3.9 Vector Properties

Question Paper

Course	DP IB Maths
Section	3. Geometry & Trigonometry
Topic	3.9 Vector Properties
Difficulty	Very Hard

Time allowed: 100

Score: /80

Percentage: /100

Question 1a

a)

Given $|a| = 6,\ |b| = 2$ and $a \times b = \begin{pmatrix} -5 \\ 2 \\ 7 \end{pmatrix}$, find the possible values of $\boldsymbol{a} \cdot \boldsymbol{b}$.

[4 marks]

Question 1b

Consider a third vector $\mathbf{c}$, where $|c| = 5$.

(b)

Given that the angle between a and c is $\frac{\pi}{3}$, find $|a \times (b + c)|$.

[3 marks]

Question 2a

The points A and B have position vectors a and b respectively.

$|a| = \sqrt{35}, b = 2i - 2j + k,\ a \times b = \begin{pmatrix} 7 \\ -11 \\ 8 \end{pmatrix}$ and θ is the angle between a and b.

(a)
Find $\cos\theta$.

[4 marks]

Question 2b

A third point C is located such that its position vector $c = \begin{pmatrix} 28 \\ -44 \\ 32 \end{pmatrix}$

(b)
Show that $c = 4a \times b$.

[2 marks]

Question 3

$ABCDE$ is a pentagon, where $A(0,4,-1)$, $D(-1,7,3)$, $\overrightarrow{AB} = \begin{pmatrix} -4 \\ 2 \\ 5 \end{pmatrix}$ and $\overrightarrow{CD} = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$.

Given that $\overrightarrow{BD} = \overrightarrow{AE}$ find the area of triangle $B\widehat{C}D$ as a percentage of the total area of the pentagon.

[8 marks]

Question 4a

Three points, $A(-3,-1)$, $B(2,2)$ and C, are located on a straight line where $\overrightarrow{AC} = \lambda\overrightarrow{AB}$. A fourth point D, is located such that $\overrightarrow{DB}$ is perpendicular to $\overrightarrow{OB}$ and $|DB| = 2\sqrt{22}$.

(a)

Find $D\widehat{B}C$.

[3 marks]

Question 4b

(b)
Given that the area of the triangle $\mathrm{BCD} = 53.1$ units^2 correct to 3 significant figures, find λ.

[4 marks]

Question 5

Consider a regular hexagon ABCDEF with sides of length $9\sqrt{2}$ units. The position vectors of A and E are $a = -11\mathrm{i} - 4\mathrm{j} + 5\mathrm{k}$ and $e = 3\mathrm{i} - 3\mathrm{j} - 12\mathrm{k}$ respectively.

Given that the coordinates of F are (r, t, t), where $r, t \in \mathbb{Z}$, $r \neq 0$. find the value of r and t.

[9 marks]

Question 6a

ABCD is a parallelogram defined by the vectors $\overrightarrow{AB} = p$ and $\overrightarrow{AD} = q$, where $p = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$ and $q = \begin{pmatrix} 2t+2 \\ t \end{pmatrix}$.

(a)
Given that the angle $B\widehat{A}D$ is acute, find the range of values for t.

[3 marks]

Question 6b

$\overrightarrow{AB}$ is enlarged by a factor of k.

(b)
Show that $kp \cdot q = k(p \cdot q)$.

[3 marks]

Question 6c

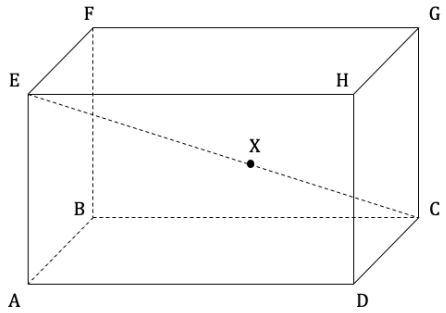
(c)
Given that $k = 7$, find the range of possible values for the area of the enlarged parallelogram.

[4 marks]

Question 7

Consider the cuboid $\mathrm{ABCDEFGH}$ as shown in the diagram below. The position vectors of A, C, D and E are $a = -5\mathrm{i} + 2\mathrm{j} - 4\mathrm{k}$, $c = 2\mathrm{i} + 3\mathrm{j} + \mathrm{k}$, $d = -\mathrm{i} + \mathrm{j}$ and $e = -3\mathrm{i} + \mathrm{j} + 2\mathrm{k}$ respectively.

X is a point located on the line $[\mathrm{EC}]$ such that $\overrightarrow{\mathrm{EX}} = \lambda\overrightarrow{\mathrm{EC}}$.



Find the shortest length $|\mathrm{HX}|$.

[7 marks]

Question 8a

(a)

Show that for any two vectors a and b, $|a \times b|^2 - (a \cdot b)^2 = |a|^2 |b|^2 \cos 2\theta$

[5 marks]

Question 8b

(b)

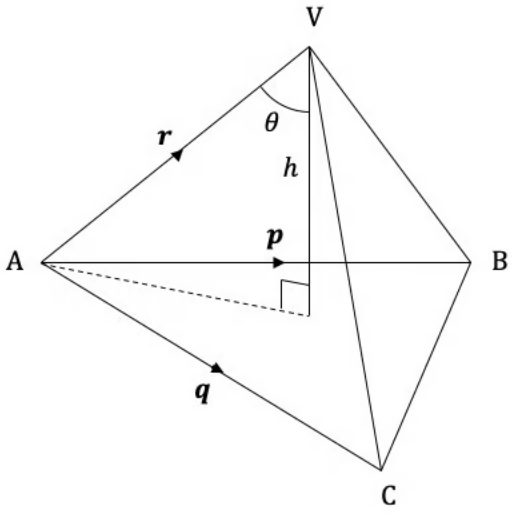
Hence show that when $\sin\theta = \frac{\sqrt{2}}{2}, |a \times b| = a \cdot b$.

[2 marks]

Question 9a

Consider a tetrahedron $ABCV$ where $\overrightarrow{AB} = p$, $\overrightarrow{AC} = q$ and $\overrightarrow{AV} = r$. The perpendicular height, h, of the tetrahedron from the base ABC makes an angle of θ with r.

This information is shown in the diagram below.



(a)

Find an expression for the volume of the tetrahedron in terms of p, q *and* r.

[4 marks]

Question 9b

(b)

Find the volume of the tetrahedron when $p = \begin{pmatrix} 4 \\ 5 \\ -2 \end{pmatrix}, q = \begin{pmatrix} 2 \\ 2 \\ -5 \end{pmatrix}, r = \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix}$

[3 marks]

Question 9c

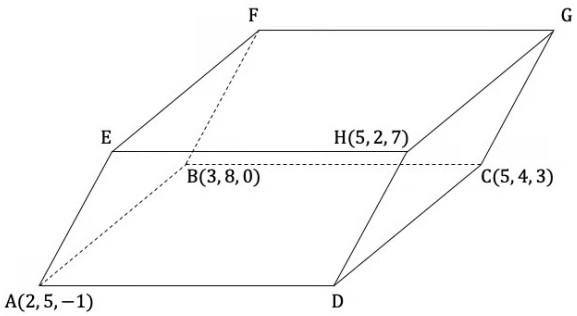
(c)

Hence find the shortest distance between vertex A and its opposite face.

[4 marks]

Question 10

Consider a parallelepiped $ABCDEFGH$ with vertices $A(2,5,-1)$, $B(3,8,0)$, $C(5,4,3)$ and $H(5,2,7)$ as seen in the diagram below.



By first finding an expression for the perpendicular height of the object, find the volume of the parallelepiped.

[8 marks]