B.

[3 marks]

Question 1b

(b) Determine if the point $C(-1, 3, 2)$ lies on the line L.

[3 marks]

Question 2

Find the vector equations of a line that is parallel to the vector $\boldsymbol{a} = 3\mathbf{i} - 4\mathbf{j} + \boldsymbol{k}$ and passes through the point $X(3, -2, 0)$.

[5 marks]

Question 3

Find the equation of the line that is perpendicular to the vector $4\boldsymbol{i} + 5\boldsymbol{j}$ and passes through the point $\mathrm{P}(7, -1)$, leaving your answer in the form $ax + by + c = 0$, where $a,\ b$ and $c \in \mathbb{Z}$.

[6 marks]

Question 4a

Consider the two lines l_1 and l_2 defined by the equations:

$$l_1: \boldsymbol{a} = \begin{pmatrix} 4 \\ 1 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -3 \\ -5 \end{pmatrix}$$

$$l_2: \boldsymbol{b} = \begin{pmatrix} 5 \\ -11 \\ 10 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 6 \\ 2 \end{pmatrix}$$

a)
Find the scalar product of the direction vectors.

[2 marks]

Question 4b

b)
Hence, find the angle, in radians, between the l_1 and l_2.

[4 marks]

Question 5a

Consider the lines l_1 and l_2 defined by:

$$l_1: \begin{cases} x = 3 - \mu \\ y = -2 + 5\mu \\ z = 4 + 2\mu \end{cases}$$

$$l_2: \boldsymbol{r} = \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 2 \\ 2 \end{pmatrix}.$$

(a) Show that the lines are not parallel.

[2 marks]

Question 5b

(b) Hence, show that the lines l_1 and l_2 do not intersect.

[5 marks]

Question 6a

Consider the line l which can be defined by both $\boldsymbol{r_1} = \begin{pmatrix} t \\ -2 \\ 5 \end{pmatrix} + \alpha \begin{pmatrix} -5 \\ 2 \\ 1 \end{pmatrix}$ and $\boldsymbol{r_2} = \begin{pmatrix} -3 \\ 6 \\ 9 \end{pmatrix} + \beta \begin{pmatrix} 15 \\ 3k \\ -3 \end{pmatrix}$.

(a) Find the value of k.

[2 marks]

Question 6b

(b) Find the value of t.

[4 marks]

Question 7a

Consider the line l_1, which can be represented by the equation $\boldsymbol{r} = \begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix}$ and l_2, which can be represented by the equation $\boldsymbol{s} = (3 - \mu)i + (1 - \mu)j + (5 + 7\mu)k$.

(a) Write down the equation for l_2 in its vector form.

[2 marks]

Question 7b

(b) Find vector product of the direction vectors of l_1 and l_2.

[2 marks]

Question 7c

(c) Hence find the angle between l_1 and l_2.

[3 marks]

Question 8a

The lines l_1 and l_2 can be defined by:

$$l_1 : \boldsymbol{r} = \begin{pmatrix} 2 \\ -5 \\ 1 \end{pmatrix} + \alpha \begin{pmatrix} 3 \\ 2 \\ k \end{pmatrix}$$

$$l_2 : \boldsymbol{s} = \begin{pmatrix} -3 \\ -4 \\ 2 \end{pmatrix} + \beta \begin{pmatrix} -11 \\ -3 \\ 5 \end{pmatrix}$$

a)
Write down the parametric equations for l_1.

[2 marks]

Question 8b

b)
Given that l_1 and l_2 intersect at point T,

(i)
find the value of k.

(ii)
determine the coordinates of the point of intersection, T.

[7 marks]

Question 9a

Consider the triangle ABC. The points A, B and C have coordinates $(4, 0, -3)$, $(2, -2, -1)$ and $(8, 1, 5)$ respectively.

M is the midpoint of [AB].

(a) Find the coordinates of the midpoint M.

[2 marks]

Question 9b

(b) Hence, find a vector equation of the line, l, that passes through points C and M.

[2 marks]

Question 9c

(c) Show that the line l is perpendicular to [AB].

[3 marks]

Question 9d

(d) Hence calculate the area of the triangle ABC.

[3 marks]