4.7 Hypothesis Testing

Question Paper

Course	DP IB Maths
Section	4. Statistics & Probability
Topic	4.7 Hypothesis Testing
Difficulty	Hard

Time allowed: 100

Score: /79

Percentage: /100

Question 1a

A new technology company, TechBright, has developed a battery that they claim has a longer lifespan than the product sold by its main competitor, Elektrik. A survey has been completed recording the battery life, in hours, of 12 batteries from each company. The results are shown in the table below.

TechBright	10.2	13.8	12.6	13.5	11.8	15.3	12.9	13.2	13.1	12.1	12.3	13.2
Elektrik	11.1	12.4	12.2	13.6	13.5	9.5	12.6	13.0	11.8	12.2	12.4	12.1

(a) State the null and alternative hypotheses.

[2 marks]

Question 1b

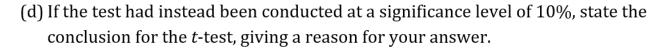(b) Perform a t-test at the 5% significance level.

[2 marks]

Question 1c

(c) State the conclusion of this test, giving a reason for your answer.

[2 marks]

Question 1d

(d) If the test had instead been conducted at a significance level of 10%, state the conclusion for the t-test, giving a reason for your answer.

[2 marks]

Question 1e

(e) Determine if TechBright is correct in claiming that their batteries have a longer lifespan than those created by Elektrik. Justify your answer.

[2 marks]

Question 2a

A football coach wants to know if the number of hours spent training by a team in the week before a match affects the outcome of the game.

Data is collected for 100 matches, with the number of hours spent training during the preceding week the match, h, recorded alongside the result of win, lose or draw. Some of this data is included in the following table:

	Win	**Lose**	**Draw**	**Total**
$0 \leq h < 8$	4		11	31
$8 \leq h < 16$		19	12	
$16 \leq h < 24$	18	4	1	23
Total	37	39		100

(a) Complete the table of observed data.

[2 marks]

Question 2b

(b) State the null hypothesis.

[1 mark]

Question 2c

(c) Calculate the expected number of losses for a team that trains between 16 and 24 hours in the week before a match if training time and match results are independent of each other.

[2 marks]

Question 2d

(d) Find the number of degrees of freedom.

[2 marks]

Question 2e

(e) Calculate the χ^2 test statistic for this data, testing at the 10% significance level.

[2 marks]

Question 2f

(f) The critical value is 7.779. State the conclusions obtained and justify your answer.

[2 marks]

Question 3a

A spinner used on a game show is divided into 6 equal parts coloured red, yellow, green, blue, purple and orange. There are reports that the spinner is not fair so an experiment is conducted to determine if this is the case. The spinner is spun 720 times.

(a) Calculate the number of times you would expect the spinner to land on red.

[1 mark]

Question 3b

The results of the experiment are shown in the table below:

	Red	**Yellow**	**Green**	**Blue**	**Purple**	**Orange**
Number of spins	117	143	122	98	115	125

(b) Write down the statistical test that can be performed to determine if the spinner is fair by comparing the observed results with the expected results.

[1 mark]

Question 3c

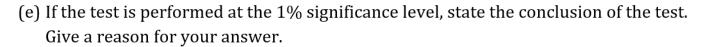(c) Write down the null and alternative hypotheses.

[2 marks]

Question 3d

(d) Perform the test, using technology, and find the p-value.

[2 marks]

Question 3e

(e) If the test is performed at the 1% significance level, state the conclusion of the test. Give a reason for your answer.

[2 marks]

Question 3f

(f) Comment on the suitability of the significance level. Justify your answer.

[2 marks]

Question 3g

A second spinner undergoes the same experiment, which results in a p-value of 0.082.

(g) Explain how the difference in the p-values could be used to comment upon the fairness of the two spinners.

[2 marks]

Question 4a

150 people are surveyed as part of an investigation into the relationship between gender and sport. The researcher asks people to choose their preferred sport for viewing from a choice of football, basketball, tennis, gymnastics and swimming. The results of the survey are shown in the table below.

	Football	Basketball	Tennis	Gymnastics	Swimming
Female	23	10	15	18	20
Male	30	14	12	1	7

A chi-squared test is performed on the data at a significance level of 5%.

(a) State the null and alternative hypotheses.

[2 marks]

Question 4b

(b) Using the table of critical values for a significance level of 5%, given below, write down the appropriate critical value.

Degrees of Freedom	Critical Value
1	3.841
2	5.991
3	7.815
4	9.488
5	11.070

[1 mark]

Question 4c

(c) Calculate the χ^2 statistic.

[2 marks]

Question 4d

(d) State the conclusion to the test, giving a reason for your answer.

[2 marks]

Question 5a

A gardener believes that sunflowers situated in areas of full sun will grow taller than sunflowers planted in partial sun. To investigate this, the gardener measures the heights, in inches, of sunflowers growing under both conditions. The results are shown in the table below.

Full sun	Partial sun
72.4	85.4
86.0	71.5
91.1	86.3
83.2	73.4
112.8	74.1
106.8	70.8
87.9	89.4
	93.1
	73.3
	76.4

A t-test is performed at a significance level of 5%.

(a) State the null and alternative hypotheses.

[2 marks]

Question 5b

(b) Calculate the p-value for the data.

[2 marks]

Question 5c

(c) Comment on the results of the test.

[2 marks]

Question 5d

(d) Comment on the result of the test if the significance level had instead been conducted at a significance level of 1%.

[2 marks]

Question 5e

(e) If the experiment were to be repeated, suggest one step that could be taken to increase the validity of the results.

[1 mark]

Question 6a

A student is investigating the relationship between a country's GDP and its literacy rate for his IA. He has classified 140 countries into low, medium and high GDP and their literacy rates into low, medium and high. His results are displayed in the table below.

	Low literacy	Medium literacy	High literacy
Low GDP	26	15	3
Medium GDP	13	21	14
High GDP	1	11	36

Perform a chi-squared test on the data at a 5% significance level to test the hypothesis that the literacy rate of a country is dependent on its GDP. The critical value is 9.488.

(a) State the null and alternative hypotheses.

[2 marks]

Question 6b

(b) Write down the number of degrees of freedom.

[1 mark]

Question 6c

(c) Calculate the p-value and the χ^2 statistic. You should justify any conclusions found.

[5 marks]

Question 6d

(d) Describe one possible issue with the way the data is presented, that might make it difficult to interpret the validity or precise implications of the test's conclusions.

[2 marks]

Question 7a

A computer game has 5 levels. At the end of each level a magic star may appear, doubling your score from that level. The game is played through all levels 200 times by a focus group. The number of times in total that the magic star appears for each game of 5 levels is recorded, and the results for all 200 games are summarised in the table below.

Number of magic stars	0	1	2	3	4	5
Frequency	7	24	70	59	38	2

The game developer wants to investigate whether the results can be modelled using the binomial distribution $B(5, 0.5)$.

(a) State the null and alternative hypotheses.

[3 marks]

Question 7b

(b) Draw a table of expected frequencies.

[3 marks]

Question 7c

(c) A chi-squared goodness of fit test is performed at the 10% significance level. State the χ^2 statistic.

[2 marks]

Question 7d

(d) The critical value for the test is 9.236. Comment on the results of the test, justifying your answer.

[2 marks]

Question 8a

In healthy adults, systolic blood pressure is normally distributed with a mean of 112 mmHg and standard deviation of 10 mmHg. A group of 200 patients take part in a clinical trial and their systolic blood pressure, p, is measured and recorded below.

Systolic blood pressure (mmHg)	Frequency
$p \leq 95$	6
$95 < p \leq 105$	59
$105 < p \leq 115$	65
$115 < p \leq 125$	63
$p > 125$	7

A chi-squared goodness of fit test at a significance level of 5% is used to determine if the sample of patients is representative of the general population in terms of their blood pressure.

(a) State the null and alternative hypotheses.

[3 marks]

Question 8b

(b) Complete the table of expected frequencies below, giving the frequencies to 4 decimal places.

Systolic blood pressure (mmHg)	Expected frequency
$p \leq 95$	
$95 < p \leq 105$	39.4796
$105 < p \leq 115$	75.1896
$115 < p \leq 125$	
$p > 125$	

[3 marks]

Question 8c

(c) State the number of degrees of freedom.

[1 mark]

Question 8d

(d) Calculate the p-value and comment on the results of the test.

[3 marks]