2.7 Polynomial Functions

Question Paper

Course	DP IB Maths
Section	2. Functions
Topic	2.7 Polynomial Functions
Difficulty	Very Hard

Time allowed: 100

Score: /76

Percentage: /100

Question 1a

Consider the function $f(x) = 2x^6 - 5x^5 + px^4 + qx^3 - 2x^2 + 20x - 8$, where p and q are constants. It is given that $(x^2 - x - 2)$ is a factor of $f(x)$.

(a)
Show that $p = 8$ and find the value of q.

[5 marks]

Question 1b

(b)
Given that $-2\mathrm{i}$ is a root of f, find all of the roots of the equation $f(x) = 0.$.

[6 marks]

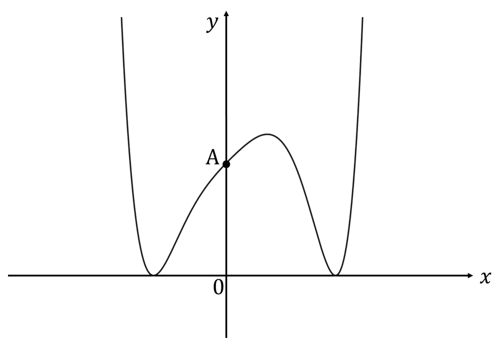
Question 2a

Consider the function

$$f(x) = \sum_{r=0}^{n} a_r x^r$$

where $a_r \in \mathbb{R}$ for $r = 0,\ 1,\ ...,\ n$.

The graph of $y = f(x)$, shown below, passes through $\mathrm{A}(0, 18)$. The roots of $f(x)$ are $\frac{3}{2}$, -1, i and $-\mathrm{i}$.



(a)
Explain why n must be even.

[2 marks]

Question 2b

(b)
Given that n is as small as possible, find an equation for $f(x)$.

[4 marks]

Question 3a

A polynomial function f is defined by $f(x) = k(m - x)^3(n - x)^2$ where $k,\ m$ and n are positive constants with $n > m$.

(a)
Sketch the graph of $y = f(x)$. Label the coordinates where the graph crosses the coordinate axes.

[4 marks]

Question 3b

(b)
Determine the maximum number of distinct real solutions to the equation $f(x) = p$, where p is a real constant.

[1 mark]

Question 3c

Consider the function $g(x) = f(ax + b)$, where a and b are positive constants. The points $(0,0)$ and $(1,0)$ lie on the graph $y = g(x)$.

(c)
Find a and b in terms of m and n.

[3 marks]

Question 4

Consider the function g defined by $g(x) = ax^3 + 4bx^2 + (4a - 3)x - 3b$, where $a,\ b \in \mathbb{R}$ are constants.

Given that $(x - 3)$ is a factor of $g(x)$, and that the sum of the roots of the equation $g(x) = 0$ is 5,

(i)
find the values of a and b, and
(ii)
hence factorise $g(x)$ fully.

[7 marks]

Question 5

Consider the function f defined by $f(x) = (2x^3 + 9x^2 + 4x - 15)(mx^2 + nx + p)$, where m, n and p are real constants.

It is given that the sum of the roots of the equation $f(x) = 0$ is $-\frac{41}{6}$, and that the product of the roots is $\frac{25}{2}$.

Find a set of values for m, n and p that satisfies the above conditions, such that $m,\ n,\ p\ \in \mathbb{Z}$..

[7 marks]

Question 6a

The equation $x^2 + (k-1)x - 2k = 0,\ k \in \mathbb{R}$ has non-real roots α and β where $\alpha^3 + \beta^3 = 5$.

(a)
Find the value of *k*.

[7 marks]

Question 6b

The equation $x^2 + px + q = 0$ has roots α^3 and β^3.

(b)
Find the values of p and q.

[2 marks]

Question 7a

Consider the polynomial function defined by

$$f(x) = \sum_{r=0}^{5} a_r x^r,$$

Where the a_r are real constants. The function has the property that $f(-x) = -f(x)$ for all values of x.

(a)
Show that $a_0 = a_2 = a_4 = 0$.

[2 marks]

Question 7b

(a)
Given that $-2+3\mathrm{i}$ is a root of the equation $f(x)=0$,
(i)
show that $2-3\mathrm{i}$ is also a root of $f(x)=0$, and

(ii)
hence find the values of a_1 and a_3 in terms of a_5.

[6 marks]

Question 8a

Consider the polynomial function $f(x)=x^4+ax^3+bx^2+cx+d$, where $a,\ b,\ c,d\ \in\mathbb{R}$. Two distinct roots of $f(x)=0$ are given by $k+k^2\mathrm{i}$ and $k^2+k\mathrm{i}$, where k is a real constant. The remainder when $f(x)$ is divided by x is 8100.

(a)
(i)
Find the two possible values of k.

(ii)
Hence find real values for and such that (x^2+px+q) is guaranteed to be a factor of $f(x)$.

[7 marks]

Question 8b

(b)

Given that $a = -12$, find the values of b and c.

[4 marks]

Question 9a

The polynomial function f is defined by

$$f(x) = 2ax^3 + (4 + 2a - a^2)x^2 - (6 + 2a + a^2)x + 3a$$

where $a \neq 0$ is a real constant.

The graph of $y = f(x)$ only intersects the x-axis at the point $\left(\frac{a}{2}, 0\right)$.

(a)
By considering the sum of the roots, use proof by contradiction to show that $f(x) = 0$ has two non-real roots.

[4 marks]

Question 9b

(b)
Find the set of possible values of a.

[5 marks]