5.3 Integration

Question Paper

Course	DP IB Maths
Section	5. Calculus
Topic	5.3 Integration
Difficulty	Medium

Time allowed: 90

Score: /71

Percentage: /100

Question 1a

A curve $y = f(x)$ passes through point $A(4, 2)$ and has a gradient of $f'(x) = 5x - 2$.

(a) Find the gradient of the curve at point A.

[2 marks]

Question 1b

(b) Find the equation of the tangent to the curve at point A.
Give your answer in the form $y = mx + c$.

[2 marks]

Question 1c

(c) Determine the equation of the curve $y = f(x)$.

[3 marks]

Question 2a

A point $P(3, 8)$ lies on the curve $y = f(x)$ that has a gradient of $f'(x) = -2x^2 + 11$.

(a) Find the gradient of the curve at point P.

[2 marks]

Question 2b

(b) Find the equation of the tangent to the curve at point P.
Give your answer in the form $y = mx + c$.

[2 marks]

Question 2c

(c) Determine the equation of the curve $y = f(x)$.

[3 marks]

Question 3a

The following table shows the x and y coordinates of five points that lie on a curve $y = f(x)$.

x	0	0.25	0.5	0.75	1
$y = f(x)$	1	2.25	4	6.25	9

(a) Estimate the area under the curve over the interval $0 \leq x \leq 1$.

[2 marks]

Question 3b

The equation of the curve was found to be $y = (2x + 1)^2$.

(b) Find the exact value of the area under the curve over the interval $0 \leq x \leq 1$.

[2 marks]

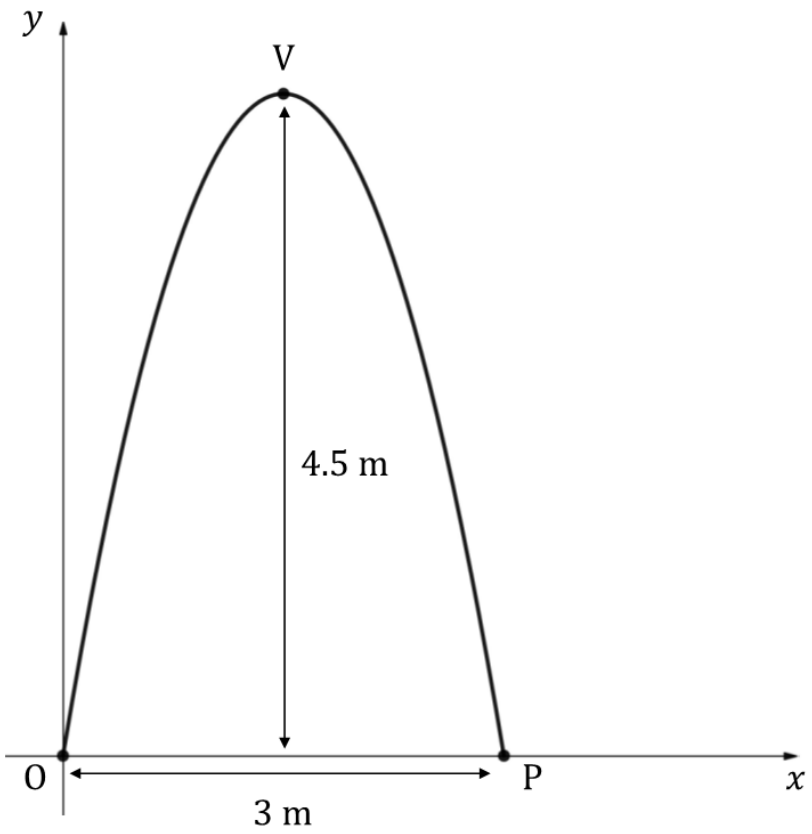
Question 3c

(c) Find the percentage error between the estimation in part (a) and the exact value in part (b). Provide a reason for the difference.

[2 marks]

Question 4a

The following diagram shows an arch that is 4.5 m tall and 3 m wide. The arch crosses the x-axis at the origin, O, and at point P, and its vertex is at point V. The arch may be represented by a curve with an equation of the form $y = x(ax + 6)$, where all units are measured in metres.



(a) Find

(i) the coordinates of P

(ii) the coordinates of V

(iii) the value of a.

[4 marks]

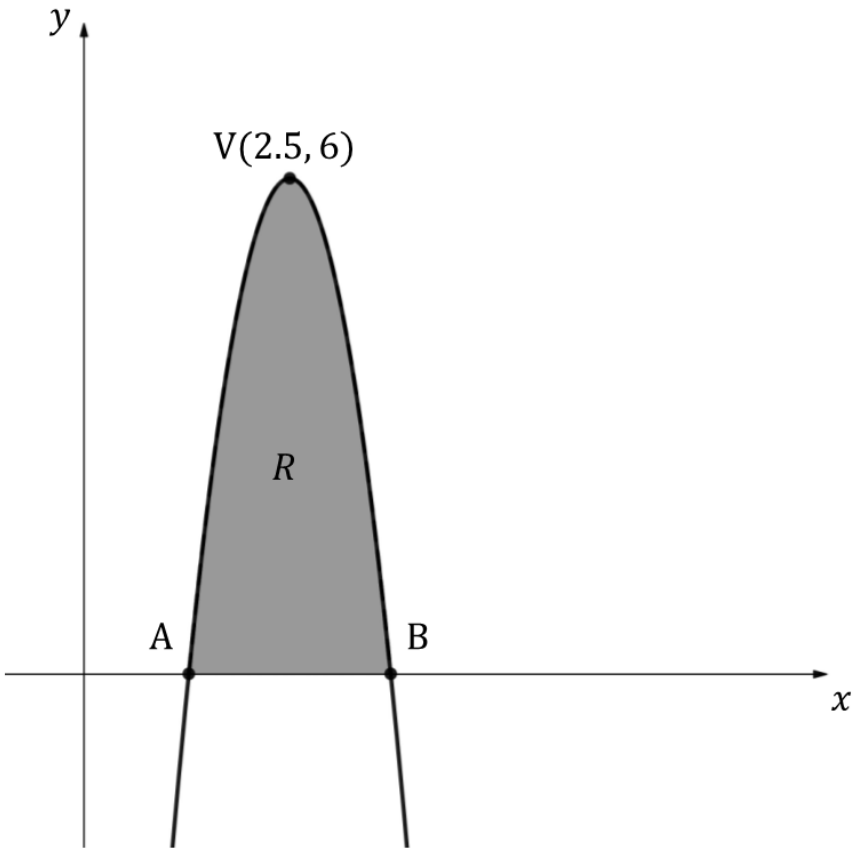
Question 4b

(b) Find the cross-sectional area under the arch.

[2 marks]

Question 5a

The diagram below shows a part of the curve $y = -4x^2 + px + q$. Points A and B represent the x-intercepts, point $\text{V}(2.5, 6)$ represents the vertex of the curve, and the shaded region R represents the area between the curve and the x-axis.



(a) Find the values of p and q.

[2 marks]

Question 5b

(b) Find the coordinates of points A and B.

[4 marks]

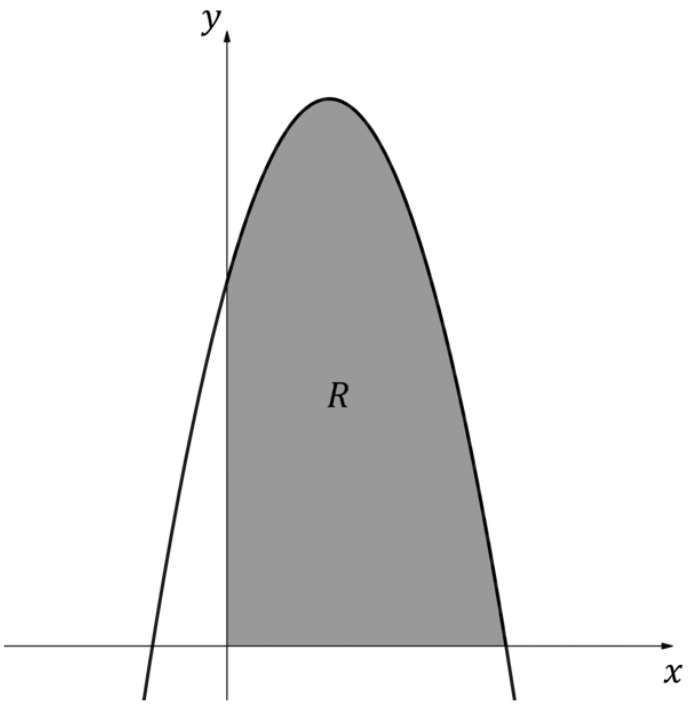
Question 5c

(c) Find the area of region R.

[2 marks]

Question 6a

The following diagram shows part of the graph of $f(x) = (5 - 2x)(2 + 3x),\ x \in \mathbb{R}$. The shaded region R is bounded by the x-axis, the y-axis and the graph of f.



(a) Write down an integral for the area of region R.

[2 marks]

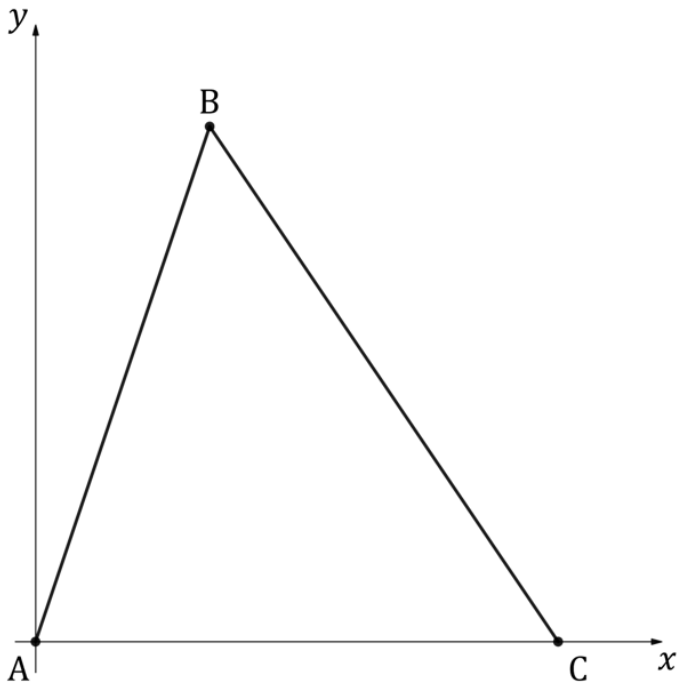
Question 6b

(b) Find the area of region R.

[1 mark]

Question 6c

The three points $A(0, 0)$, $B(4, h)$ and $C(9, 0)$ define the vertices of a triangle.



(c) Find the value of h, the y-coordinate of B, given that the area of the triangle is equal to the area of region R.

[2 marks]

Question 7a

A rice farm sells x kg of rice every week.

It is known that $\frac{dP}{dx} = -0.02x + 6, x \geq 0,$ where P is the weekly profit, in dollars ($), from the sale of x kg of rice.

(a) Find the amount of rice, in kg, that should be sold each week to maximise the profit.

[3 marks]

Question 7b

The profit from selling 250 kg of rice is \$480.

(b) Find $P(x)$.

[5 marks]

Question 8a

A paint company sells x hundred of litres of paint every week.

It is known that $\frac{dP}{dx} = -1.9x + 145, x \geq 0,$ where P is the weekly profit, in euros (€), from the sale of x hundred litres of paint.

(a) Find the number of litres that should be sold each week to maximise the profit.

[3 marks]

Question 8b

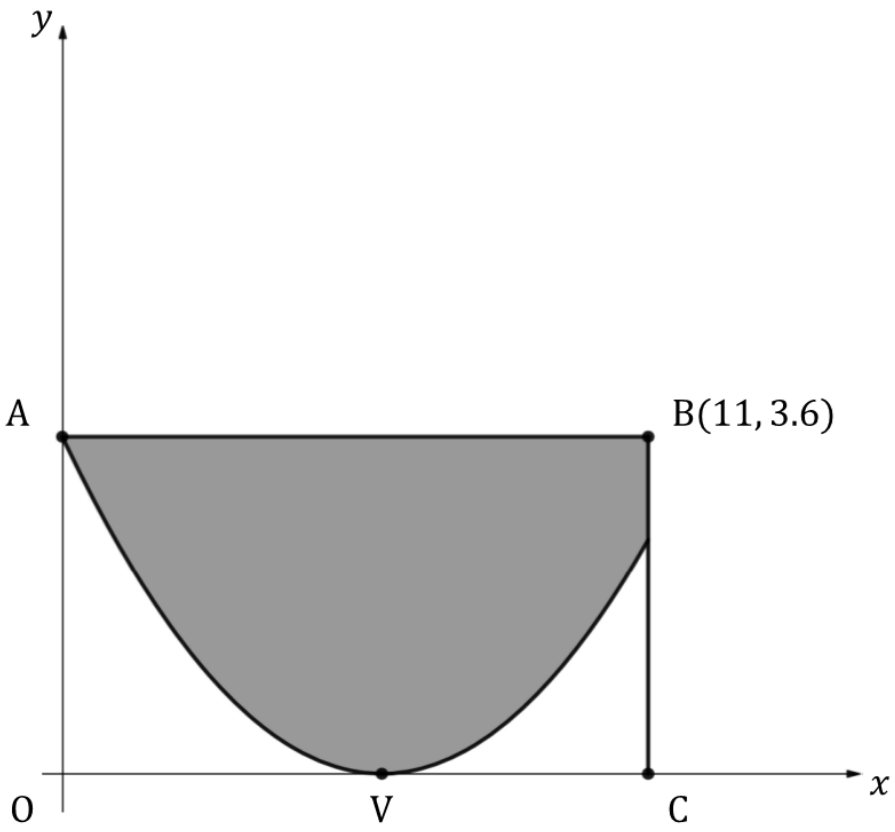
The profit from selling 7000 litres of paint is €5000.

(b) Find $P(x)$.

[5 marks]

Question 9a

A river has a cross-sectional area shown by the shaded region of the diagram below, where the x and y values are in metres. The riverbed (the curved part of the region shown) has an equation of the form $y = q(x - 6)^2$. Point O is the origin, and points O, A, B and C are the vertices of a rectangle. Point V, the deepest point of the riverbed, is situated on the x-axis.



(a) Find

(i) the coordinates of V

(ii) the area of the rectangle OABC.

[3 marks]

Question 9b

(b) Determine the value of q.

[2 marks]

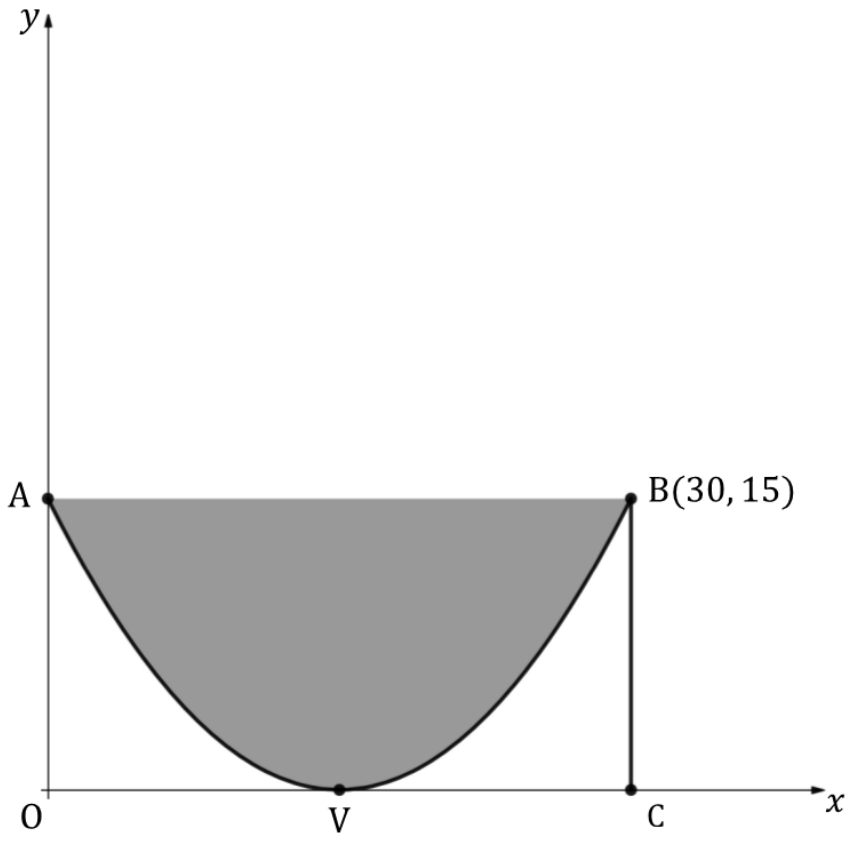
Question 9c

(c) Find the cross-sectional area of the riverbed.

[3 marks]

Question 10a

A trough has a cross-sectional area shown by the shaded region of the diagram below, where the x and y values are in centimetres. The curved bottom of the trough has an equation in the form $y = r(x - 15)^2$. Point O is the origin, and points O, A, B and C are the vertices of a rectangle. Point V, the deepest point of the trough, is situated on the x-axis.



(a) Determine the value of r.

[2 marks]

Question 10b

(b) Find the cross-sectional area of the trough.

[4 marks]

Question 10c

The length of the trough is 1.2 m.

(c) Find the volume of the trough. Give your answer in cm^3.

[2 marks]