IB Physics DP

9. Wave Phenomena (HL only)

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CONTENTS

9.1 Simple Harmonic Motion

9.1.1 The Defining Equation of SHM

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The Defining Equation of SHM

- **Simple harmonic motion (SHM)** is a fundamental oscillation which appears naturally in various phenomena, including:
  - The swing of a pendulum
  - A mass on a spring
  - Guitar strings
  - Vibrations of molecules

- SHM is defined as:

  A type of oscillation in which the acceleration on a body is proportional to its displacement, but acts in the opposite direction

- The conditions for an object to oscillate in SHM are that it shows:
  - Periodic oscillations
  - Acceleration **proportional** to its displacement
  - Acceleration in the **opposite direction** to its displacement

- SHM is **isochronous**, meaning that the **time period stays constant**
  - Therefore frequency must also be constant
- The period of simple harmonic oscillations can be calculated using the equation:

$$T = \frac{2\pi}{\omega}$$

- Where:
  - T = the time period of the oscillator (s)
  - ω = angular frequency (rad s^{-1})

- The acceleration of an oscillator can be calculated using the **defining equation of SHM**:

$$a = -\omega^2 x$$

- Where:
  - a = acceleration of the oscillator (m s^{-2})
  - x = displacement of the oscillator from its equilibrium position (m)

- The equation shows that:
  - The acceleration reaches its maximum value when the displacement is at a maximum i.e. $x = x_0$
  - The minus sign shows that when the object is displacement to the **right**, the direction of the acceleration is to the **left**

- An object in SHM will have a restoring force acting on it to return it to its equilibrium position
- This restoring force will be directly proportional, but in the **opposite direction**, to the displacement of the object from the equilibrium position
  - **Note:** the restoring force and acceleration act in the **same direction**

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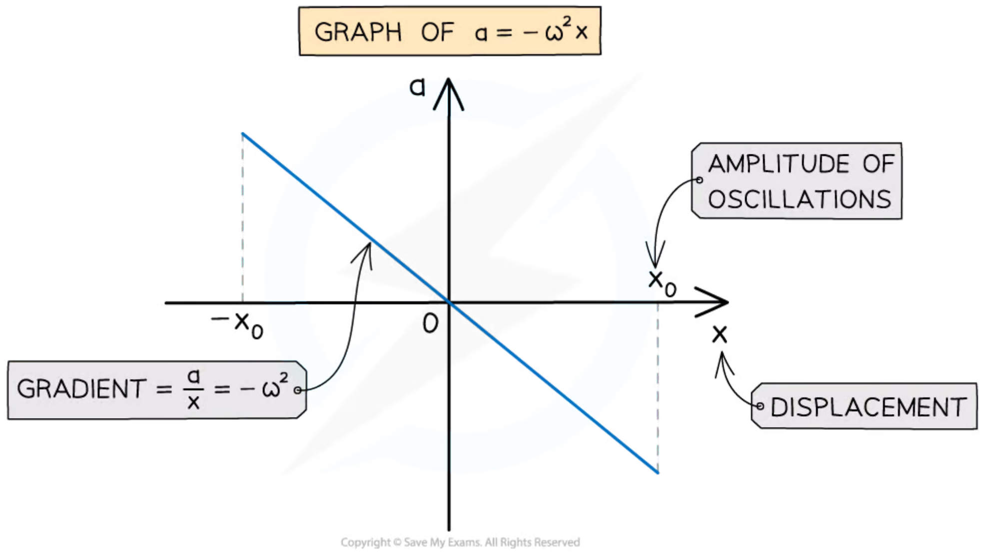

The acceleration of an object in SHM is directly proportional to the negative displacement

Calculating Displacement of an Oscillator

- The graph of acceleration against displacement is a straight line through the origin sloping downwards (similar to y = –x)
- Key features of the graph:
  - The gradient is equal to $-\omega^2$
  - The maximum and minimum displacement *x* values are the amplitudes $-x_0$ and $+x_0$

- A solution to the SHM acceleration equation is the displacement equation:

$$x = x_0 \sin(\omega t)$$

- Where:
  - *x* = displacement of the oscillator (m)
  - x_0 = maximum displacement or amplitude (m)
  - *t* = time (s)

- This equation can be used to find the position of an object in SHM with a particular angular frequency and amplitude at a moment in time
  - **Note:** This version of the equation is only relevant when an object begins oscillating from the equilibrium position (x = 0 at t = 0)

- The displacement will be at its maximum when sin (ωt) equals 1 or – 1, when $x = x_0$
- If an object is oscillating from its amplitude position ($x = x_0$ or $x = -x_0$ at t = 0) then the displacement equation will be:

$$x = x_0 \cos(\omega t)$$

- This is because the cosine graph starts at a maximum, whilst the sine graph starts at 0

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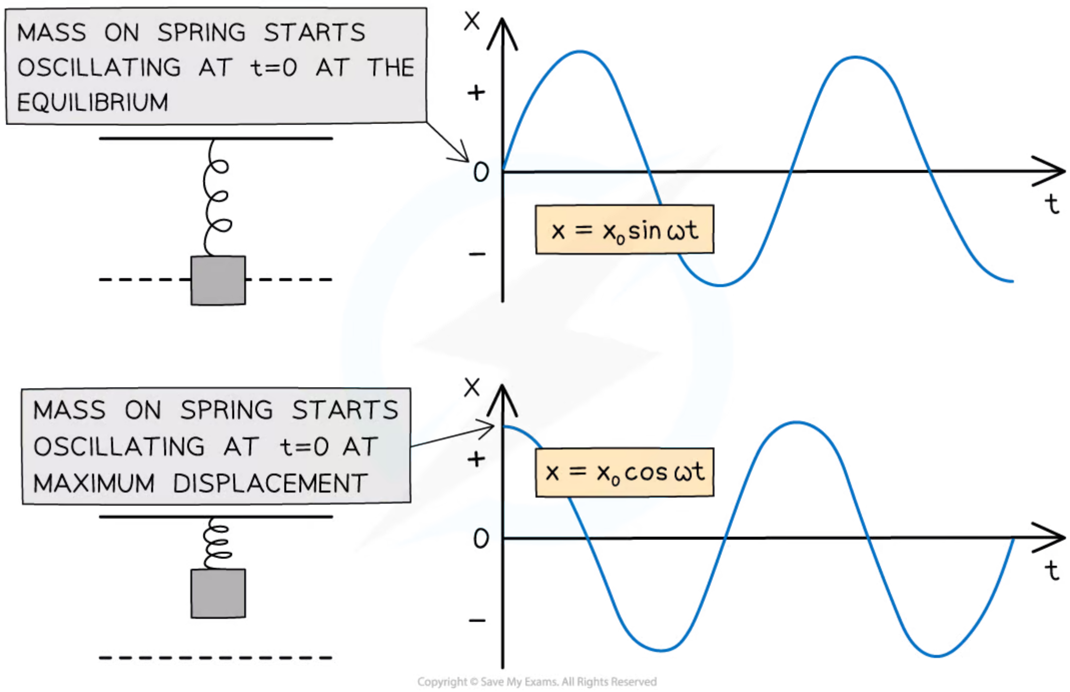

These two graphs represent the same SHM. The difference is the starting position

Equations & Graphs for SHM

- A summary of the equations and graphs of simple harmonic motion are shown in the table
- Note that the equations differ depending on the starting point of the oscillator
  - The derivation of these equations are found lower down the page

Summary table of equations and graphs for displacement, velocity and acceleration

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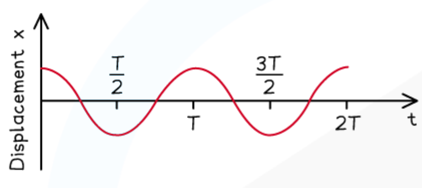
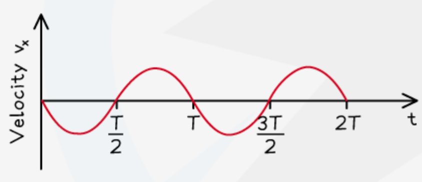
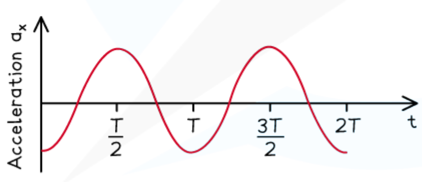
		Object starts in centre of motion	Object starts at extremes of motion
Displacement		$x = x_0 \sin \omega t$	$x = x_0 \cos \omega t$
Velocity		$v = \omega x_0 \cos \omega t$ $v = \pm \omega \sqrt{(x_0^2 - x^2)}$	$v = -\omega x_0 \sin \omega t$ $v = \pm \omega \sqrt{(x_0^2 - x^2)}$
Acceleration		$a = -x_0 \omega^2 \sin \omega t$ $a = -\omega^2 x$	$a = -x_0 \omega^2 \cos \omega t$ $a = -\omega^2 x$

Worked Example

A mass is suspended from a fixed point by means of a spring. The stationary mass is pulled vertically downwards through a distance of 4.3 cm and then released at $t = 0$. The mass is observed to perform simple harmonic motion with a period of 0.8 s.

Calculate the displacement, x, in cm of the mass at time $t = 0.3$ s.

Step 1: List the known quantities

- Maximum displacement, $x_0 = 4.3$ cm
- Period of oscillation, $T = 0.8$ s
- Time interval, $t = 0.3$ s

Step 2: Write down the SHM displacement equation

- Since the mass is released at $t = 0$ at its maximum displacement, the displacement equation will involve the cosine function:

$$x = x_0 \cos(\omega t)$$

Step 3: Write the equation linking the angular frequency, ω

$$\omega = \frac{2\pi}{T}$$

Step 4: Combine the two equations and substitute in the values:

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- **Note:** the calculator must be in **radians** mode

$$x = x_0 \cos\left(\frac{2\pi t}{T}\right) = 4.3 \times \cos\left(\frac{2\pi \times 0.3}{0.8}\right)$$

$$x = -3.041 = \mathbf{-3.0\ cm}\ (2\ s.f.)$$

- The negative value means the mass is 3.0 cm on the opposite side of the equilibrium position to where it started i.e. 3.0 cm above it

Calculating Velocity of an Oscillator

- The velocity, v, of an oscillation can be found by differentiating the displacement with respect to time::

$$v = \frac{dx}{dt}$$

- To differentiate the sine and cosine functions:
  - The derivative of $y = \sin x$ is $\frac{dy}{dx} = \cos x$
  - The derivative of $y = \sin(ax)$ is $\frac{dy}{dx} = a\cos(ax)$
  - The derivative of $y = \cos x$ is $\frac{dy}{dx} = -\sin x$
  - The derivative of $y = \cos(ax)$ is $\frac{dy}{dx} = -a\sin(ax)$

- When the oscillation begins from the **equilibrium position**:

$$x = x_0 \sin(\omega t)$$

- Since the derivative of $\sin(\omega t)$ is $\frac{d}{dt}(\sin(\omega t)) = \omega\cos(\omega t)$:

$$v = x_0 \frac{d}{dt}(\sin(\omega t)) = x_0(\omega\cos(\omega t))$$

- Therefore, the velocity when the oscillation begins from the **equilibrium position** is given by:

$$v = \omega x_0 \cos(\omega t)$$

- When the oscillation begins from the **maximum displacement**:

$$x = x_0 \cos(\omega t)$$

- Since the derivative of $\cos(\omega t)$ is $\frac{d}{dt}(\cos(\omega t)) = -\omega\sin(\omega t)$:

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$$v = x_0 \frac{d}{dt}(\cos(\omega t)) = x_0(-\omega \sin(\omega t))$$

- Therefore, the velocity when the oscillation begins from the **maximum displacement** is given by:

$$v = -\omega x_0 \sin(\omega t)$$

- Since the maximum value of $\sin(\omega t)$ or $\cos(\omega t)$ is 1, **maximum speed** is given by:

$$v_{max} = \omega x_0$$

- This is the maximum speed of the oscillation regardless of its starting point

Calculating Acceleration of an Oscillator

- The acceleration, *a*, of an oscillator can be found by differentiating the displacement with respect to time twice:

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt}$$

- When the oscillation begins from the **equilibrium position**:

$$x = x_0 \sin(\omega t)$$

$$v = \omega x_0 \cos(\omega t)$$

- Since the derivative of $\cos(\omega t)$ is $\frac{d}{dt}(\cos(\omega t)) = -\omega \sin(\omega t)$:

$$a = \omega x_0 \frac{d}{dt}(\cos(\omega t)) = \omega x_0(-\omega \sin(\omega t))$$

- Therefore, the velocity when the oscillation begins from the **equilibrium position** is given by:

$$a = -\omega^2 x_0 \sin(\omega t)$$

- When the oscillation begins from the **maximum displacement**:

$$x = x_0 \cos(\omega t)$$

$$v = -\omega x_0 \sin(\omega t)$$

- Since the derivative of $\sin(\omega t)$ is $\frac{d}{dt}(\sin(\omega t)) = \omega \cos(\omega t)$:

$$a = -\omega x_0 \frac{d}{dt}(\sin(\omega t)) = -\omega x_0(\omega \cos(\omega t))$$

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- Therefore, the velocity when the oscillation begins from the **maximum displacement** is given by:

$$a = -\omega^2 x_0 \cos(\omega t)$$

- Since the maximum value of $\sin(\omega t)$ or $\cos(\omega t)$ is 1, **maximum acceleration** is given by:

$$a_{max} = \omega^2 x_0$$

- This is the maximum acceleration of the oscillator regardless of its starting point

Displacement-Velocity Relation for SHM

- A useful relation between the displacement and velocity of a simple harmonic oscillator can be derived using the expressions from the equilibrium position:

$$x = x_0 \sin(\omega t)$$

$$v = \omega x_0 \cos(\omega t)$$

- The key to deriving this expression is to use the trigonometric identity:

$$\sin^2(\omega t) + \cos^2(\omega t) = 1$$

- To get started, square both expressions:

$$x^2 = x_0^{\ 2} \sin^2(\omega t)$$

$$v^2 = \omega^2 x_0^{\ 2} \cos^2(\omega t)$$

- According to the trig identity, $\cos^2(\omega t) = 1 - \sin^2(\omega t)$ so:

$$v^2 = \omega^2 x_0^{\ 2}(1 - \sin^2(\omega t))$$

$$v^2 = \omega^2 x_0^{\ 2} - \omega^2 x_0^{\ 2} \sin^2(\omega t)$$

$$v^2 = \omega^2 x_0^{\ 2} - \omega^2 \left[x_0^{\ 2} \sin^2(\omega t)\right]$$

$$v^2 = \omega^2 x_0^{\ 2} - \omega^2 x^2$$

$$v^2 = \omega^2 \left(x_0^{\ 2} - x^2\right)$$

- Squaring both sides of the equation leads to the displacement-velocity relation for SHM:

$$v = \pm\,\omega\sqrt{x_0^{\ 2} - x^2}$$

- **Note:** The same relation will be achieved if the expressions for the maximum displacement are used:

$$x = x_0 \cos(\omega t)$$

$$v = -\omega x_0 \sin(\omega t)$$

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Exam Tip

The defining equation of SHM shows acceleration, as a positive value, and displacement, $-x$ as a negative one. This reminds us that acceleration and displacement are **vector** quantities and are always in the opposite direction to each other in SHM.

Since displacement is a vector quantity, remember to keep the minus sign in your solutions if they are negative. Getting the marks will depend on keeping your positive and negative numbers distinct from each other! Also remember that your calculator must be in **radians** mode when using the cosine and sine functions. This is because the angular frequency ω is calculated in rad s^{-1}, **not** degrees.

9.1.2 Energy Changes in SHM

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Energy Changes in SHM

- During simple harmonic motion, energy is constantly exchanged between two forms:
  - **Kinetic** energy
  - **Potential** energy

- The potential energy could be in the form of:
  - Gravitational potential energy (for a pendulum)
  - Elastic potential energy (for a horizontal mass on a spring)
  - Or both (for a vertical mass on a spring)

- The speed of an oscillator is at a maximum when displacement $x = 0$, so:

  The kinetic energy of an oscillator is at a maximum when the displacement is zero

- This is because kinetic energy is equal to $\frac{1}{2}mv^2$ so when the oscillator moves at maximum velocity (at the equilibrium position) it reaches its maximum value of kinetic energy

- Therefore, the kinetic energy is zero at maximum displacement $x = x_0$, so:

  The potential energy of an oscillator is at a maximum when the displacement (both positive and negative) is at a maximum, $x = \pm x_0$

- This is because the kinetic energy is transferred to potential energy as the height above the equilibrium position increases

- A simple harmonic system is therefore constantly converting between kinetic and potential energy
- When one increases, the other decreases and vice versa, therefore:

  The total energy of a simple harmonic system always remains constant

- The total energy is, therefore, equal to the sum of the kinetic and potential energies

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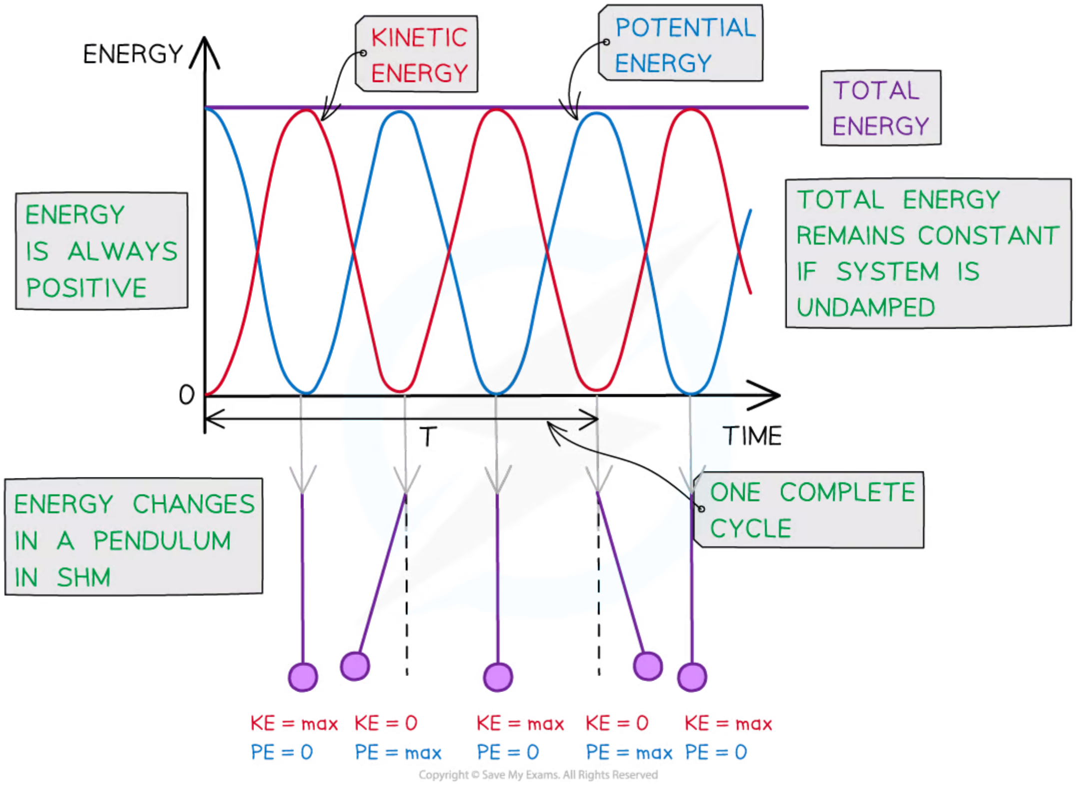

The kinetic and potential energy of an oscillator in SHM vary periodically

The key features of the energy-time graph are:

- Both the kinetic and potential energies are represented by periodic functions (sine or cosine) which are varying in opposite directions to one another
- When the potential energy is 0, the kinetic energy is at its maximum point and vice versa
- The **total energy** is represented by a **horizontal straight line** directly above the curves at the maximum value of both the kinetic or potential energy
- Energy is **always positive** so there are no negative values on the y axis
- **Note:** kinetic and potential energy go through **two** complete cycles during one **period** of oscillation
    - This is because one complete oscillation reaches the maximum displacement **twice** (positive and negative)

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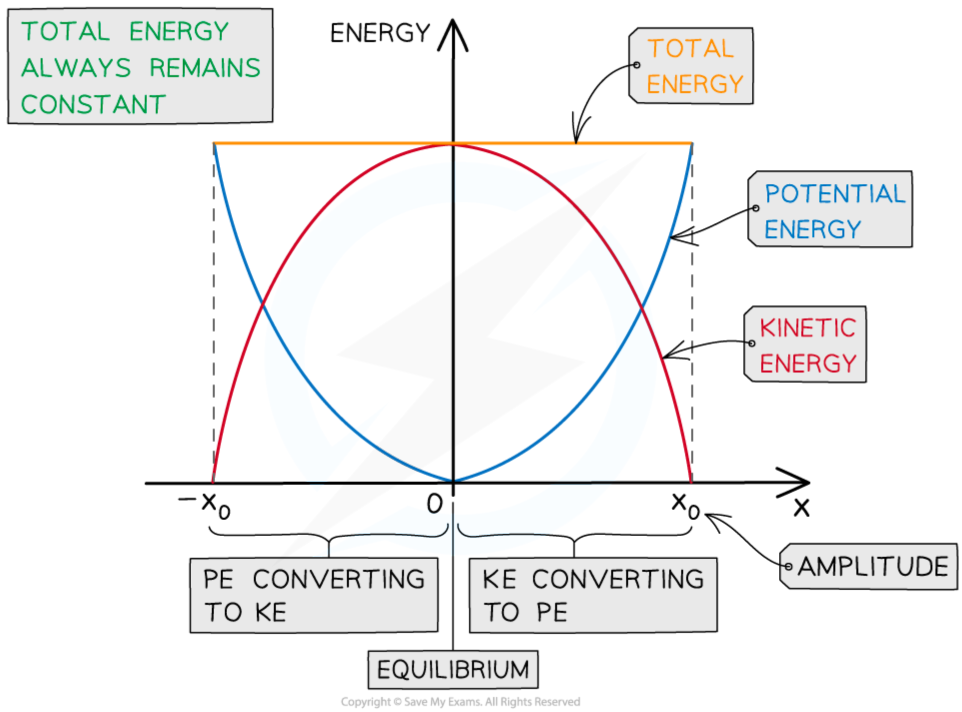

Potential and kinetic energy v displacement in half a period of an SHM oscillation

- **The key features of the energy-displacement graph:**
    - Displacement is a vector, so, the graph has both **positive** and **negative** x values
    - The **potential energy** is always at a maximum at the amplitude positions x_0 and zero at the equilibrium position (x = 0)
    - This is represented by a **'U' shaped curve**
    - The **kinetic energy** is the opposite: it is zero at the amplitude positions x_0 and maximum at the equilibrium position, x = 0
    - This is represented by a **'n' shaped curve**
    - The total energy is represented by a **horizontal straight line** above the curves

Calculating Energy in Simple Harmonic Motion

- Using the expression for the velocity, v, of a simple harmonic oscillator:

$$v = -\omega x_0 \sin(\omega t)$$

- The kinetic energy, E_K, of an oscillator can be written as:

$$E_K = \frac{1}{2}mv^2$$

$$E_K = \frac{1}{2}m(-\omega x_0 \sin(\omega t))^2$$

$$E_K = \frac{1}{2}m\omega^2 x_0^2 \sin^2(\omega t)$$

- Since the maximum value of $\sin(\omega t)$ or $\cos(\omega t)$ is 1, **maximum kinetic energy** is given by:

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$$E_{K(max)} = \frac{1}{2} m\omega^2 {x_0}^2$$

- When the kinetic energy of the system is at a maximum, the potential energy is zero
  - Hence this represents the **total energy of the system**
- The total energy, E_T, of a system undergoing simple harmonic motion is, therefore, defined by:

$$E_T = \frac{1}{2} m\omega^2 {x_0}^2$$

- Where:
  - E_T = total energy of a simple harmonic system (J)
  - m = mass of the oscillator (kg)
  - ω = angular frequency (rad s^{-1})
  - x_0 = amplitude (m)

- **Note:** The same expression for total energy will be achieved if the other expression for velocity is used:

$$v = \omega x_0 \cos(\omega t)$$

- An expression for the potential energy of a simple harmonic oscillator can be derived using the expressions for velocity and displacement

$$x = x_0 \sin(\omega t)$$

$$v = \omega x_0 \cos(\omega t)$$

- The key to deriving this expression is to use the trigonometric identity:

$$\sin^2(\omega t) + \cos^2(\omega t) = 1$$

- In a simple harmonic oscillation, the total energy of the system is equal to:

Total energy = Kinetic energy + Potential energy

$$E_T = E_K + E_P$$

- The **potential energy** of an oscillator can be written as:

$$E_P = E_T - E_K$$

$$E_P = \frac{1}{2} m\omega^2 {x_0}^2 - \frac{1}{2} mv^2$$

$$E_P = \frac{1}{2} m\omega^2 {x_0}^2 - \frac{1}{2} m(-\omega x_0 \sin(\omega t))^2$$

$$E_P = \frac{1}{2} m\omega^2 {x_0}^2 - \frac{1}{2} m\omega^2 {x_0}^2 \sin^2(\omega t)$$

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- Taking out a factor of $\frac{1}{2}m\omega^2{x_0}^2$ gives:

$$E_P = \frac{1}{2}m\omega^2{x_0}^2\left(1 - \sin^2(\omega t)\right)$$

$$E_P = \frac{1}{2}m\omega^2{x_0}^2\cos^2(\omega t)$$

$$E_P = \frac{1}{2}m\omega^2\left[x_0\cos(\omega t)\right]^2$$

- Since $x = x_0\sin(\omega t)$, the potential energy of the system can be written as:

$$E_P = \frac{1}{2}m\omega^2x^2$$

- Since the maximum potential energy occurs at the maximum displacement of the oscillation, i.e. $x = x_0$:

$$E_{P(max)} = \frac{1}{2}m\omega^2{x_0}^2$$

- Therefore, it can be seen that:

$$E_T = E_{K(max)} = E_{P(max)}$$

Kinetic Energy-Displacement Relation for SHM

- Using the displacement-velocity relation for SHM:

$$v = \pm\,\omega\sqrt{{x_0}^2 - x^2}$$

- Substituting into the equation for kinetic energy:

$$E_K = \frac{1}{2}mv^2$$

$$E_K = \frac{1}{2}m\left(\omega\sqrt{{x_0}^2 - x^2}\right)^2$$

- This leads to the kinetic energy-displacement relation for SHM:

$$E_K = \frac{1}{2}m\omega^2\left({x_0}^2 - x^2\right)$$

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Exam Tip

You may be expected to draw as well as interpret energy graphs against time or displacement in exam questions. Make sure the sketches of the curves are as even as possible and **use a ruler** to draw straight lines, for example, to represent the total energy.

9.1.3 Calculating Energy Changes in SHM

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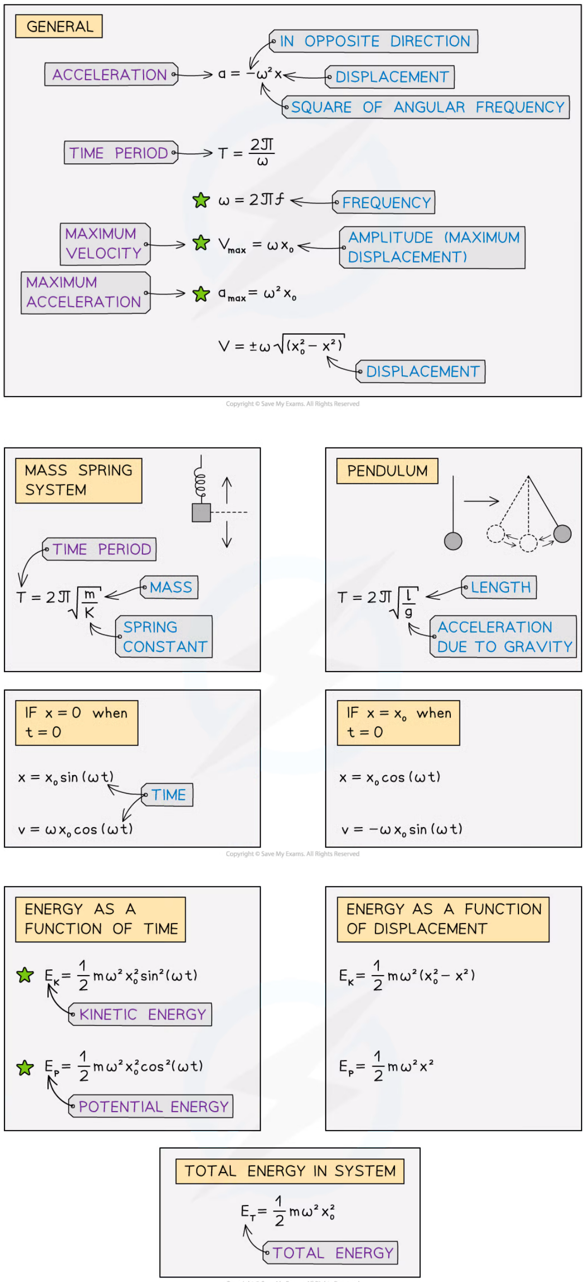
Calculating Energy Changes in SHM

- There are many equations to learn in the topic of simple harmonic motion
- The key equations are summarised below:



$$a = -\omega^2 x$$
$$T = \frac{2\pi}{\omega}$$
$$\omega = 2\pi f$$
$$v_{max} = \omega x_0$$
$$a_{max} = \omega^2 x_0$$
$$v = \pm\omega\sqrt{(x_0^2 - x^2)}$$
$$T = 2\pi\sqrt{\frac{m}{k}}$$
$$T = 2\pi\sqrt{\frac{L}{g}}$$
$$x = x_0 \sin(\omega t)$$
$$v = \omega x_0 \cos(\omega t)$$
$$x = x_0 \cos(\omega t)$$
$$v = -\omega x_0 \sin(\omega t)$$
$$E_K = \frac{1}{2} m\omega^2 x_0^2 \sin^2(\omega t)$$
$$E_P = \frac{1}{2} m\omega^2 x_0^2 \cos^2(\omega t)$$
$$E_K = \frac{1}{2} m\omega^2 (x_0^2 - x^2)$$
$$E_P = \frac{1}{2} m\omega^2 x^2$$
$$E_T = \frac{1}{2} m\omega^2 x_0^2$$

A summary of the equations related to simple harmonic motion. The green stars indicate equations which are not included in the IB data booklet

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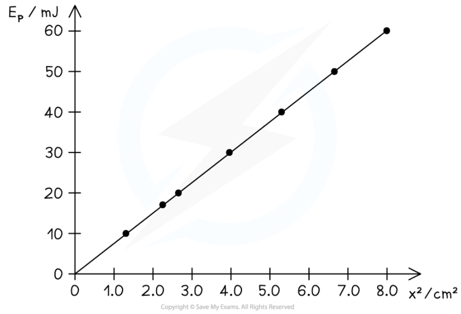
Worked Example

The graph shows the potential energy, E_P, for a particle oscillating with SHM. The particle has mass 45 g.



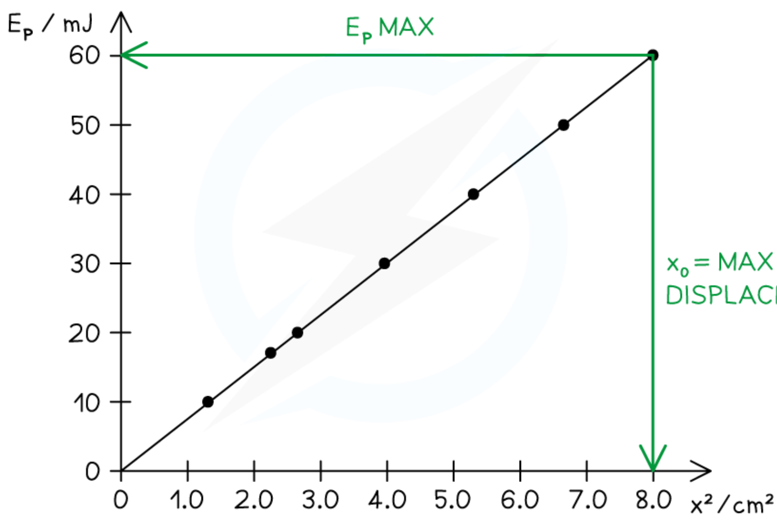
(a)
Use the graph to determine the amplitude and hence, the period of oscillation.
(b)
Calculate the maximum speed which the particle achieves.

Part (a)

Step 1: List the known quantities

- Mass of the particle, m = 45 g = 45 × 10^{-3} kg

Step 2: Use the graph to determine the maximum potential energy of the particle



- Maximum potential energy, E_{Pmax} = 60 mJ = 60 × 10^{-3} J

Step 3: Determine the amplitude of the oscillation

- The amplitude of the motion, x_0, is the maximum displacement
- At the maximum displacement, the particle is at its highest point, hence this is the position of maximum potential energy

- From the graph, when $E_P = E_{Pmax}$:

$$x_0^2 = 8.0 \text{ cm} = 0.08 \text{ m}$$

- Therefore, the amplitude is:

$$x_0 = \sqrt{0.08} = 0.28 \text{ m}$$

Step 4: Write down the equation for the potential energy of an oscillator

$$E_P = \frac{1}{2} m\omega^2 {x_0}^2$$

Step 5: Rearrange the equation for angular velocity, ω:

$$\omega^2 = \frac{2E_P}{m{x_0}^2}$$

$$\omega = \sqrt{\frac{2E_P}{m{x_0}^2}}$$

Step 6: Substitute the known values and calculate ω

$$\omega = \sqrt{\frac{2 \times (60 \times 10^{-3})}{(45 \times 10^{-3}) \times 0.08}}$$

$$\omega = 5.77 \text{ rad s}^{-1}$$

Step 7: Write down the relation between period, *T*, and angular velocity, ω

$$T = \frac{2\pi}{\omega}$$

Step 8: Determine the time period of the oscillation

$$T = \frac{2\pi}{5.77} = 1.1 \text{ s}$$

Part (b)

Step 1: List the known quantities

- Angular velocity, $\omega = 5.77 \text{ rad s}^{-1}$
- Amplitude, $x_0 = 0.28 \text{ m}$

Step 2: Write down the equation for the maximum speed of an oscillator

$$v_{max} = \omega x_0$$

Step 3: Substitute in the known quantities

$$v_{max} = 5.77 \times 0.28 = 1.6 \text{ m s}^{-1}$$

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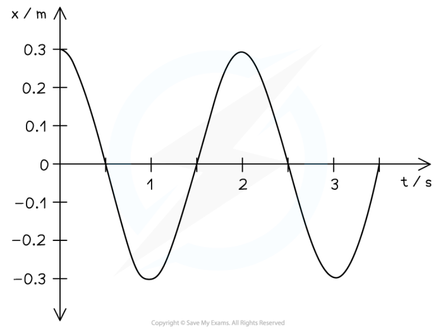
Worked Example

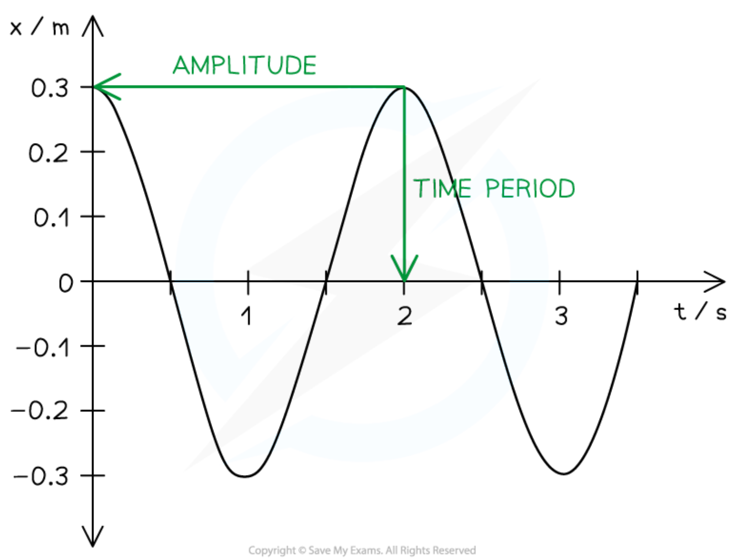
A student investigated the behaviour of a 200 g mass oscillating on a spring, and produced the graph shown.



(a)
Determine the values for amplitude and time period
(b)
Hence find the maximum kinetic energy of the oscillating mass.

Part (a)

Step 1: Read the values of amplitude and time period from the graph



Step 2: State the values of amplitude and time period

- Amplitude, x_0 = 0.3 m
- Time period, T = 2.0 s

Part (b)

Step 1: List the known quantities

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- Mass of the oscillator, m = 200 g = 0.2 kg

Step 2: Write down the equation for the maximum speed of an oscillator

$$v_{max} = \omega x_0$$

Step 3: Write down the equation relating angular speed and time period

$$\omega = \frac{2\pi}{T}$$

Step 4: Combine the two equations and calculate the maximum speed

$$v_{max} = \frac{2\pi x_0}{T} = \frac{2\pi \times 0.3}{2.0}$$

$$v_{max} = 0.942 \text{ m s}^{-1}$$

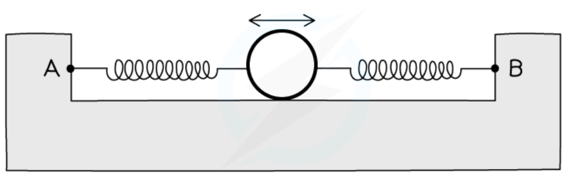
Step 5: Use the maximum speed to calculate the maximum kinetic energy of the oscillating mass

$$E_{Kmax} = \frac{1}{2} m v_{max}^2$$

$$E_{Kmax} = 0.5 \times 0.2 \times 0.942^2 = 0.1 \text{ J}$$

Worked Example

A ball of mass 23 g is held between two fixed points **A** and **B** by two stretch helical springs, as shown in the diagram below.



The ball oscillates along the line **AB** with simple harmonic motion of frequency 4.8 Hz and amplitude 1.5 cm.

Calculate the total energy of the oscillations.

Step 1: Write down all known quantities

- Mass, m = 23 g = 23×10^{-3} kg
- Amplitude, x_0 = 1.5 cm = 0.015 m
- Frequency, f = 4.8 Hz

Step 2: Write down the equation for the total energy of SHM oscillations:

$$E = \frac{1}{2} m\omega^2 {x_0}^2$$

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Step 3: Write an expression for the angular frequency

$$\omega = 2\pi f$$

Step 4: Substitute values into the SHM energy equation

$$E = \frac{1}{2} m(2\pi f)^2 {x_0}^2$$

$$E = 0.5 \times (23 \times 10^{-3}) \times (2\pi \times 4.8)^2 \times (0.015)^2$$

$$E = 2.354 \times 10^{-3} = 2.4 \text{ mJ (2 s.f.)}$$

Exam Tip

There are a large number of equations associated with SHM. Most of them are given in the data booklet which you will be given to use in the exam

Make sure you are familiar with the equations, as you will probably need to use several different ones to solve the longer questions.

YOUR NOTES
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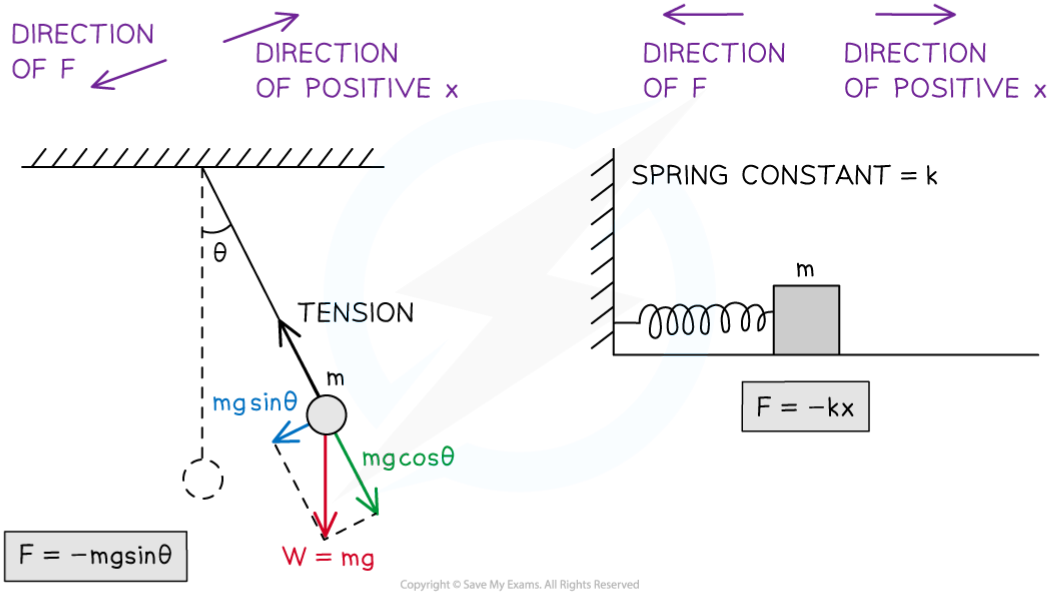
Period of a Simple Pendulum & a Mass-Spring System

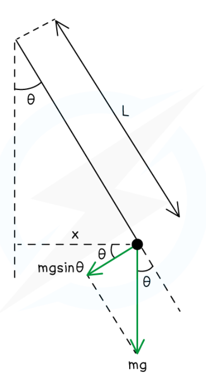
- Two examples of simple harmonic oscillators are:
    - A simple pendulum
    - A mass-spring system
- Considering equations related to the restoring force leads to expressions for the **time periods** of these scenarios
    - These relationships are useful for investigating simple harmonic motion experimentally



For a pendulum, the restoring force is provided by the component of the bob's weight that is perpendicular to the tension in the pendulum's string. For a mass-spring system, the restoring force is provided by the force of the spring

Time Period of a Simple Pendulum

- A simple pendulum is a type of simple harmonic oscillator
    - The pendulum consists of a string and a bob (a weight, generally spherical) at the end
- An oscillating pendulum can be modelled as simple harmonic motion when the angle of oscillation is small



Forces on a pendulum when it is displaced. Assuming θ < 10°, the small angle approximation can be used to describe the time period of a simple pendulum such as this.

YOUR NOTES
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- The restoring force, F, which returns an oscillating pendulum bob to the equilibrium position is:

$$F = -mg \sin \theta$$

- Where:
  - m = the mass of the pendulum bob (kg)
  - g = acceleration due to gravity (m s^{-2})
  - θ = angle between the bob and the vertical (°)
- Using Newton's Second Law:

$$F = ma = -mg \sin \theta$$

- Both sides can be divided by m to give an expression for the acceleration, a:

$$a = -g \sin \theta$$

- In this case, the small-angle approximation can be used, this is where **$\sin \theta \simeq \theta$**
  - This is assuming the angle the pendulum makes with the vertical is less than 10°
- The expression for acceleration then becomes:

$$a = -g\theta$$

- The displacement, x, is equal to the length of the arc made by the bob, $x = L\theta$
  - Where L = length of the pendulum (m)
- Rearranging this for θ and substituting it into the acceleration equation gives:

$$a = -g\left(\frac{x}{L}\right)$$

- This equation shows:
  - For small values of x, the condition for SHM is satisfied as restoring force, F is proportional to $-x$
  - For large values of x, the acceleration of a simple pendulum is not proportional to the displacement

- Using the defining equation of SHM:

$$a = -\omega^2 x$$

- Where ω = angular frequency of the oscillation (rad s^{-1})
- Equating with the previously derived expression for acceleration leads to:

$$-\omega^2 x = -g\left(\frac{x}{L}\right)$$

$$\omega^2 = \frac{g}{L}$$

YOUR NOTES
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$$\omega = \sqrt{\frac{g}{L}}$$

- The equation relating angular frequency, ω, and time period, T, is:

$$\omega = \frac{2\pi}{T}$$

- Finally, combining these expressions leads to an equation for the **time period of a simple pendulum**:

$$\frac{2\pi}{T} = \sqrt{\frac{g}{L}}$$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Time Period of a Mass-Spring System

- A mass-spring system is another type of simple harmonic oscillator
- The restoring force, F, which returns a mass-spring system to its equilibrium position is given by:

$$\boldsymbol{F = -kx}$$

- Where:
  - x = extension of the spring (m)
  - k = spring constant (N m^{-1})
- Using Newton's Second Law:

$$F = ma = -kx$$

- Rearranging for the acceleration, a:

$$a = -\frac{k}{m}x$$

- The defining equation of SHM is given by $\boldsymbol{a = -\omega^2 x}$
- Where:
  - m = mass (kg)
  - ω = angular frequency (rad s^{-1})
- Combining these expressions for acceleration gives:

$$-\omega^2 x = -\frac{k}{m}x$$

- This can be simplified to give an expression for ω:

$$\omega^2 = \frac{k}{m}$$

YOUR NOTES
↓

$$\omega = \sqrt{\frac{k}{m}}$$

- The equation relating angular frequency, ω, and time period, T, is:

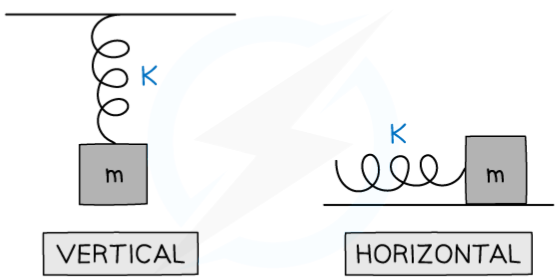
$$\omega = \frac{2\pi}{T}$$

- Finally, combining these expressions leads to an equation for the **time period of a mass-spring system**:

$$\frac{2\pi}{T} = \sqrt{\frac{k}{m}}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

- This equation applies for both a **horizontal** or **vertical** mass-spring system



A mass-spring system can be either vertical or horizontal. The time period equation applies to both.

- The equation shows that:
  - The **higher** the spring constant k, the **stiffer** the spring and hence, the **shorter** the time period of oscillation
  - The time period (and hence, frequency) of a mass-spring system is independent of the force of gravity
- A consequence of this is that oscillations would have the **same time period** on Earth and the Moon

Worked Example

A swinging pendulum with a length of 80.0 cm has a maximum angle of displacement of 8°.

Determine the angular frequency of the oscillation.

Step 1: List the known quantities

- Length of the pendulum, L = 80 cm = 0.8 m

- Acceleration due to gravity, $g = 9.81$ m s^{-2}

Step 2: Write down the relationship between angular frequency, ω, and period, T

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Step 3: Write down the equation for the time period of a simple pendulum

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- This equation is valid for this scenario since the maximum angle of displacement is less than 10°

Step 4: Equate the two equations and rearrange for ω

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Step 5: Substitute the values to calculate ω

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Step 6: State the final answer to the correct number of significant figures

$$\omega = 3.5 \text{ rad s}^{-1}$$

Worked Example

Calculate the frequency of a mass of 2.0 kg attached to a spring with a spring constant of 0.9 N m^{-1} oscillating with simple harmonic motion.

Step 1: Write down the known quantities

- Mass, $m = 2.0$ kg
- Spring constant, $k = 0.9$ N m^{-1}

Step 2: Write down the equation for the time period of a mass-spring system

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Step 3: Write down the equation relating time period, T, and frequency, f

$$T = \frac{1}{f}$$

Step 4: Combine the equations and rearrange for frequency, f

$$\frac{1}{f} = 2\pi\sqrt{\frac{m}{k}}$$

YOUR NOTES

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Step 5: Substitute in the values to calculate frequency, *f*

$$f = \frac{1}{2\pi}\sqrt{\frac{0.9}{2}} = 0.1068 \text{ Hz}$$

Step 6: State the final answer to the correct number of significant figures

$$f = 0.11 \text{ Hz}$$

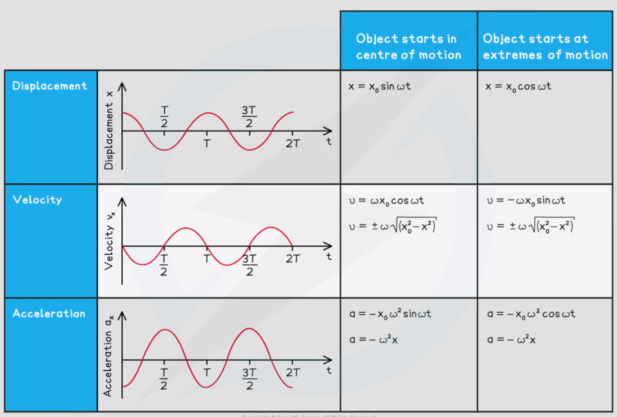
Exam Tip

Another area of physics where you may have seen the spring constant *k* is in Hooke's Law, where $F = kx$.

Exam questions commonly merge these topics together, so make sure you're familiar with the Hooke's Law equation too.

The motion of both pendula and mass-spring systems can be described in graphical and mathematical forms. As with other forms of motion, you should become familiar with both

Make sure to pay particular attention to the difference between the graph shapes produced when the oscillator starts at the **equilibrium position** or **maximum displacement**



Graphs and equations can be used to describe different aspects of oscillations

9.2 Single-Slit Diffraction

9.2.1 The Nature of Single-Slit Diffraction

YOUR NOTES
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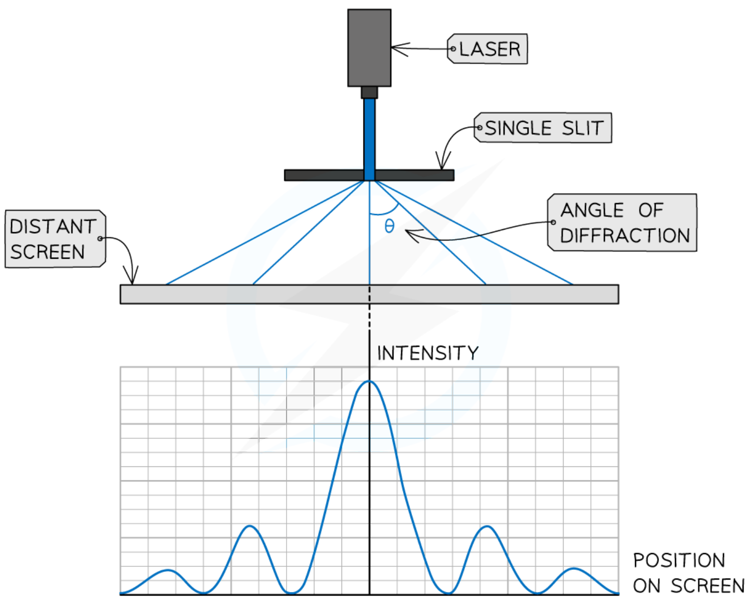
The Nature of Single-Slit Diffraction

- When plane waves are incident normally on a single slit, a **diffraction pattern** is produced
  - This is represented as a series of light and dark **fringes** which show the areas of **maximum** and **minimum** intensity
- If a laser emitting blue light is directed at a single slit, where the slit width is larger than the wavelength of the light, it will spread out as follows:



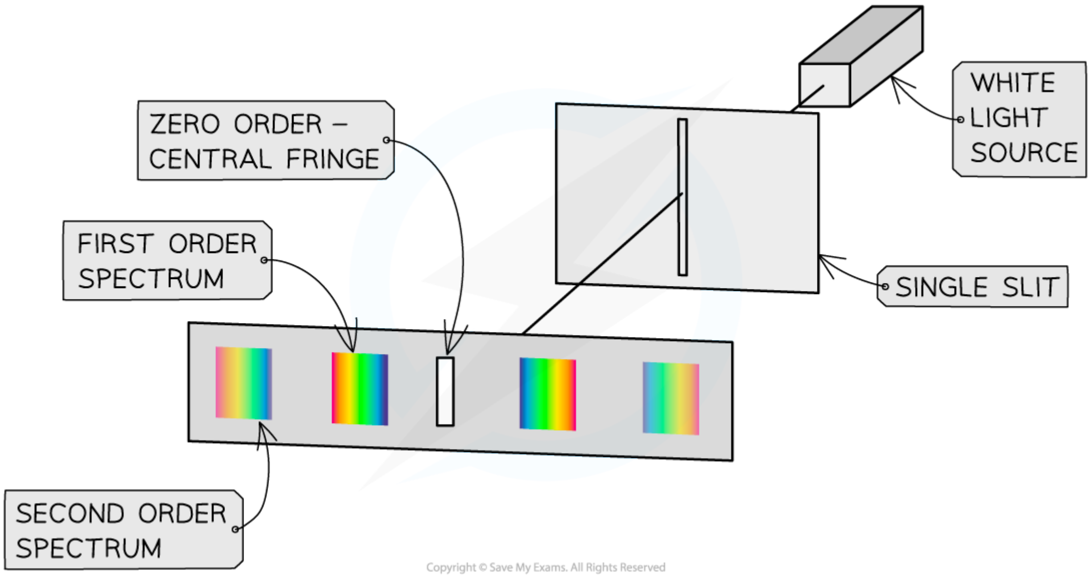
The intensity pattern of blue laser light diffracted through a single slit

- The features of the single-slit diffraction pattern using monochromatic light are:
  - A **central** maximum with a **high intensity**
  - Equally spaced subsidiary maxima, successively smaller in intensity and half the width of the central maximum

Single Slit Diffraction of White Light

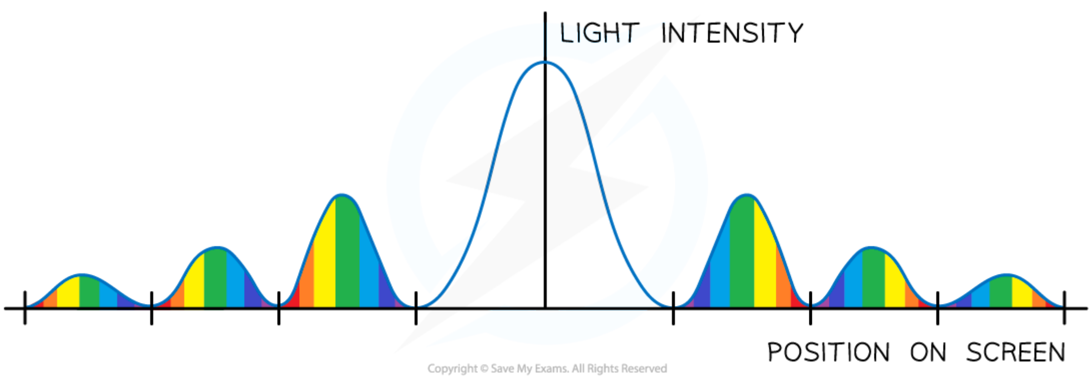
- When white light is incident on a slit, separate diffraction patterns can be observed for each wavelength making up the white light

YOUR NOTES



Single slit diffraction of a white light source

- If the laser were to be replaced by a non-laser source emitting **white light**:
  - The central maximum would be **white**
  - All maxima would be composed of a **spectrum**
  - The shortest wavelength (violet / blue) would appear **nearest** to the central maximum
  - The longest wavelength (red) would appear **furthest** from the central maximum
  - The fringe spacing would be smaller and the maxima would be wider



Qualitative treatment of the variation of the width of the central diffraction maximum with wavelength and slit width. Red light is diffracted the most, blue light is diffracted the least.

Exam Tip

Be mindful that **all** waves undergo diffraction, so questions about diffraction may involve sound, ultrasound, electromagnetic waves, or even waves on water

YOUR NOTES
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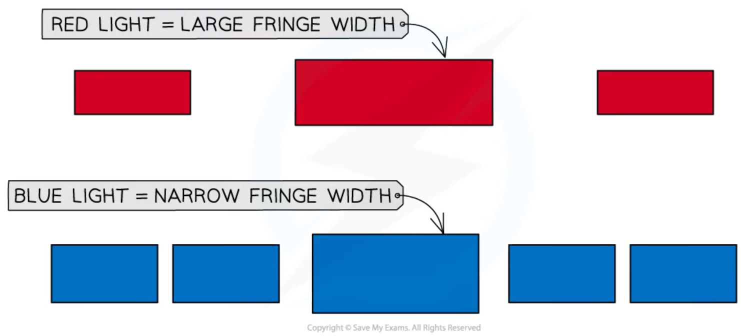
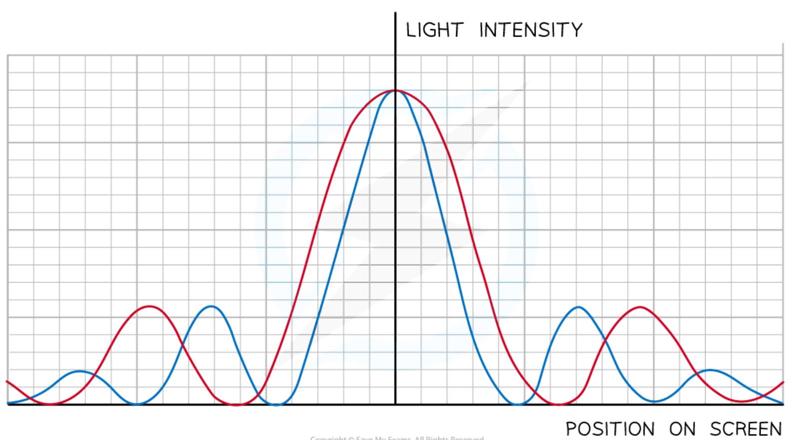
Slit Width

- The **angle of diffraction** is directly proportional to the **wavelength** of the light
  - This means that the **width** of the **bright maxima**, or fringe, is also proportional to the wavelength
- **Red** light – which has the **longest wavelength** of visible light – will produce a diffraction pattern with **wide fringes**
- **Blue** light – which has a much **shorter wavelength** – will produce a diffraction pattern with **narrow fringes**



Fringe width depends on the wavelength of the light

- Therefore, if the blue laser were to be replaced with a red laser:
  - The wavelength of red light is longer so the light would diffract **more**
  - The intensity fringes would therefore be **wider**



The intensity pattern of red laser light shows longer wavelengths diffract more than shorter blue wavelengths

- If the slit was made narrower:
  - The intensity would **decrease**
  - The fringe spacing would be **wider**

YOUR NOTES
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Exam Tip

When drawing diffracted waves, take care to keep the wavelength (the distance between each wavefront) constant. It is only the amplitude of the wave that changes when diffracted.

9.2.2 Intensity of Interference Maxima & Minima

YOUR NOTES ↓

Intensity of Interference Maxima & Minima

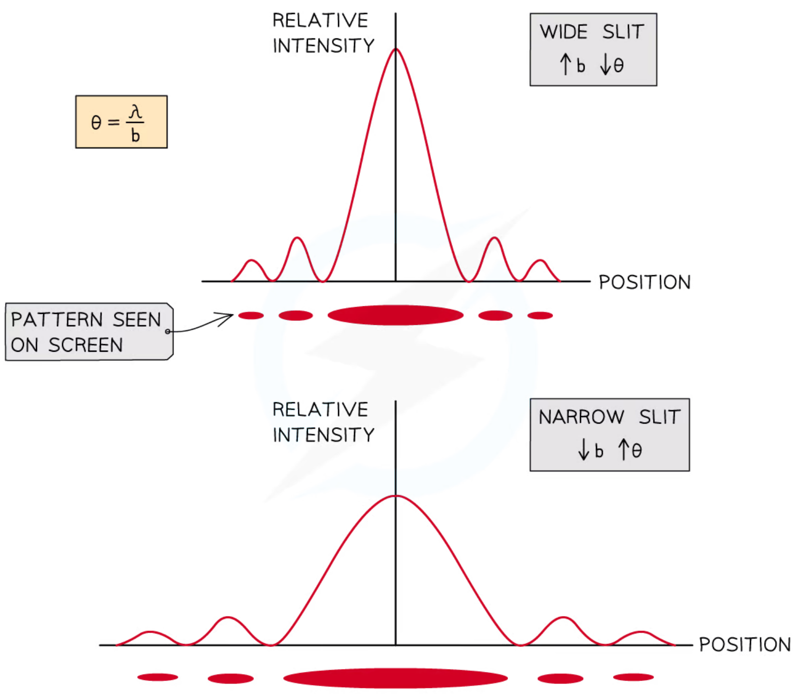
- Using different sources of monochromatic light demonstrates that:
  - Increasing the **wavelength** increases the **width** of the fringes
- The **angle of diffraction** of the **first minima** can be found using the equation:

$$\theta = \frac{\lambda}{b}$$

- Where:
  - θ = the angle of diffraction (radians)
  - λ = wavelength (m)
  - b = slit width (m)

- This equation explains why **red light** produces wider maxima
  - It is because the **longer** the **wavelength**, λ, the **larger** the **angle of diffraction**, θ
- It also explains the coloured fringes seen when **white light** is diffracted
  - It is because **red light** (longer λ) will **diffract more** than **blue light** (shorter λ)
  - This creates fringes which are blue nearer the centre and red further out

- It also explains why wider slits cause the maxima to be narrower
  - It is because the **wider** the **slit**, b, the **smaller** the **angle of diffraction**, θ

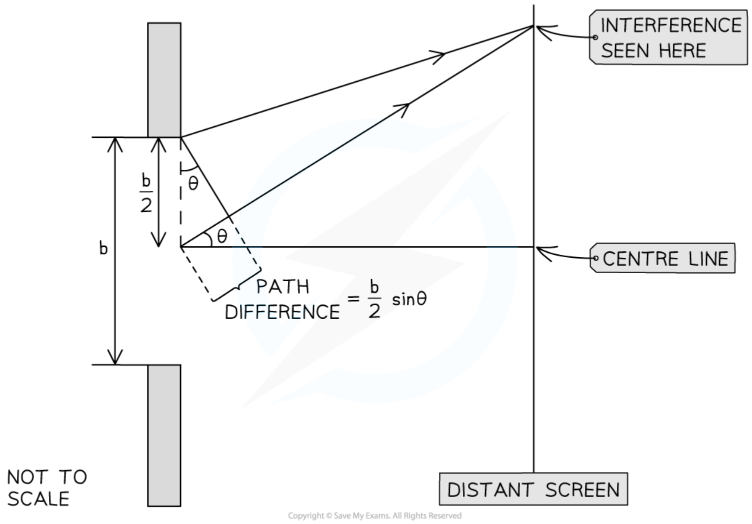


Slit width and angle of diffraction are inversely proportional. Increasing the slit width leads to a decrease in angle of diffraction, hence the maxima appear narrower

Single Slit Geometry

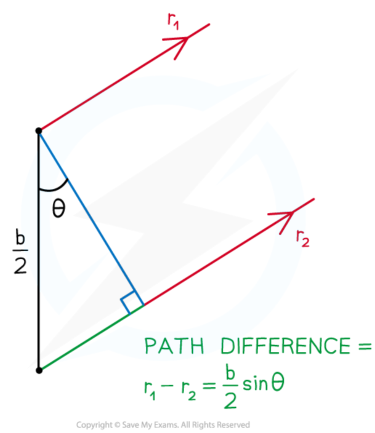
YOUR NOTES

- The diffraction pattern made by waves passing through a slit of width *b* can be observed on a screen placed a large distance away



The geometry of single-slit diffraction

- If the distance, *D*, between the slit and the screen is considerably larger than the slit width, $D >> b$:
  - The light rays can be considered as a set of plane wavefronts that are **parallel** to each other



Determining the path difference using two parallel waves

- For two paths, r_1 and r_2, travelling parallel to each other at an angle, θ, between the normal and the slit, the path difference will be:

$$\text{path difference} = r_1 - r_2 = \frac{b}{2}\sin\theta$$

- For a minima, or area of destructive interference:

The path difference must be a half-integral multiple of the wavelength

YOUR NOTES
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path difference = $\frac{n\lambda}{2}$

- Equating these two equations for path difference:

$$\frac{n\lambda}{2} = \frac{b}{2}\sin\theta$$

$$n\lambda = b\sin\theta$$

- Where n is a non-zero integer number, n = 1, 2, 3...
- Since the angle θ is small, the small-angle approximation may be used: $\sin\theta \approx \theta$

$$n\lambda = b\theta$$

- Therefore, the first minima, n = 1, occurs at:

$$\lambda = b\theta$$

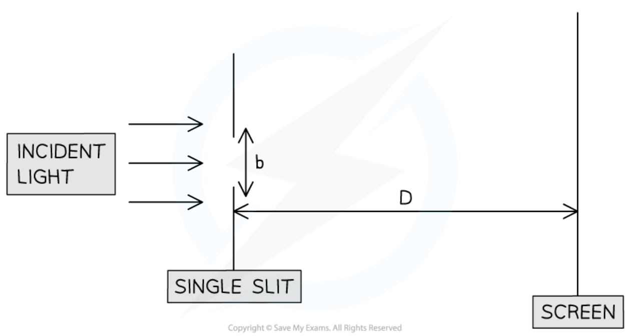
- This leads to the equation for **angle of diffraction** of the first minima:

$$\theta = \frac{\lambda}{b}$$

Worked Example

A group of students are performing a diffraction investigation where a beam of coherent light is incident on a single slit with width, *b*.

The light is then incident on a screen which has been set up a distance, *D*, away.



A pattern of light and dark fringes is seen.

The teacher asks the students to change their set-up so that the width of the first bright maximum **increases**.

Suggest three changes the students could make to the set-up of their investigation which would achieve this.

Step 1: Write down the equation for the angle of diffraction

YOUR NOTES
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$$\theta = \frac{\lambda}{b}$$

- The width of the fringe is related to the size of the angle of diffraction, θ

Step 2: Use the equation to determine the factors that could increase the width of each fringe

Change 1

- The angle of diffraction, θ, is inversely proportional to the slit width, b

$$\theta \propto \frac{1}{b}$$

- Therefore, **reducing the slit width** would increase the fringe width

Change 2

- The angle of diffraction, θ, is directly proportional to the wavelength, λ

$$\theta \propto \lambda$$

- Therefore, **increasing the wavelength** of the light would increase the fringe width

Change 3

- The distance between the slit and the screen will also affect the width of the central fringe
- A larger distance means the waves must travel further hence, will spread out more
- Therefore, **moving the screen further away** would increase the fringe width

YOUR NOTES
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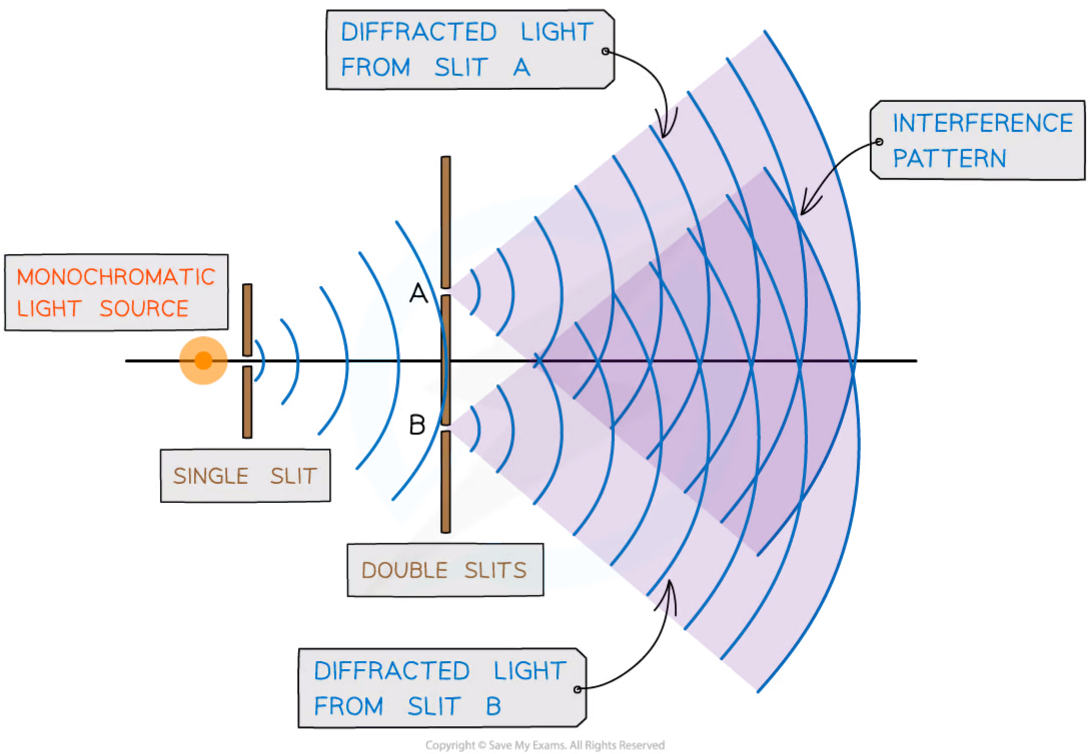
9.3 Interference

9.3.1 Young's Double-Slit Experiment

Young's Double-Slit Experiment

- Young's double-slit experiment demonstrates how light waves can produce an **interference pattern**
- The setup of the experiment is shown below:



Young's double-slit experiment arrangement

- When a monochromatic light source is placed behind a single slit, the light is diffracted producing two light sources at the double slits **A** and **B**
- Since both light sources originate from the same primary source, they are coherent and will therefore create an observable interference pattern
    - Both diffracted light from the double slits create an interference pattern made up of **bright** and **dark fringes**
- The distance between the fringes can be calculated using the double-slit equation:

YOUR NOTES
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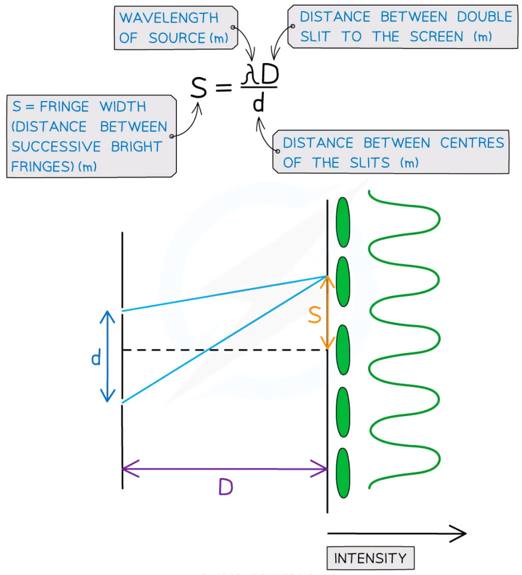


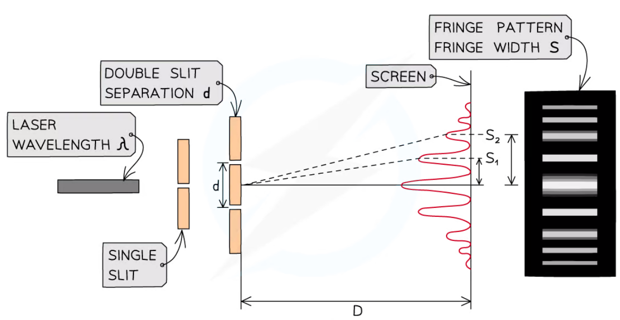
Double slit interference equation with d, s and D represented on a diagram

Investigating Young's Double-Slits Experimentally

The overall aim of this experiment is to investigate the relationship between the distance between the slits and the screen, *D*, and the fringe width, *s*

- Independent variable = Fringe width, *s*
- Dependent variable = Distance between the slits and the screen, *D*
- Control variables
  - Laser wavelength, λ
  - Slit separation, *d*

Method



The setup of apparatus required to measure the fringe width s for different values of D

1. Set up the apparatus by fixing the laser and the slits to a retort stand and place the screen so that *D* is 0.5 m, measured using the metre ruler
2. Darken the room and turn on the laser
3. Measure from the central fringe across many fringes using the vernier callipers (or ideally, a travelling microscope) and divide by the number of fringe widths to find the fringe width, *s*
4. Increase the distance *D* by 0.1 m and repeat the procedure, increasing it by 0.1 m each time up to around 1.5 m

YOUR NOTES
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5. Repeat the experiment twice more and calculate and record the mean fringe width, *s*, for each distance *D*

- An example table might look like this:

DISTANCE BETWEEN SLITS AND SCREEN (D / m) · FRINGE WIDTH (S / m)

D / m	S / m 1st READING	S / m 2nd READING	S / m 3rd READING	S / m MEAN
0.5				
0.6				
0.7				
0.8				
0.9				
1.0				
1.1				
1.2				
1.3				
1.4				
1.5				

Analysing the Results

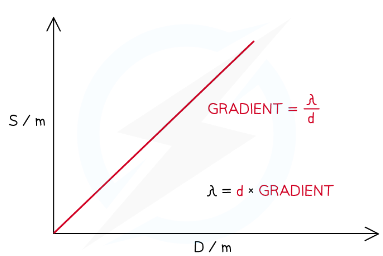
- The fringe spacing equation is given by:

$$s = \frac{\lambda D}{d}$$

- Where:
  - *s* = the distance between each fringe (m)
  - λ = the wavelength of the laser light (m)
  - *D* = the distance between the slit and the screen (m)
  - *d* = the slit separation (m)

- Comparing this to the equation of a straight line: y = mx
  - y = *s* (m)
  - x = *D* (m)
  - Gradient = $\frac{\lambda}{d}$ (unitless)

- Plot a graph of *s* against *D* and draw a line of best fit
- The wavelength of the laser light is equal to the gradient multiplied by the slit separation, as shown by the graph:



Evaluating the Experiment

Systematic errors:

YOUR NOTES
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- Ensure the use of the set square to avoid parallax error in the measurement of the fringe width
- The distance between fringes is very small due to the short wavelength of visible light
  - A monochromatic light source must be used so that the fringes easier to observe

Random errors:

- The fringe spacing can be subjective depending on its intensity on the screen, therefore, take multiple measurements of s (between 3–8) and find the average
- Use a Vernier scale to record distances s to reduce percentage uncertainty
  - Use a travelling microscope, if available, for the greatest accuracy
- Reduce the uncertainty in s by measuring across all visible fringes and dividing by the number of fringes
- Conduct the experiment in a darkened room, so the fringes are clear

Safety Considerations

- Lasers should be Class 2 and have a maximum output of no more than 1 mW
- Do not allow laser beams to shine into anyone's eyes
- Remove reflective surfaces from the room to ensure no laser light is reflected into anyone's eyes

YOUR NOTES
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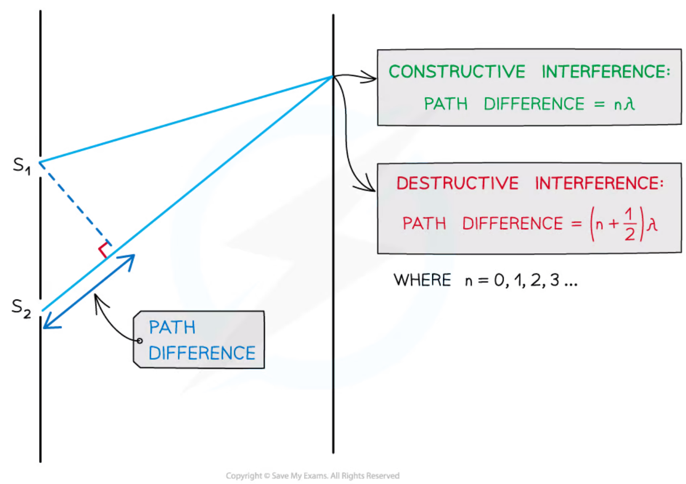
9.3.2 Two-Slit Interference Patterns

Two-Slit Interference Patterns

- For two-source interference fringes to be observed, the sources of the wave must be:
  - **Coherent** (constant phase difference)
  - **Monochromatic** (single wavelength)

- When two waves interfere, the resultant wave depends on the **phase difference** between the two waves
  - This is proportional to the **phase difference** between the waves which can be written in terms of the wavelength λ of the wave

- As seen from the diagram, the wave from slit S_2 has to travel slightly further than that from S_1 to reach the same point on the screen
  - The difference in this distance is the **path difference**



Interference is caused by the variation in path length between the two slits

- For **constructive** interference (or maxima), the difference in wavelengths will be an **integer number of whole wavelengths**
- The condition for constructive interference can be written as:

path difference = $n\lambda$ (where n = 0, 1, 2, 3 ...)

- When waves undergo **constructive interference** their amplitudes add to form a resultant combined wave at the point where they meet

---

- For **destructive** interference (or minima) it will be an **integer number of whole wavelengths plus a half wavelength**
- The condition for destructive interference can be written as:

path difference = $\left(n + \frac{1}{2}\right)\lambda$ (where n = 0, 1, 2, 3 ...)

YOUR NOTES
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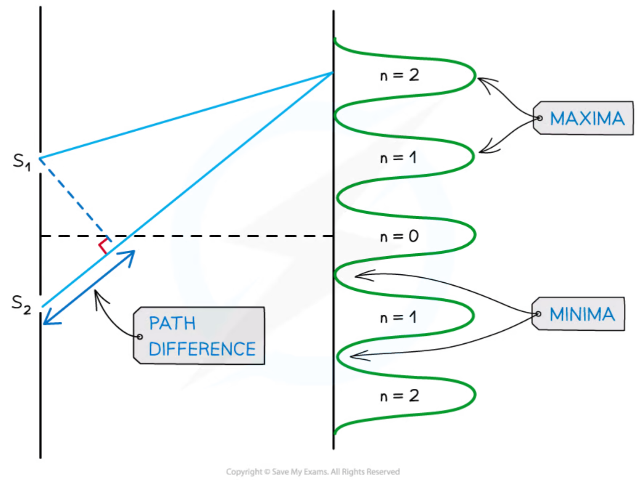
- *n* is the order of the maxima (bright fringes) / minima (dark fringes)
  - *n* = 0 is taken from the middle, *n* = 1 is the next peak and so on
- When waves undergo **destructive interference** their amplitudes subtract to form a resultant combined wave at the point where they meet

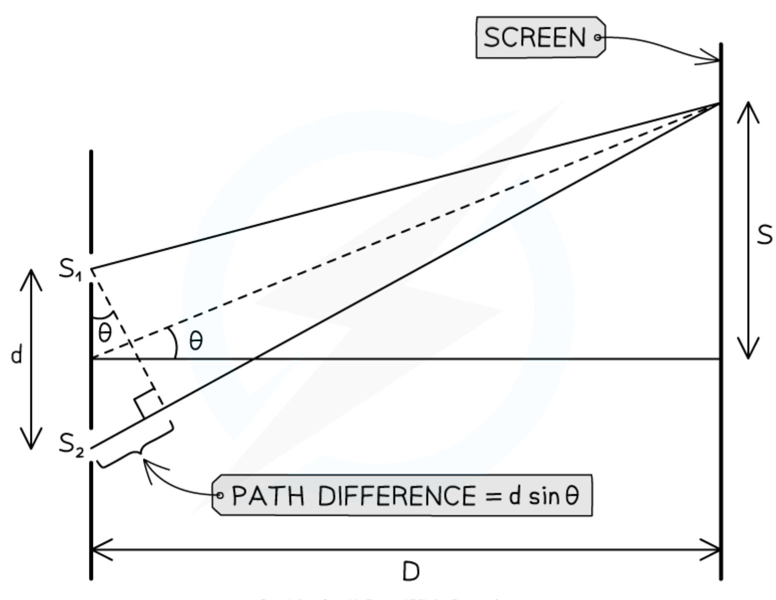


Interference pattern of light waves shown with orders of maxima and minima

- An **interference pattern** shows the intensity of light at different distances away from the central maxima
  - For a double-slit **interference pattern** the intensity of the light is the same for all maxima
- This is different to the double-slit **diffraction pattern** that is observed on a screen (These are explained in 9.3.3 Diffraction Grating Patterns)

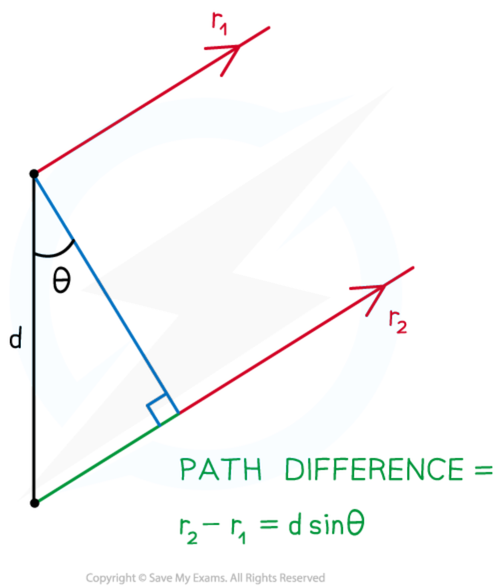
Double-Slit Diffraction Geometry

- The diffraction pattern made by waves passing through two slits of separation *d* can be observed on a screen placed a large distance, *D*, away



YOUR NOTES

- If the distance, D, between the slits and the screen is considerably larger than the slit separation, $D >> d$:
  - The light rays can be considered as a set of plane wavefronts that are **parallel** to each other



- For two paths, r_1 and r_2, travelling parallel to each other at an angle, θ, between the normal and the slit, the path difference will be:

  path difference = $r_2 - r_1 = d \sin \theta$

- For **constructive** interference:

  path difference = $n\lambda$

- Therefore, bright fringes will occur when:

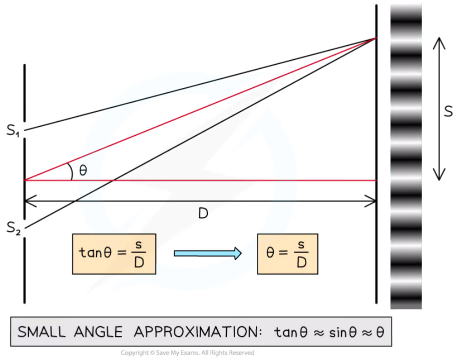
$$n\lambda = d\sin \theta$$

- For **destructive** interference:

  path difference = $\left(n + \frac{1}{2}\right)\lambda$

- Therefore, dark fringes will occur when:

$$\left(n + \frac{1}{2}\right)\lambda = d\sin \theta$$



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- The distance between fringes, s, can be related to the distance to the screen, D, using trigonometry:

$$\tan\,\theta = \frac{s}{D}$$

- Since the angle θ is small, the small-angle approximation may be used: $\tan\,\theta \approx \sin\,\theta \approx \theta$

$$\theta = \frac{s}{D}$$

- Using the condition for constructive interference and the small-angle approximation:

$$n\lambda = d\sin\,\theta$$

$$n\lambda = d\theta$$

- Rearrange the equation for θ:

$$\theta = \frac{n\lambda}{d}$$

- Combining the two equations gives:

$$\frac{n\lambda}{d} = \frac{s}{D}$$

- Rearranging for s gives an equation for the distance between two maxima:

$$s = \frac{n\lambda D}{d}$$

Exam Tip

The equations for constructive and destructive interference above are the same versions as found in your data booklet - don't let other versions of these equations confuse you!

Constructive and destructive interference can be calculated with alternative equations for specific values of n

YOUR NOTES
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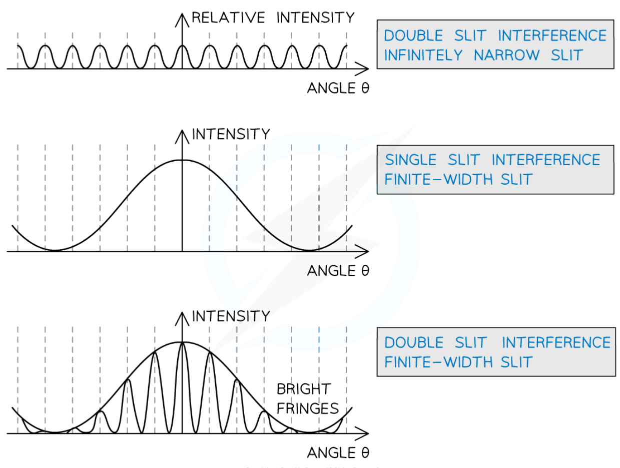
Intensity Graphs

- In single slit diffraction, the observed interference pattern shows:
  - A **central maximum** with the highest intensity
  - Subsequent maxima with **half the width** of the central maximum
  - The **intensity** of each subsequent maxima **decreases**

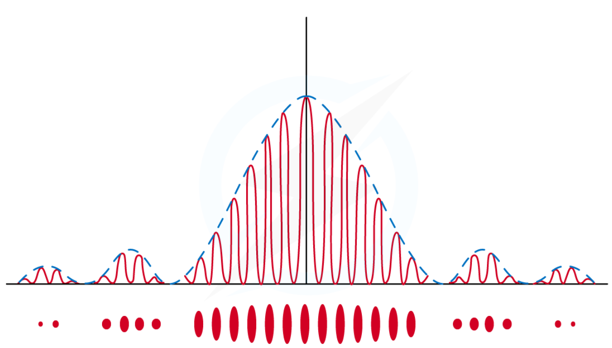
- In double slit diffraction, the observed interference pattern shows:
  - The fringes are equally bright, meaning they have **equal intensity**
  - The angle increases by a **constant** amount, as the fringes are **equally separated**
- **Note:** this is only the case for double slit diffraction for slit widths of negligible size



Intensity graphs for single and double slit interference when the slit is infinitely narrow compared to a slit of finite width

- The fringes due to the double slits are much closer together than in the single slit case
  - This is because the distance between the slits is **greater than their widths**

- The resulting interference pattern is a combination of the double-slit and single slit interference patterns
  - This is known as **modulation**



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A modulated double-slit interference pattern with the single slit diffraction pattern is still visible in the "envelope" (shown by the dotted line). This occurs when the slit width is significant.

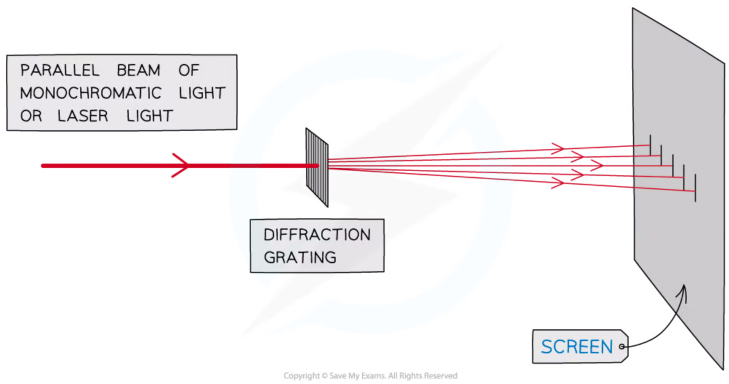
- The intensity of the resulting fringe pattern has been **modulated** by the interference of the two diffracted beams, as the effect of the single slits is significant
- This is assuming that:
  - The slit width is not negligible
  - The distance between the slits is much greater than their width

9.3.3 Diffraction Grating Patterns

YOUR NOTES

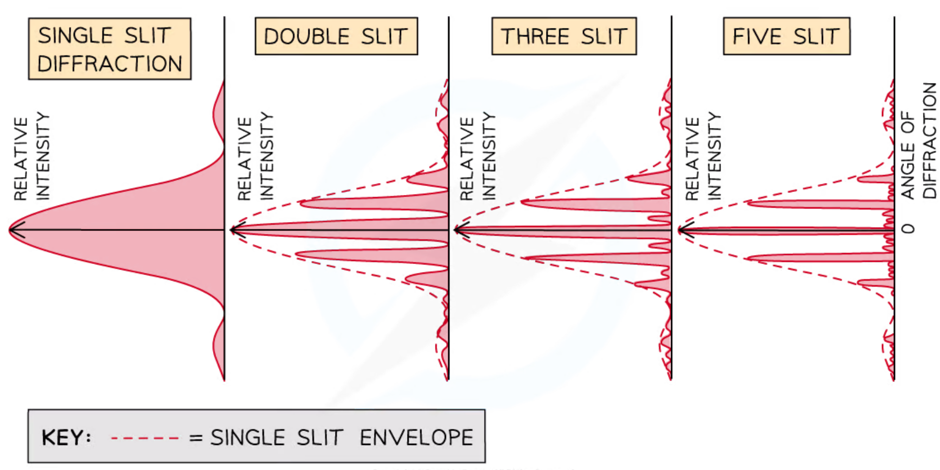
Diffraction Grating Patterns

- A diffraction grating is a plate consisting of a **very large** number of parallel, identical, close-spaced slits
    - When monochromatic light is incident on a grating, a pattern of narrow bright fringes is produced on a screen



Diffraction grating with multiple slits, usually described in terms of 'slits per metre'

- The **number of slits** dictates both the **interference** and **diffraction patterns** which are seen
- On the screen a **diffraction pattern** is observed
    - The **interference pattern** is a measure of the intensity of the light at different angles of diffraction away from the central maxima
    - For a double slit arrangement this is shown in (9.3.2 Two-Slit Interference Patterns)
- On the **diffraction pattern** as the number of slits increases:
    - Between the maxima, secondary maxima appear
    - The central maxima and subsequent bright fringes become narrower



Diffraction patterns for light interfering through different numbers of slits

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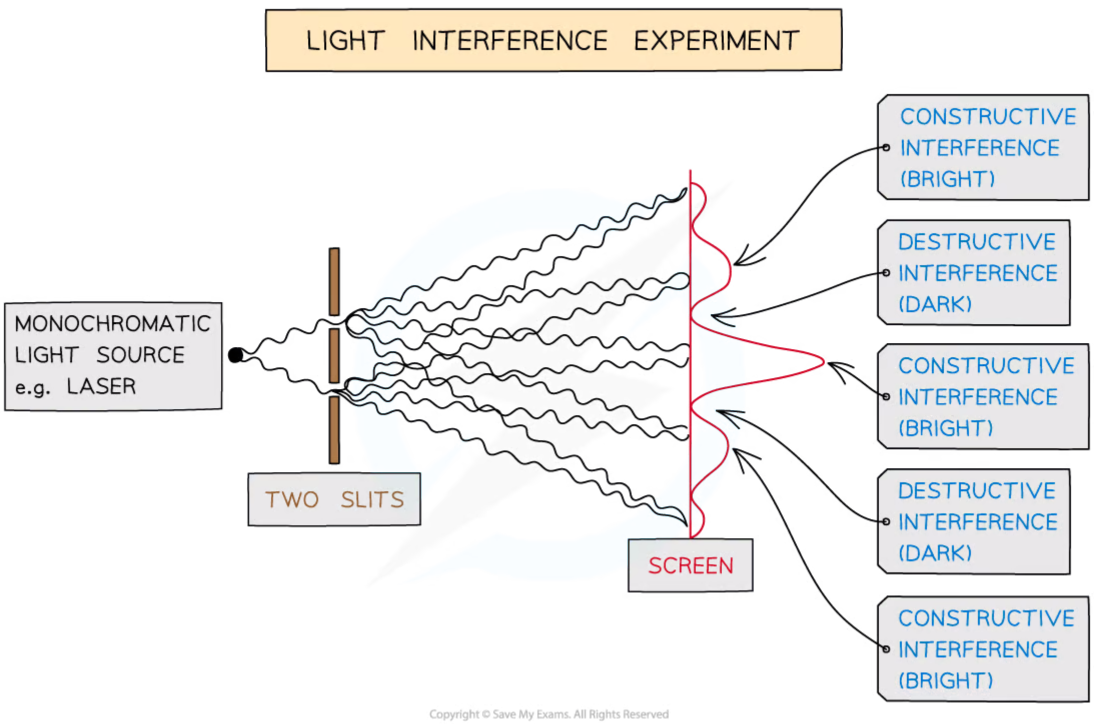
- When there are 3 slits, 1 secondary maxima can be seen between the primary maxima
- When there are 5 slits, 3 secondary maxima can be seen between the primary maxima
    - Therefore, with N slits (when N > 2), there are **(N – 2) secondary maxima**
- Once the number of slits increases to **N > 20**:
    - The primary maxima will become thinner and sharper (since slit width, $d \propto \frac{1}{N}$)
    - The (N – 2) secondary maxima will become unobservable

- On a **diffraction pattern**, as the number of slits increases:
    - The intensity of the central and other larger maxima increases
    - Since the overall amount of light being let through each slit is increased, the pattern increases in intensity by a factor of $N^2 I_0$
        - Where I_0 is the intensity of the central maximum of a single slit diffraction pattern
- It is important to recognise the difference between **interference** and **diffraction patterns**



The diffraction pattern for light passing through a double-slit is the pattern observed on the screen

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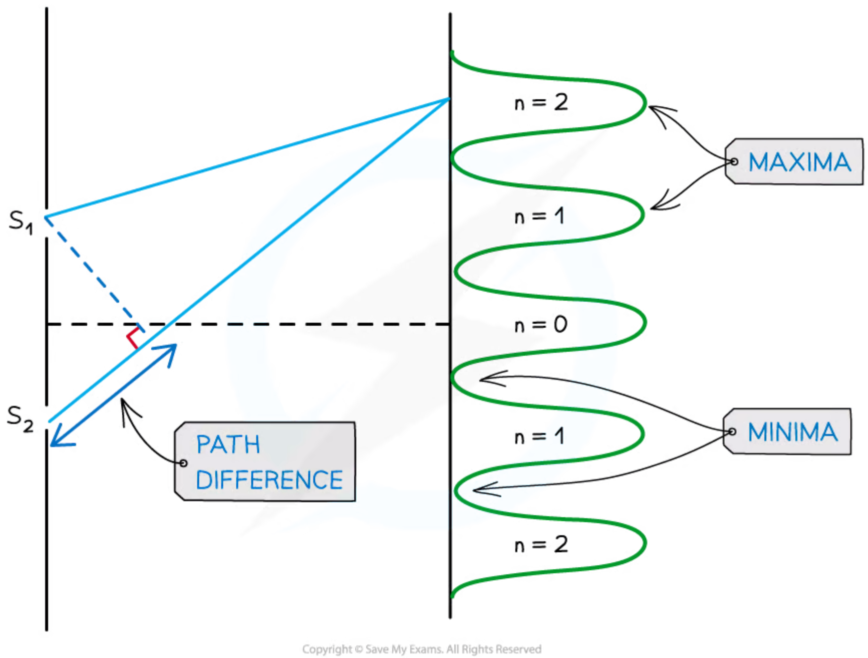

The interference pattern for light passing through a double-slit is different to the diffraction pattern

Investigating Interference by Diffraction Grating

The overall aim of this experiment is to calculate the wavelength of the laser light using a diffraction grating

- Independent variable = Distance between maxima, h
- Dependent variable = The angle between the normal and each order, θ_n (where n = 1, 2, 3 etc)
- Control variables:
  - Distance between the slits and the screen, D
  - Laser wavelength λ
  - Slit separation, d

Method

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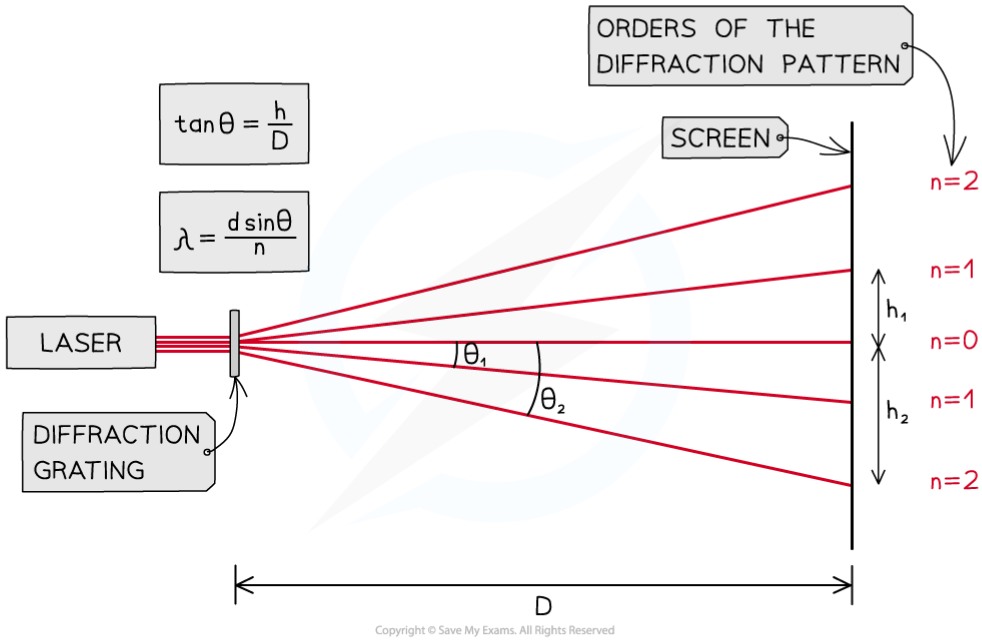


The setup of apparatus required to measure the distance between maxima h at different angles θ

1. Place the laser on a retort stand and the diffraction grating in front of it
2. Use a set square to ensure the beam passes through the grating at normal incidence and meets the screen perpendicularly
3. Set the distance *D* between the grating and the screen to be 1.0 m using a metre ruler
4. Darken the room and turn on the laser
5. Identify the zero-order maximum (the central beam)
6. Measure the distance *h* to the nearest two first-order maxima (i.e. n = 1, n = 2) using a vernier calliper
7. Calculate the mean of these two values
8. Measure distance *h* for increasing orders
9. Repeat with a diffraction grating with a different number of slits per mm

- An example table might look like this:

DIFFRACTION ORDER — DISTANCE BETWEEN MAXIMA — ANGLE BETWEEN MAXIMA

n	h / m 1st READING	h / m 2nd READING	h / m MEAN	θ / °
1				
2				
3				
4				
5				

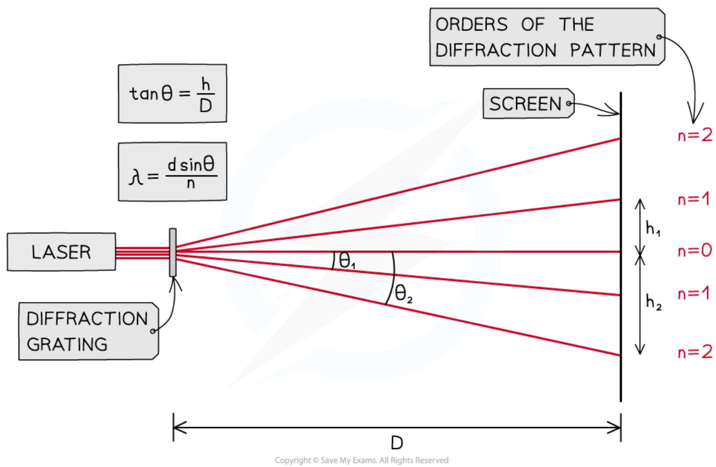
Analysing the Results

YOUR NOTES ↓

The diffraction grating equation is given by:

$$n\lambda = d\sin\theta$$

- Where:
  - n = the order of the diffraction pattern
  - λ = the wavelength of the laser light (m)
  - d = the distance between the slits (m)
  - θ = the angle between the normal and the maxima



- The distance between the slits is equal to:

$$d = \frac{1}{N}$$

- Where
  - N = the number of slits per metre (m^{-1})

- Since the angle is not small, it must be calculated using trigonometry with the measurements for the distance between maxima, h, and the distance between the slits and the screen, D

$$\tan\theta = \frac{h}{D} \quad \rightarrow \quad \theta = \tan^{-1}\left(\frac{h}{D}\right)$$

- Calculate a mean θ value for each order
- Calculate a mean value for the wavelength of the laser light and compare the value with the accepted wavelength
  - This is usually 635 nm for a standard school red laser

Evaluating the Experiments

Systematic errors:

- Ensure the use of the set square to avoid parallax error in the measurement of the fringe width
- Using a grating with more lines per mm will result in greater values of h
  - This lowers its percentage uncertainty

Random errors:

- The fringe spacing can be subjective depending on its intensity on the screen, therefore, take multiple measurements of *h* (between 3–8) and find the mean
- Use a Vernier scale to record distances *h* to increase precision and therefore reduce percentage uncertainty
- Reduce the uncertainty in *h* by measuring across all visible fringes and dividing by the number of fringes
- Increase the grating to screen distance *D* to increase the fringe separation (although this may decrease the intensity of light reaching the screen)
- Conduct the experiment in a darkened room, so the fringes are clear

YOUR NOTES

Safety Considerations

- Lasers should be Class 2 and have a maximum output of no more than 1 mW
- Do not allow laser beams to shine into anyone's eyes
- Remove reflective surfaces from the room to ensure no laser light is reflected into anyone's eyes

Worked Example

A student investigates the interference patterns produced by two different diffraction gratings. One grating used was marked 100 slits / mm, the other was marked 300 slits / mm. The distance between the grating and the screen is measured to be 3.75 m.The student recorded the distance between adjacent maxima after passing a monochromatic laser source through each grating. These results are shown in the tables below.

300 slits/mm	h_1/cm	h_2/cm	Average h/cm	Cumulative Total h/cm
n=0 to 1	71.7	71.5	71.6	71.6
n=1 to 2	79.8	79.8	79.8	151.4

100 slits/mm	h_1/cm	h_2/cm	Average h/cm	Cumulative Total h/cm
n=0 to 1	23.8	24.0	23.9	23.9
n=1 to 2	24.0	25.0	24.5	48.4

Calculate the mean wavelength of the laser light and compare it with the accepted value of 635 nm. Assess the percentage uncertainty in this result.

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Step 1: Calculate the distance between the slits

For 300 slits / mm: $d = \frac{1}{N} = \frac{1}{300 \times 10^3}$

For 100 slits / mm: $d = \frac{1}{N} = \frac{1}{100 \times 10^3}$

Step 2: Calculate the mean angle for each order

$$\theta = \tan^{-1}\left(\frac{h}{D}\right)$$

- For 300 slits / mm:

$$\theta_1 = \tan^{-1}\left(\frac{71.6}{375}\right) = 10.81°$$

$$\theta_2 = \tan^{-1}\left(\frac{151.4}{375}\right) = 21.99°$$

- For 100 slits / mm:

$$\theta_1 = \tan^{-1}\left(\frac{23.9}{375}\right) = 3.647°$$

$$\theta_2 = \tan^{-1}\left(\frac{48.4}{375}\right) = 7.354°$$

Step 3: Use the grating equation to determine the wavelengths for each order

$$n\lambda = d \sin\theta$$

- For 300 slits / mm:

$$n = 1: \ \lambda = \frac{1}{300 \times 10^3} \times \sin 10.81° = 6.25 \times 10^{-7} = 625 \text{ nm}$$

$$n = 2: \ \lambda = \frac{1}{2 \times (300 \times 10^3)} \times \sin 21.99° = 6.24 \times 10^{-7} = 624 \text{ nm}$$

- For 100 slits / mm:

$$n = 1: \ \lambda = \frac{1}{100 \times 10^3} \times \sin 3.647° = 6.36 \times 10^{-7} = 636 \text{ nm}$$

$$n = 2: \ \lambda = \frac{1}{2 \times (100 \times 10^3)} \times \sin 7.354° = 6.40 \times 10^{-7} = 640 \text{ nm}$$

Step 4: Calculate the mean wavelength

$$\text{Mean } \lambda = \frac{625 + 624 + 636 + 640}{4} = 631.25 = 631 \text{ nm}$$

Step 5: Determine the percentage uncertainty in this value

- The difference between the calculated and accepted value is:

$$635 - 631 = 4 \text{ nm}$$

$$\% \text{ uncertainty} = \frac{4}{635} \times 100\% = 0.6\%$$

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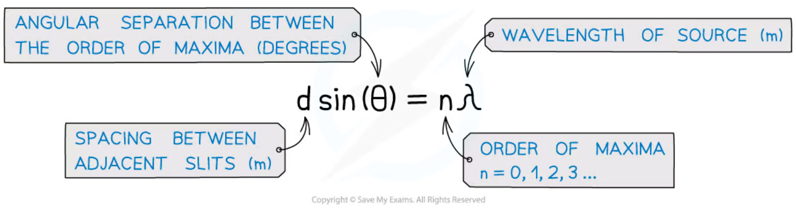
9.3.4 The Diffraction Grating Equation

The Diffraction Grating Equation

- The angles at which the maxima of intensity (constructive interference) are produced can be deduced by the diffraction grating equation:

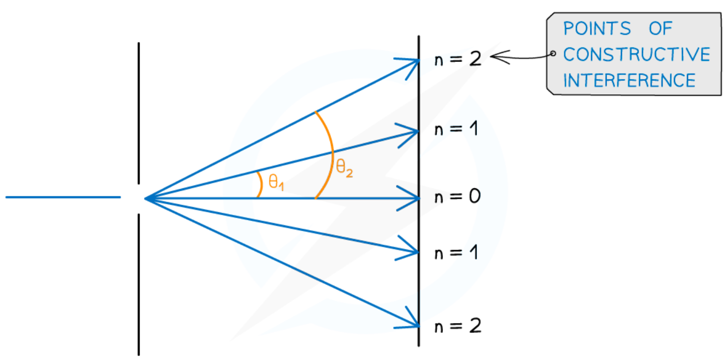


- The **lines per m** (or per mm, per nm etc.) on the grating is usually represented by the symbol *N*
- Therefore, the spacing between each slit, *d*, can be calculated from *N* using the equation:

$$d = \frac{1}{N}$$

Angular Separation

- The angular separation of each maxima is calculated by rearranging the grating equation to make θ the subject
- The angle θ is taken from the centre meaning the higher orders are at greater angles



Angular separation

- The angular separation between two angles is found by subtracting the smaller angle from the larger one
- The angular separation between the first and second maxima n_1 and n_2 is $\boldsymbol{\theta_2 - \theta_1}$

Orders of Maxima

- The maximum angle to see orders of maxima is when the beam is at right angles to the diffraction grating
  - This means $\theta = 90^{\circ}$ and $\sin\theta = 1$
- The highest order of maxima visible is therefore calculated by the equation:

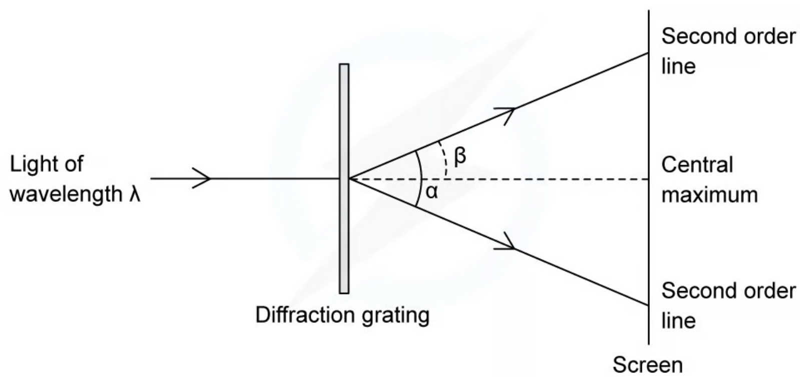
YOUR NOTES

$$n = \frac{d}{\lambda}$$

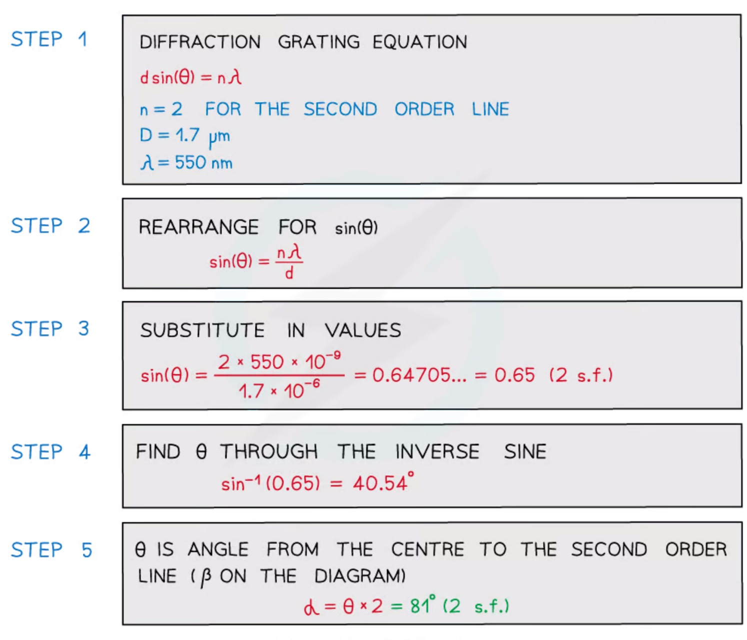
- Note that since *n* must be an integer, if the value is a decimal it must be rounded **down**
  - E.g If *n* is calculated as 2.7 then *n* = 2 is the highest order visible

Worked Example

An experiment was set up to investigate light passing through a diffraction grating with a slit spacing of 1.7 μm. The fringe pattern was observed on a screen. The wavelength of the light is 550 nm.



Calculate the angle α between the two second-order lines.



Derivation of the Diffraction Grating Equation

- When light passes through the slits of the diffraction grating, the path difference at the zeroth order maximum is zero

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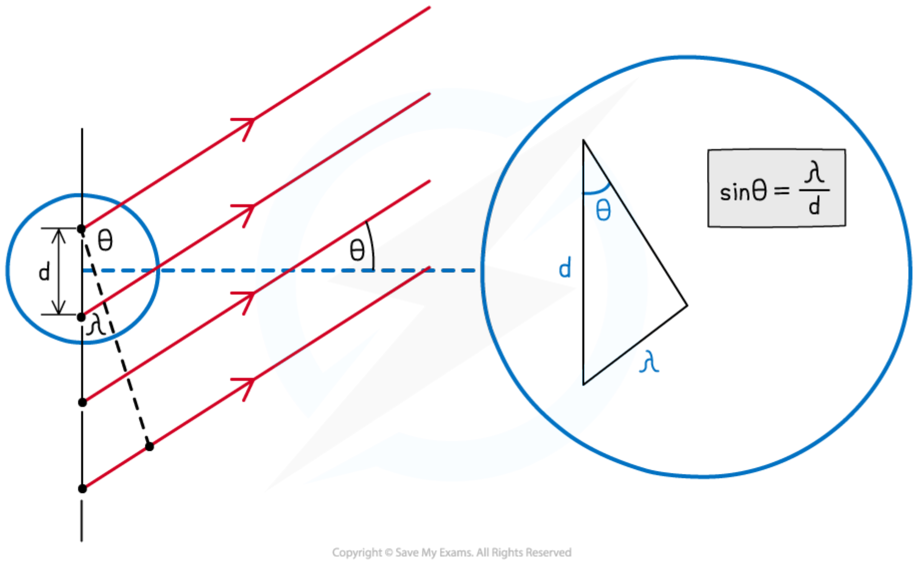
- At the first-order maxima (n = 1), there is constructive interference, hence the path difference is λ
  - Therefore, at the nth order maxima, the path difference is equal to $n\lambda$



Using this diagram and trigonometry, the diffraction grating equation can be derived

- Using trigonometry, an expression for the first order maxima can be written:

$$\sin\,\theta = \frac{\lambda}{d}$$

- Where:
  - θ = the angle between the normal and the maxima
  - λ = the wavelength of the light (m)
  - d = the slit separation (m)

- This means, for n = 1:

$$\sin\,\theta_1 = \frac{\lambda}{d}$$

- Similarly, for n = 2, where the path difference is 2λ:

$$\sin\,\theta_2 = \frac{2\lambda}{d}$$

- Therefore, in general, where the path difference is $n\lambda$:

$$\sin\,\theta_n = \frac{n\lambda}{d}$$

- A small rearrangement leads to the equation for the diffraction grating:

$$\boldsymbol{d\sin\theta_n = n\lambda}$$

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Exam Tip

Take care that the angle θ is the correct angle taken from the centre and **not** the angle taken between two orders of maxima.

9.3.5 Thin Film Interference

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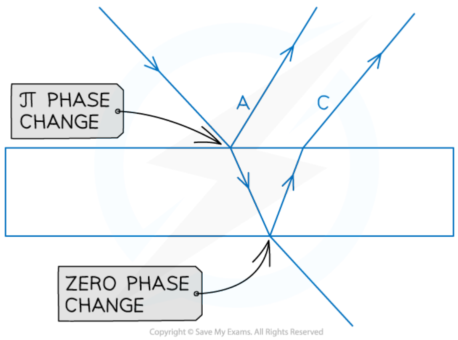
Thin Film Interference

- Thin film interference causes the iridescence seen in:
  - Nature on peacock feathers
  - Glossy flower petals
  - Soap bubbles
  - The shiny side of a CD
  - Thin layers - or films - of oil on water
- This phenomenon occurs when light waves reflecting off the top and bottom surfaces of a thin film interfere with one another

The colourful pattern observed on a CD is a result of thin film interference

Conditions for Thin Film Interference

- To see the interference light must be incident on a material which:
  - Is very **thin**
  - Has a **higher refractive index** than the medium surrounding it
  - Also **transmits** light
- The effect is caused by the reflection of waves from the top and bottom surfaces of the thin film

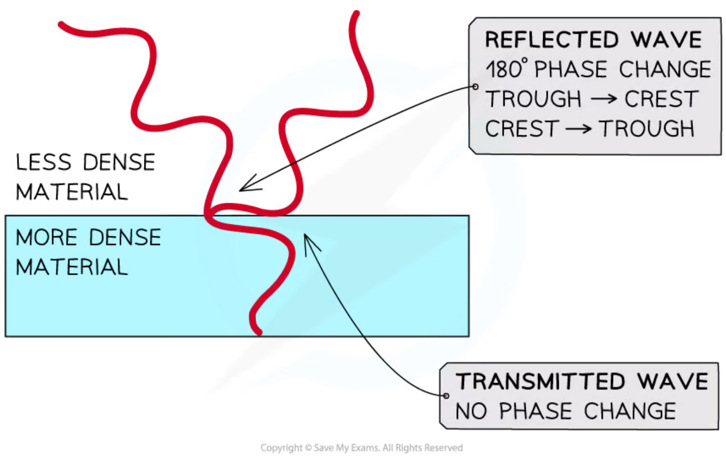


Phase changes at the top and bottom of a thin film

Light incident on the top surface of the thin film (A)

- Part of the light wave reflects at a boundary between a less-dense and a more-dense medium (e.g. Air to Oil where $n_{air} < n_{oil}$), and a **phase change** is seen, such that:

A wave reflected at a boundary with a medium of a higher refractive index undergoes a phase change of half a wavelength ($\frac{\lambda}{2}$), 180° or π rad



A wave incident on the boundary with a more dense material is both reflected and transmitted. The transmitted wave continues without a phase change and the reflected wave has a 180° phase change. What was the peak on the incident wave becomes the trough on the reflected wave.

- You need to be aware of thin-film interference for a wave incident on oil and water
- Light is **reflected** and **transmitted** from a boundary with a **less dense** to a **more dense** material
  - Air to Water
  - Water to Oil
- Light is **transmitted only** from a boundary with a **more dense** to **less dense** material
  - Oil to Air
- A less dense material has a lower **refractive index (n)**
  - The refractive index of air is 1

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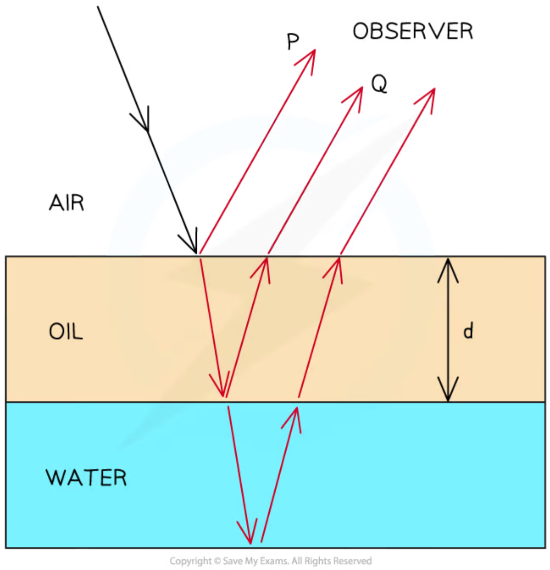
- The refractive index of water is 1.33
- The refractive index of oil is 1.4–1.5
- Normally in thin-film interference situations: $n_{air} < n_{water} < n_{oil}$



Light is reflected and transmitted at the boundary from a less dense to a more dense material. Light is transmitted only at the boundary from a more dense to a less dense material. Hence, in this diagram P and Q exist but the third unlabelled ray does not.

Light transmitting through the thin film (B)

- Part of the light wave also **refracts** as it enters the thin film and is **transmitted** through the material to the bottom surface
- The light wave is now travelling through a **denser medium**, so it travels at a **slower speed** and has a **shorter wavelength**

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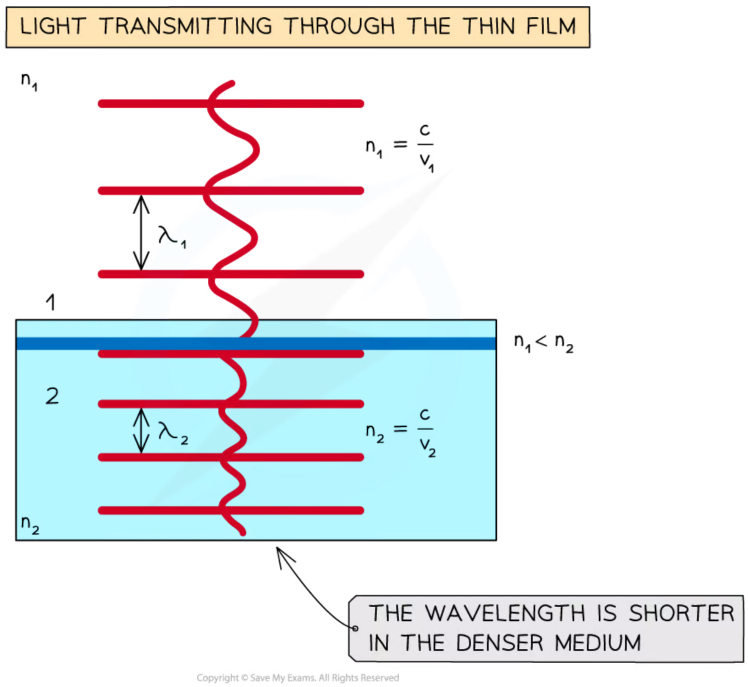
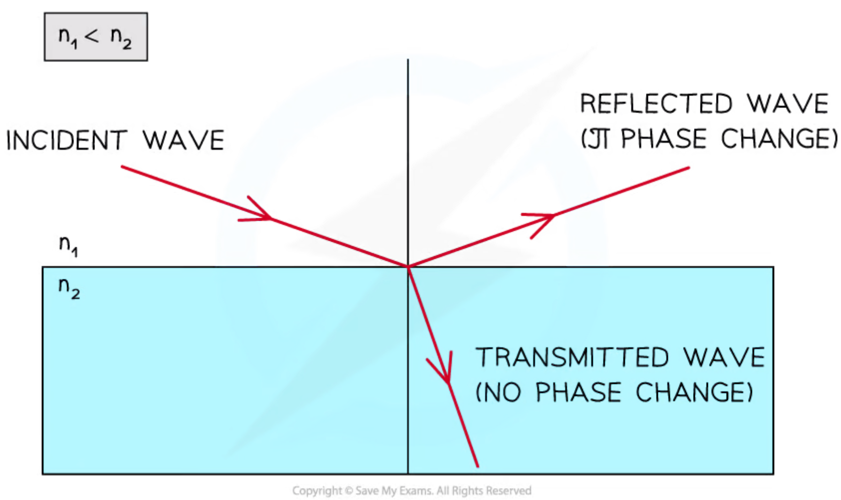


When the light ray enters a denser medium the wavelength becomes shorter.

Light incident on the bottom surface of the thin film (C)

- If the bottom surface is at a boundary with a more dense material (a higher refractive index e.g. oil-water where $n_{oil} < n_{water}$) then both of the following will occur:
  - **Reflection** with a **π phase change** back into the thin film
  - **Refraction** of the **transmitted light** into the next medium
    - This is the same situation as the light wave incident on the top surface of thin film



Light is transmitted with no phase change between the boundary of a less dense to a more dense material and reflected with a phase change.

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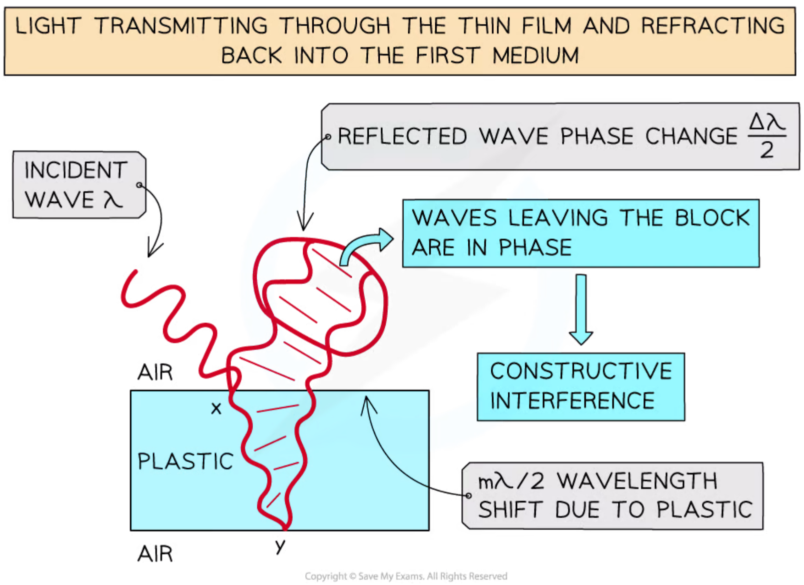
- If the bottom surface is at a boundary with a less dense material (a lower refractive index e.g. oil-air where $n_{air} < n_{oil}$) then the following will occur:
  - **Transmission** with refraction will occur out of the material
  - **No reflection** will occur at the boundary

Light transmitting through the thin film and refracting back into the first medium (D)

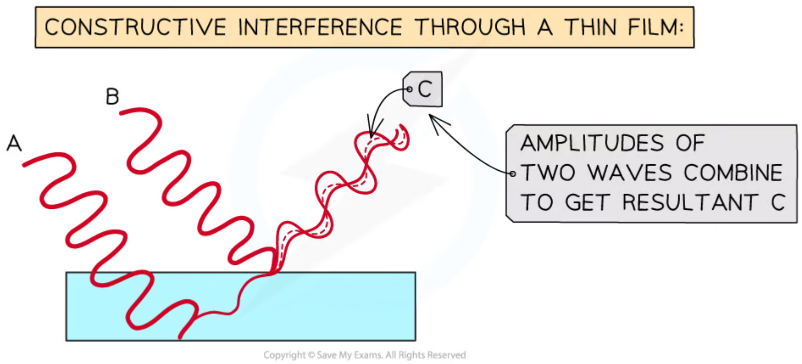
- Light travelling through the thin film undergoes a **wavelength shift** due to the density of the plastic
- The wavelength shift is a multiple (normally labelled *m*) of a half-integer multiple of wavelengths
- The thinner the film, the smaller the value of *m*



Light reflecting off the thin-film air boundary undergoes a wavelength shift related to the density of the thin-film. Light reflecting off the air-thin film boundary undergoes a phase change of half a wavelength. This results in constructive interference and a bright light seen by the observer.

Observing Constructive Interference

- Constructive interference will be seen as brighter colours by the observer
  - The light intensity has increased making the colour appear brighter
- The total path difference of the two waves reflected and refracted off the top surface of the thin film must be a multiple of wavelengths for constructive interference to be seen by the observer



The combination of the phase change due to the reflection off the air-film surface and the phase change within the film here, results in constructive interference seen by an observer because the path difference is a multiple number of wavelengths.

- According to the data booklet, the formula for the path difference for constructive interference is:

$$2dn = (m + \frac{1}{2})\lambda$$

- Where:
  - d = thickness of the film
  - n = refractive index of medium
  - m = an integer related to the refractive index and thickness of the medium
    - The thinnest film exists when $m = 0$
  - λ = wavelength of the light in air
- Constructive interference occurs when two waves are out of phase by an integer number of wavelengths The wave reflected from A has undergone a phase change so now has a path difference of $\frac{\lambda}{2}$
- The wave coming from C inside the film must have a phase difference of a half-integer multiple of wavelengths So, the total phase difference would be $\frac{\lambda}{2}$ + multiple of $\frac{\lambda}{2}$
- In the formula for the path difference for constructive interference: $2dn = (m + \frac{1}{2})\lambda$
- So, the thinnest film exists when $m = 0$
  - So the multiple of $\frac{1}{2}\lambda$ would just be one
  - When m = 1 the multiple would be $\frac{3}{2}\lambda$
  - A common question is to be asked to calculate the thickness of the **thinnest film**

Observing Destructive Interference

- Darker colours are observed through destructive interference
  - The light intensity has decreased making the colour appear darker

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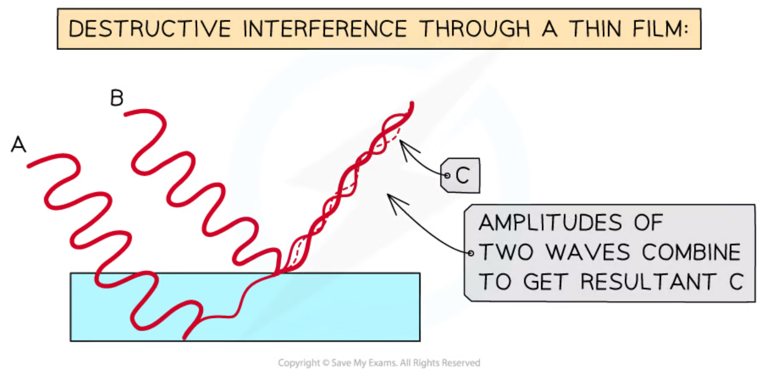
- The total path difference of the two waves reflected and refracted off the top surface of the thin film must be a half-integer multiple of wavelengths for destructive interference to be seen by the observer



The combination of the phase change due to the reflection off the air-film surface and the phase change within the film here, results in destructive interference seen by an observer because the path difference is a half integer multiple number of wavelengths.

- According to the data booklet, the formula for the path difference for destructive interference is:

$$2dn = m\lambda$$

- Where:
  - d = thickness of the film
  - n = refractive index of medium
  - m = an integer related to the refractive index and thickness of the medium
  - λ = wavelength of the light in air

- Destructive interference occurs when two waves are out of phase by a half-integer number of wavelengths The wave reflected from A has undergone a phase change so now has a path difference of $\frac{\lambda}{2}$
- The wave coming from C inside the film must have a phase difference of a half-integer multiple of wavelengths So, the total phase difference would be $\frac{\lambda}{2}$ + multiple of $\frac{\lambda}{2} = \frac{m\lambda}{2}$
- In the formula for the path difference for destructive interference: $dn = \frac{m\lambda}{2}$
- So, the thinnest film exists when $m = 1$
- Other films exist with destructive interference for only odd values of m
  - When $m = 2$ for example constructive interference and not destructive takes place as $\frac{2\lambda}{2} = \lambda$

Distance Travelled inside the film

- Light travels the same distance within a thin film whether it undergoes constructive or destructive interference

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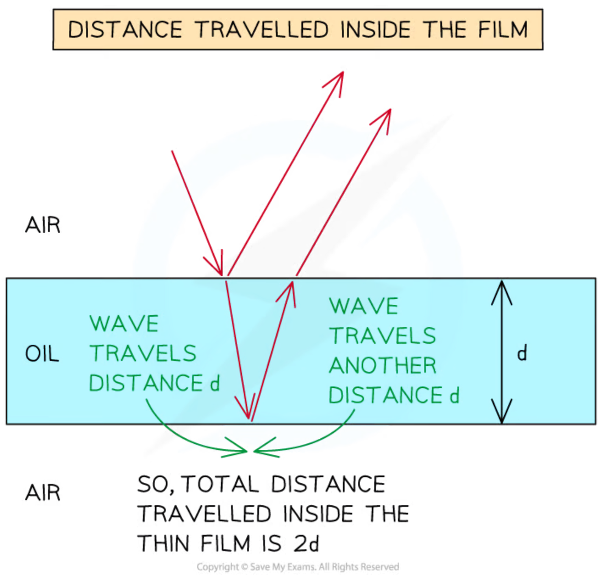
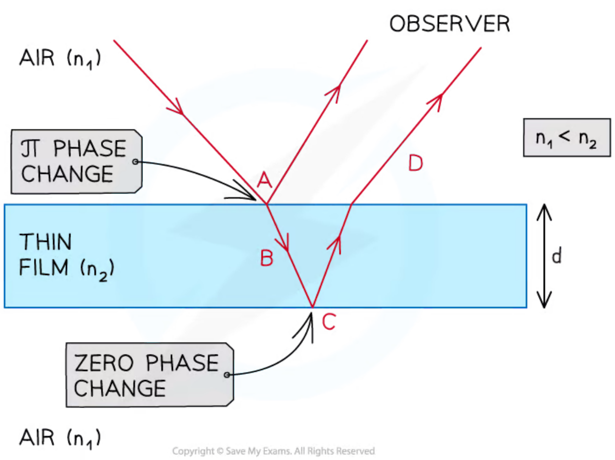
- Light enters from the top surface of the thin film, passes through the thin film of thickness *d*
- Light is then reflected off the bottom surface travelling a further distance of *d* to the original surface
- So, the total distance travelled whether the light interferes constructively or destructively is 2*d*



The reflected wave travels a total distance of twice the thickness of the film



Model the thin film as a parallel-sided, rectangular 'slice' with a thickness of 'd'

Uses of Thin Film Interference

- Thin films can prevent light from being reflected
- Due to the conservation of energy, this increases the light which is transmitted through a medium
- This effect is utilised in:
  - **Camera lenses** - these often have a coating to prevent reflection

- **Solar cells** - this is to ensure a high proportion of incident light can be captured

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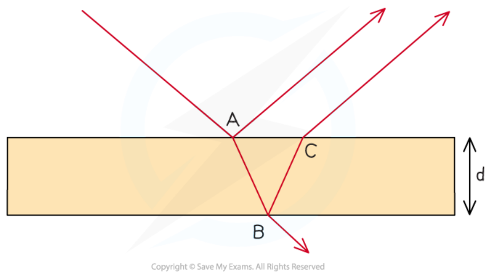
Solving Problems Using Thin Films

The Mathematics of Constructive and Destructive Interference

- Consider a thin film, such as a soap bubble, modelled as a very thin, parallel-sided rectangle, with thickness *d* and refractive index higher than that of the surrounding air, i.e. n > 1
  - If the thin film is sitting on top of a different denser medium e.g. water then the following situations will not apply because of the phase change obtained from the reflection at the oil-water boundary



A thin film can be modelled as a parallel-sided, rectangular 'slice' with a thickness of 'd'

- Constructive interference occurs when:

The thickness of the coating is $\frac{\lambda}{4}$

- This is because the light from the bottom of the film has travelled an extra distance $\frac{\lambda}{2}$ (there and back)
- Since the first ray has undergone a phase change of π, or half a wavelength, $\frac{\lambda}{2}$, the condition for **constructive interference** is:

$$\text{Path difference, } 2d = \left(m + \frac{1}{2}\right)\lambda_0$$

- Where:
  - d = thickness of the film (m)
  - m = an integer (generally taken as m = 0)
  - λ_0 = wavelength of light in soap (m)
- The value of λ_0 can be obtained using the relationship:

$$\lambda_0 = \frac{\lambda}{n}$$

- Where:
  - λ = wavelength of light in a vacuum (m)
  - n = refractive index of the film
- The condition for constructive interference can, therefore, be rewritten in terms of λ as:

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$$2d = \left(m + \frac{1}{2}\right)\frac{\lambda}{n}$$

- Rearranging this gives:

$$2dn = \left(m + \frac{1}{2}\right)\lambda$$

- Destructive interference occurs when:

The thickness of the coating is $\frac{\lambda}{2}$

- This is because the light from the bottom of the film has travelled an overall distance of λ
- This time, the condition for **destructive interference** is:

Path difference, $2d = m\lambda_0$

- The value of λ_0 can be obtained using the relationship:

$$\lambda_0 = \frac{\lambda}{n}$$

- The condition for destructive interference can, therefore, be rewritten in terms of λ as:

$$2d = m\frac{\lambda}{n}$$

- Rearranging this gives:

$$2dn = m\lambda$$

Worked Example

A camera lens has a reflective coating applied to ensure that as much of the light falling on the lens is transmitted, with minimal reflection.

The lens has refractive index of 1.72 and the coating a refractive index of 1.31.

Estimate the thickness of coating required to minimise reflection of visible light. You can assume an average wavelength of 540 nm.

Step 1: List the known quantities

- Refractive index of coating, n = 1.31
- Wavelength, λ = 540 nm

Step 2: Use the data booklet to find the conditions for thin-film interference

Constructive Interference	Destructive Interference
$2dn = \left(m + \frac{1}{2}\right)\lambda$	$2dn = m\lambda$

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Step 3: Consider the phase changes at the boundaries

- Firstly, at the boundary with the coating, which has higher optical density, **phase change = π**
- Secondly, at the boundary with the lens, which has higher optical density, **phase change = π**

Step 4: Determine the correct equation to use

- To prevent reflection, this requires destructive interference, so the equation to use is:

$$2dn = \left(m + \frac{1}{2}\right)\lambda$$

- For a minimum thickness, m = 0

Step 5: Rearrange to make thickness, *d*, the subject and calculate

$$d = \frac{\lambda}{4n}$$

$$d = \frac{540 \times 10^{-9}}{4 \times 1.31} = 100 \text{ nm}$$

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9.4 Resolution

9.4.1 The Rayleigh Criterion

Diffracting Aperture

- A circular aperture allows a cone of light to enter a region behind the aperture
- Examples of circular aperture include:
  - A lens within an optical device such as a camera
  - The pupil of an eye

- A circular aperture allows light to **act like a point source** once passing through
- Placing two point sources **near each other** or viewing those sources **too far away**, will make them appear to be a single **unresolved** source of light
- Consider car headlights which are distant on a highway:
  - Initially, when the car is far away, the headlights appear as one point source
  - It is not until the car comes closer that the two individual headlights can be resolved individually

- Light from any object passing through a circular aperture, including the human eye, will **diffract** and create a diffraction pattern upon the **detector** inside
  - In the case of the human eye, for example, the detector is the retina

- Each of these diffraction patterns need space on the detector to be resolved
  - If they are too close, then they will appear as one single source

- Resolution is defined as:

  The capability of an imaging system to be able to tell if two sources are independent and produce individual images of those two sources

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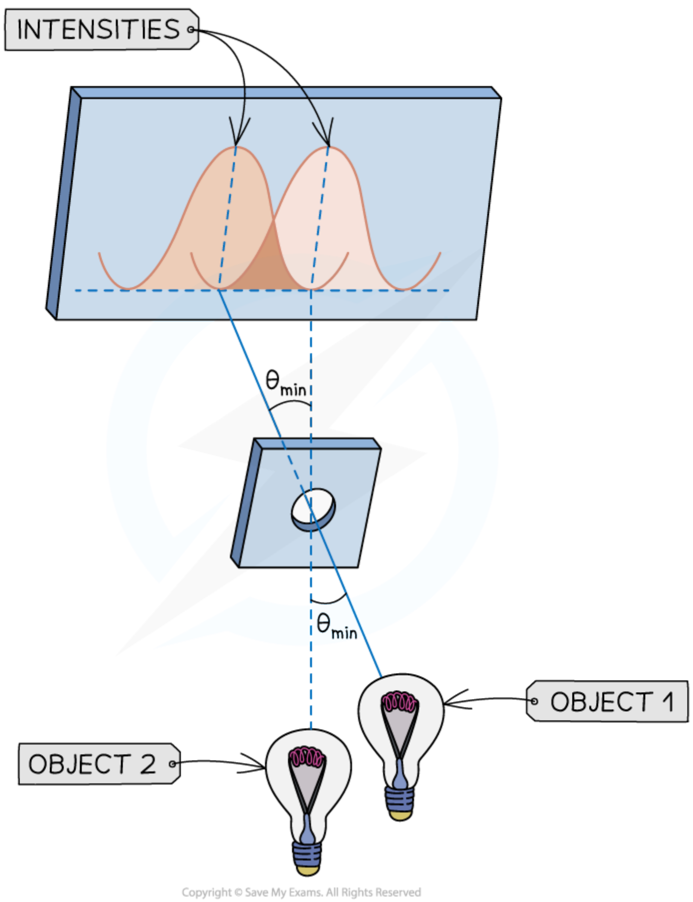

Two light sources outside a circular aperture produce diffraction patterns that have a minimum angle of resolution determined by the Rayleigh criterion

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The Rayleigh Criterion

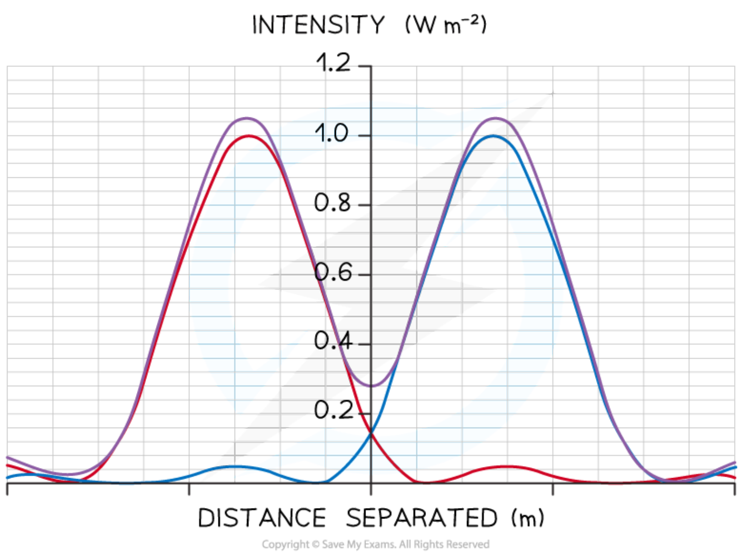
- The **Rayleigh Criterion** describes the limit of resolution of a system to separate two sources

Two sources that only just be resolved. The red is a single source, the blue is the other source and the purple line represents their combined intensity

- The Rayleigh Criterion states that:

  Two sources are able to be just resolved if the principal maximum from one diffraction pattern is aligned with the first minimum of the other diffraction pattern

- Two sources that can be fully resolved:

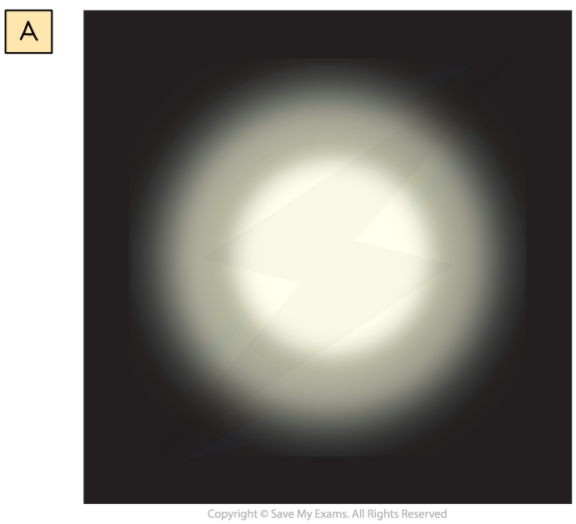


Two sources that can be fully resolved. The red is a single source, the blue is the second source and the purple line represents their combined intensity

- Visually, two sources that **could clearly** be resolved would look like the above
- Two sources that cannot be resolved:

Two sources that cannot be resolved. The red is a single source, the blue is the second source and the purple line represents their combined intensity

- Visually, two sources that could not be resolved:



Visually, two sources that could not be resolved

- Visually, two sources that are only just resolved:

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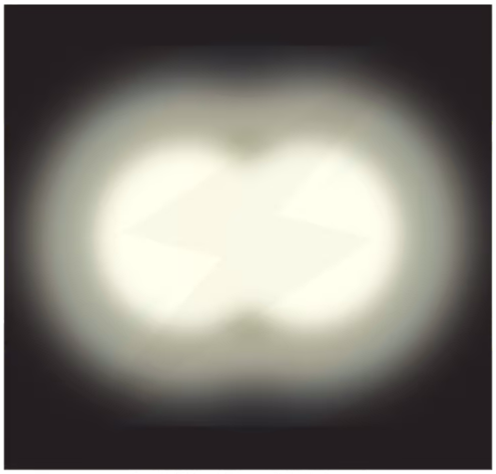

Visually, two sources that could only just be resolved

- This above example is the **limiting case** of the Rayleigh criterion

C

Visually, two sources that could clearly be resolved would look like the above

- If the two sources are separated **further** apart than maximum to minimum, they **can** be resolved
  - If the two sources are brought **closer** together, they **cannot** be resolved

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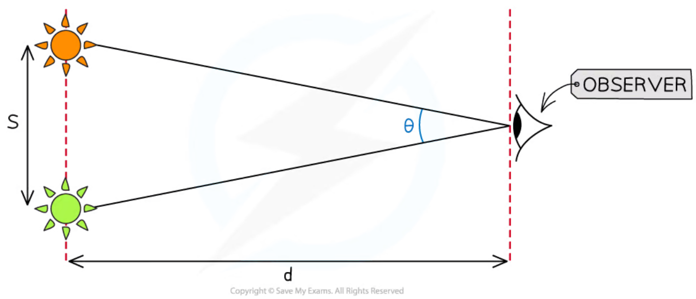
9.4.2 Rayleigh Criterion Calculations

Rayleigh Criterion Calculations

- The Rayleigh Criterion can be mathematically described by considering **angular separation** and **single-slit diffraction**
- Angular separation can be calculated using the equation:

$$\theta = \frac{s}{d}$$

- Where:
  - θ = angular separation (rad)
  - s = distance between the two sources (m)
  - d = distance between the sources and the observer (m)



Angular separation, θ, is equal to the separation, s, of two sources divided by the distance, d, between the sources and the observer

- In single slit diffraction, the first minimum occurs when the angle of diffraction is:

$$\theta = \frac{\lambda}{b}$$

- Where:
  - θ = the angle of diffraction (radians)
  - λ = the wavelength of the light (m)
  - b = the slit width (m)

- According to the Rayleigh criterion, the two sources through a single slit would be **just resolvable** when the angle is equal to that of the **first diffraction minimum** or larger
  - With the **circular aperture**, the value is multiplied by a **factor** of **1.22**

- For a **circular aperture**, the Rayleigh criterion is:

$$\theta = 1.22\frac{\lambda}{b}$$

- Where
  - θ = the angle of diffraction (radians)
  - λ = the wavelength of the light (m)
  - b = the diameter of the circular aperture (m)

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- When the angular separation is larger, or equal to, the Rayleigh criterion, then the two sources can be resolved
- Therefore, for a circular slit, **resolution** occurs when:
  - The angular separation ≥ The angle of diffraction
- Mathematically, the **condition for the resolution of two sources** can be written as:

$$\frac{s}{d} \geq 1.22\frac{\lambda}{b}$$

Worked Example

A student looks at a helicopter in the night sky with one eye closed and can just resolve two lights as individual sources. The wavelength of both sources is 530 nm. The approximate diameter of the student's pupil is 6.0 mm. The distance from the student to the helicopter is 6.0 km.

Determine the minimum distance between the lights.

Step 1: List the known quantities

- Wavelength of light sources, λ = 530 nm = 530×10^{-9} m
- Student pupil diameter, b = 6.0 mm = 6.0×10^{-3} m
- Distance from the helicopter light sources to the student's eye, d = 6.0 km = 6×10^{3} m

Step 2: Select the relevant equation

- Since the lights can just be resolved, this is Rayleigh's criterion
- The equation needed is:

$$\frac{s}{d} \geq 1.22 \times \frac{\lambda}{b}$$

- As the situation is when the lights can just be resolved, this can be written as:

$$\frac{s}{d} = 1.22 \times \frac{\lambda}{b}$$

Step 3: Rearrange equation and input values

- Rearrange for the distance between the sources, s:

$$s = 1.22 \times \frac{\lambda}{b} \times d$$

$$s = 1.22 \times \frac{530 \times 10^{-9}}{6 \times 10^{-3}} \times (6 \times 10^{3}) \approx 0.65\ m$$

Step 4: State the final answer

- The approximate distance between the light sources on the helicopter, s = **65 cm**

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Worked Example

A student views a car in the distance. It has headlights which are 1.5 m apart. The wavelength of light from the car headlights is 500 nm and the pupil diameter of the student is 4.0 mm.

Estimate the maximum distance at which the two headlights could be resolved by the student.

Step 1: List the known values

- Wavelength of light sources, $\lambda = 500$ nm $= 500 \times 10^{-9}$ m
- Student pupil diameter, $b = 4.0$ mm $= 4.0 \times 10^{-3}$ m
- Distance of separation between headlights, $s = 1.5$ m

Step 2: Select the relevant equation

- Since the answer will occur when Rayleigh's criterion is met, the equation needed is:

$$\frac{s}{d} = 1.22 \times \frac{\lambda}{b}$$

Step 3: Rearrange equation and input values

- Rearrange for the distance between the sources and the observer, *d*:

$$s = 1.22 \times \frac{\lambda}{b} \times d$$

$$d \times \frac{\lambda}{b} = \frac{s}{1.22}$$

$$d = \frac{s \times b}{1.22 \times \lambda}$$

$$d = \frac{1.5 \times (4.0 \times 10^{-3})}{1.22 \times (500 \times 10^{-9})} = 9836\ m$$

Step 4: State the final answer

- The maximum distance where the student could resolve the headlights, *d* = **9800 m (2 s.f.)**

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Exam Tip

You might be curious where the factor of 1.22 comes from, however, the derivation of this is beyond the scope of the IB DP Physics course so just make sure you know how to use it in your calculations

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Resolvance of Diffraction Gratings

- In order to know if a diffraction grating is able to resolve two wavelengths, the **resolving power** of the diffraction grating must be found
  - This is based on the Rayleigh criterion as applied to diffraction gratings and their output
- The resolving power, R, of a diffraction grating is given by:

$$R = \frac{\lambda}{\Delta\lambda}$$

- Where:
  - R = the resolving power of the grating (no unit)
  - λ = the wavelength of incident light (m)
  - $\Delta\lambda$ = the smallest difference in wavelength that the grating can resolve (m)
- The resolving power is also equal to

$$R = N \times m$$

- Where:
  - N = the total number of slits on the diffraction grating (or those illuminated by an incident beam of light)
  - m = the order of diffraction
- Therefore, combining the two equations gives:

$$R = \frac{\lambda}{\Delta\lambda} = N \times m$$

Worked Example

A student is using a diffraction grating to resolve two emission wavelengths from calcium in the 3rd order of the spectrum. These wavelengths are 164.99 nm and 165.20 nm.

Determine the minimum number of lines per mm needed if a beam of width 0.25 mm is incident upon the diffraction grating.

Step 1: List the known values
- Order of diffraction, $m = 3$
- Wavelength, $\lambda_1 = 164.99$ nm
- Wavelength, $\lambda_2 = 165.20$ nm
- Beam width = 0.25 mm

Step 2: Determine values for λ and $\Delta\lambda$
- The value of the incident wavelength, λ, can be determined from the mean of the two wavelengths:

$$\lambda = \frac{164.99 + 165.20}{2} = 165.095 \text{ nm}$$

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- The difference between the two wavelengths, $\Delta\lambda$, is:

$$\Delta\lambda = \lambda_2 - \lambda_1 = 165.20 - 164.99 = 0.21 \text{ nm}$$

Step 3: Find the resolving power of these wavelengths
- The resolving power can be calculated using:

$$R = \frac{\lambda}{\Delta\lambda}$$

Step 4: Input the relevant values

$$R = \frac{\lambda}{\Delta\lambda} = \frac{165.095}{0.21} = 786.2$$

Step 5: Find the number of lines illuminated
- The equation relating resolving power and number of slits is given by:

$$R = N \times m$$

- Rearranging for N and substituting the values for R and m:

$$N = \frac{R}{m} = \frac{786.2}{3} = 262.1$$

- The value 262.1 is the number of lines illuminated for the beam of 0.25 mm

Step 6: Find the number of lines needed per mm for this situation
- Since 262.1 is the number of lines illuminated for the beam of 0.25 mm, then 4 times more lines must be illuminated to account for 1 mm:

$$262.1 \times 4 = 1048.4 \text{ lines per mm}$$

State 7: State the final answer

- The minimum amount of lines needed per mm to resolve the relevant calcium lines is: **1049 lines per mm**

Exam Tip

In the worked example, the answer may look as though it has been rounded incorrectly but we are looking for the actual number of lines here, not fractions of lines

Rounding down to 1048 would leave the grating nearly half a line short, which can't happen, so always **round up to the nearest whole number**

9.5 Doppler Effect

9.5.1 The Doppler Effect

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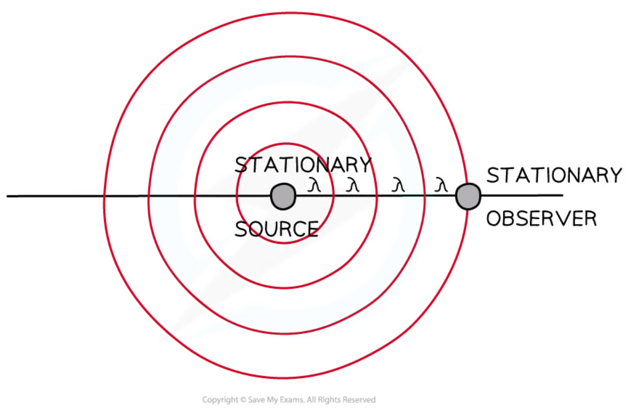
The Doppler Effect

- When a source of sound, such as the whistle of a train or the siren of an ambulance, moves **away** from an observer:
  - It appears to **decrease** in frequency, i.e. it sounds **lower** in pitch
  - Although, the **source** of the sound remains at a **constant** frequency
- This frequency change due to the relative motion between a source of sound or light and an observer is known as the **Doppler effect** (or **Doppler shift**)
- When the observer and the source of sound (e.g. ambulance siren) are both **stationary**:
  - The waves appear to remain at the **same** frequency for both the observer and the source



Stationary source and observer

- When the source starts to move **towards** the observer, the wavelength of the waves is **shortened**
  - The sound, therefore, appears at a **higher** frequency to the observer

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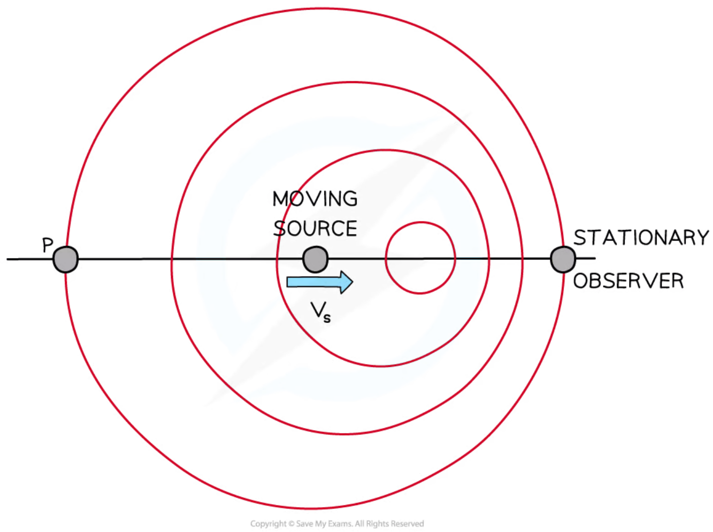

Moving source and stationary observer

- Notice how the waves are closer together between the source and the observer compared to point **P** and the source
- This also works if the source is moving away from the observer
  - If the observer was at point **P** instead, they would hear the sound at a lower frequency due to the wavelength of the waves **broadening**
- The frequency is **increased** when the source is moving **towards** the observer
- The frequency is **decreased** when the source is moving **away** from the observer

Worked Example

A cyclist rides a bike ringing their bell past a stationary observer. Which of the following accurately describes the doppler shift caused by the sound of the bell?

	Wavelength	Frequency	Sound pitch
A	Shorter	Higher	Lower
B	Longer	Lower	Higher
C	Shorter	Lower	Higher
D	Longer	Lower	Lower

ANSWER: D

- If the cyclist is riding past the observer, the wavelength of sound waves are going to become longer
  - This rules out options A and C
- A longer wavelength means a lower frequency (from the wave equation)

- Lower frequency creates a lower sound pitch
  - Therefore, the answer is row D

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Representing The Doppler Effect

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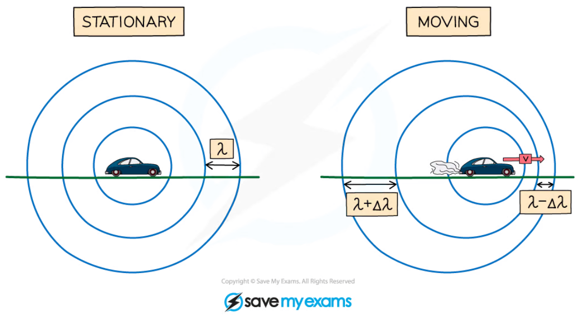
- Wavefront diagrams help visualize the Doppler effect for moving wave sources and stationary observers



Wavefronts are even in a stationary object but are squashed in the direction of the moving wave source

- A moving object will cause the **wavelength**, λ, (and frequency) of the waves to change:
  - The **wavelength** of the waves **in front** of the source **decreases** (λ – Δλ) and the **frequency increases**
  - The wavelength **behind** the source **increases** (λ + Δλ) and the **frequency decreases**
  - This effect is known as the **Doppler effect** or **Doppler shift**

- Note: Δλ means '**change in wavelength**'
- The Doppler shift is observed by all waves including sound and light
- When the source starts to move **towards** the observer, the wavelength of the waves is **shortened**
  - For sound waves, sound, therefore, appears at a **higher** frequency to the observer
  - For light waves, the light shifts towards **blue** due to its higher frequency

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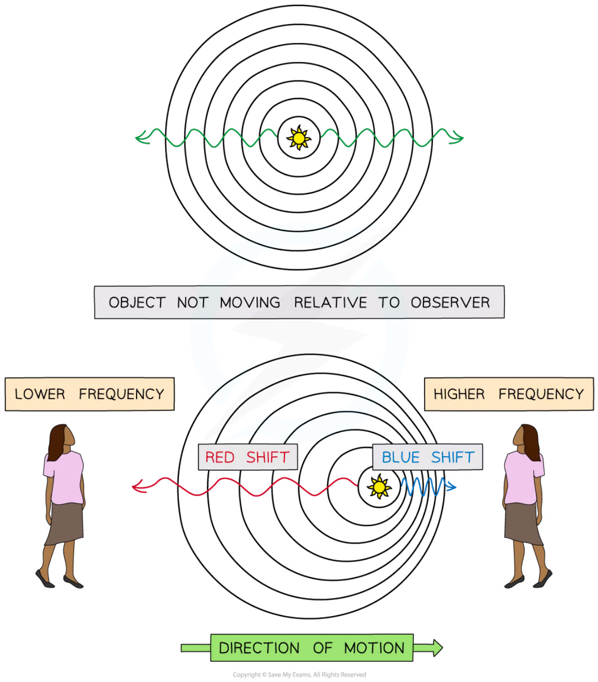

Representing red-shifted and blue-shifted light

- When the source starts to move **away from** the observer, the wavelength of the wave **broadens**
  - For sound waves, sound therefore appears at a **lower** frequency to the observer
  - For light waves, the light shifts towards **red** due to its lower frequency
- When the source starts to move **towards** the observer, the wavelength of the wave **shortens**
  - For sound waves, sound therefore appears at a **higher** frequency to the observer
  - For light waves, the light shifts towards **blue** due to its higher frequency
- This is because red light has a **longer wavelength** than blue light

9.5.2 Uses of The Doppler Effect

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Uses of The Doppler Effect

- The Doppler effect is important in many key areas of science including:
  - **Radar** readings for moving objects
  - Measuring the **rate of blood flow** within a patient
  - Finding planet **orbits** around distant stars
  - Mapping the **expansion** of the universe

Redshift of EM Radiation

- On Earth, the Doppler effect of sound can be easily observed when sound waves move past an observer at a notable speed
- In space, the Doppler effect of light can be observed when spectra of distant stars and galaxies are observed, this is known as:
  - **Redshift** if the object is moving away from the Earth, or
  - **Blueshift** if the object is moving towards the Earth

- Redshift is defined as:

  The fractional increase in wavelength (or decrease in frequency) due to the source and observer receding from each other

- For **non-relativistic** galaxies, Doppler redshift can be calculated using:

$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta f}{f} = \frac{v}{c}$$

- Where:
  - $\Delta\lambda$ = shift in wavelength (m)
  - λ = wavelength emitted from the source (m)
  - Δf = shift in frequency (Hz)
  - f = frequency emitted from the source (Hz)
  - v = speed of recession (m s^{-1})
  - c = **speed of light** in a vacuum (m s^{-1})

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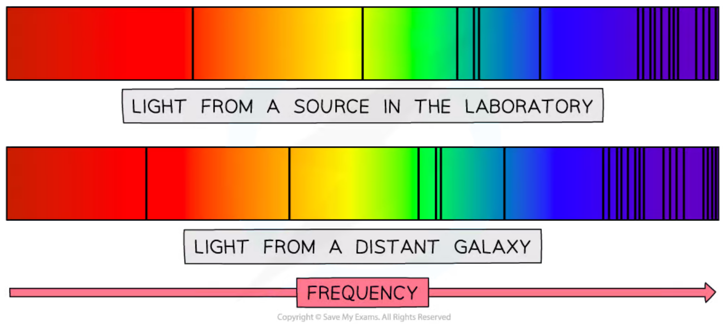
Worked Example

The spectra below show dark absorption lines against a continuous visible spectrum.



A particle line in the spectrum of light from a source in the laboratory has a frequency of 4.570×10^{14} Hz. The same line in the spectrum of light from a distant galaxy has a frequency of 4.547×10^{14} Hz.

Calculate the speed of the distant galaxy in relation to the Earth. Determine whether it is moving towards or away from the Earth.

Step 1: Write down the known quantities

- Received frequency, $f_r = 4.547 \times 10^{14}$ Hz
- Original frequency, $f_0 = 4.570 \times 10^{14}$ Hz
- Shift in frequency, $\Delta f = (4.547 - 4.570) \times 10^{14} = -2.3 \times 10^{12}$ Hz
- Speed of light, $c = 3.0 \times 10^8$ m s^{-1}

Step 2: Write down the Doppler redshift equation

$$\frac{\Delta f}{f_0} = \frac{v}{c}$$

Step 3: Rearrange for speed *v*, and calculate

$$v = \frac{c\Delta f}{f_0} = \frac{(3.0 \times 10^8) \times (2.3 \times 10^{12})}{4.570 \times 10^{14}} = 1.5 \times 10^6 \text{ m s}^{-1}$$

Step 4: Write a concluding sentence

- The observed frequency is **less** than the emitted frequency (the light from a laboratory source), therefore, the source is **receding**, or moving away, from the Earth at **1.5×10^6 m s^{-1}**

An Expanding Universe

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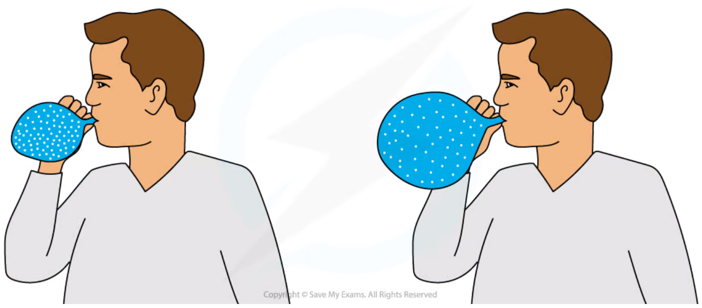
- After the discovery of Doppler redshift, astronomers began to realize that almost all the galaxies in the universe are receding
- This led to the idea that the space between the Earth and the galaxies must be **expanding**
- This expansion stretches out the light waves as they travel through space, shifting them towards the red end of the spectrum
- The more red-shifted the light from a galaxy is, the **faster** the galaxy is moving away from Earth



- The expansion of the universe can be compared to dots on an inflating balloon
  - As the balloon is inflated, the dots all move away from each other
  - In the same way as the rubber stretches when the balloon is inflated, space itself is **stretching out between galaxies**
  - Just like the dots, the galaxies move away from each other, however, they themselves do not move

9.5.3 The Doppler Equation

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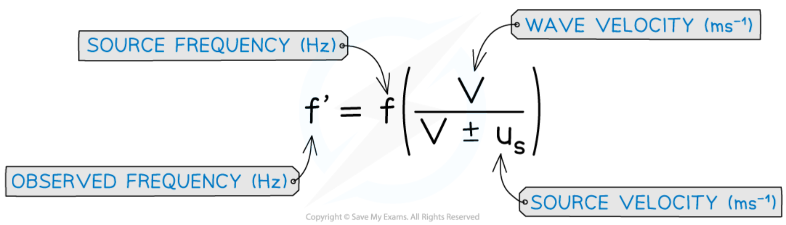
The Doppler Equation

Calculating Doppler Shift

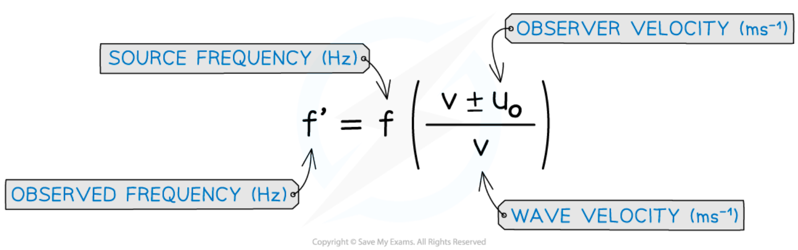
- When a source of sound waves moves relative to a stationary observer, the observed frequency can be calculated using the equation below:



Doppler shift equation for a moving source

- The wave velocity for sound waves is 340 ms^{-1}
- The ± depends on whether the source is moving towards or away from the observer
  - If the source is moving **towards** the observer, the denominator is $v - u_s$
  - If the source is moving **away** from the observer, the denominator is $v + u_s$

- When a source of sound waves remains stationary, but the observer is moving relative to the source, the observed frequency can be calculated using the equation below:



Doppler shift equation for a moving observer

- The ± depends on whether the observer is moving towards or away from the source
  - If the observer is moving **towards** the source, the numerator is $v + u_O$
  - If the observer is moving **away** from the source, the numerator is $v - u_O$

- These equations can also be written in terms of wavelength
  - For example, the equation for a moving source is shown below:

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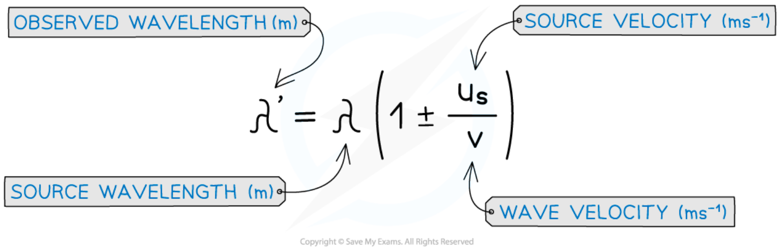


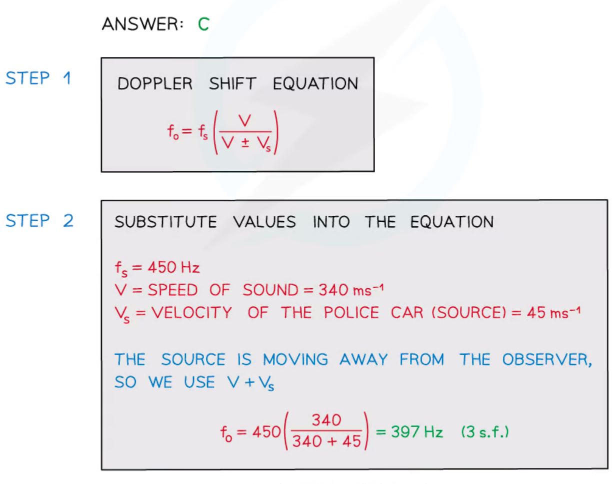
Doppler shift equation for a moving source in terms of wavelength

- The ± depends on whether the source is moving **towards or away** from the observer
    - If the source is moving **towards**, the term in the brackets is $1 - \frac{u_S}{v}$
    - If the source is moving **away**, the term in the brackets is $1 + \frac{u_S}{v}$

Worked Example

A police car siren emits a sound wave with a frequency of 450 Hz. The car is traveling away from an observer at speed of 45 m s^{-1}. The speed of sound is 340 m s^{-1}. Which of the following is the frequency the observer hears?

A. 519 Hz **B.** 483 Hz **C.** 397 Hz **D.** 358 Hz



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Worked Example

A bank robbery has occurred and the alarm is sounding at a frequency of 3 kHz. The thief jumps into a car which accelerates and reaches a constant speed. As he drives away at a constant speed, the frequency decreases to 2.85 kHz. The speed of sound is 340 m s^{-1}. Determine at what speed must he be driving away from the bank.

Step 1: List the known quantities

- Source frequency: f = 3 kHz = 3000 Hz
- Observed frequency: f' = 2.85 kHz = 2850 Hz
- Speed of sound: v = 340 m s^{-1}
- The observer is moving away from a stationary source of sound

Step 2: write the doppler shift equation

$$f' = f\left(\frac{v - u_o}{v}\right)$$

Step 3: rearrange to find the desired quantity

$$\frac{f'}{f} = \left(\frac{v - u_o}{v}\right)$$

$$\frac{f'}{f} \times v = v - u_o$$

$$\left(\frac{f'}{f} \times v\right) + u_o = v$$

$$u_o = v - \left(\frac{f'}{f} \times v\right)$$

Step 4: substitute in values

$$u_o = v - \left(\frac{f'}{f} \times v\right) = 340 - \left(\frac{2850}{3000} \times 340\right) = 340 - 323 = 17\ ms^{-1}$$

Step 5: State final answer

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- The bank robber must be driving away at a constant speed of **17 ms^{-1}** based on the change in frequency heard

Calculating Doppler Shift of Light

- Doppler shift can be calculated with relation to a light emitting source
  - For example, a galaxy moving towards or away from Earth
- Doppler shift for light is **complicated**, however if the **speed** of the observer or source is **small (non-relativistic)** compared to the speed of light, then this equation becomes **simpler**
- The Doppler shift for a light-emitting non-relativistic source is described using the equation:

$$\frac{\text{CHANGE IN FREQUENCY}}{\text{REFERENCE FREQUENCY}} = \frac{\text{CHANGE IN WAVELENGTH}}{\text{REFERENCE WAVELENGTH}} = \frac{\text{VELOCITY OF A GALAXY}}{\text{SPEED OF LIGHT}}$$

$$\frac{\Delta f}{f_0} = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{\Delta\lambda}{\lambda_0} \approx \frac{v}{c}$$

Doppler shift equation relating wavelength change for a moving source

- Where:
  - Δf = change in frequency in Hertz (Hz)
  - f_0 = reference frequency in Hertz (Hz)
  - λ = observed wavelength of the source in metres (m)
  - λ_0 = reference wavelength in metres (m)
  - $\Delta\lambda$ = change in wavelength in metres (m)
  - v = velocity of a galaxy in metres per seconds (m/s)
  - c = the speed of light in metres per second (m/s)

- This means that the change in wavelength, $\Delta\lambda$:

$$\Delta\lambda = \lambda - \lambda_0$$

- This equation can be used to calculate the velocity of a galaxy if its wavelength can be measured and compared to a reference wavelength
- Since the fractions have the same units on the numerator (top number) and denominator (bottom number), the Doppler shift has **no units**

Worked Example

Light emitted from a star has a wavelength of 435 × 10^{-9} m. A distance galaxy emits the same light but has a wavelength of 485 × 10^{-9} m. Calculate the speed at which the galaxy is moving relative to Earth. The speed of light = 3 × 10^{8} m/s.

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Step 1: List the known quantities

- Observed wavelength, $\lambda = 485 \times 10^{-9}$ m
- Reference wavelength, $\lambda_0 = 435 \times 10^{-9}$ m

Step 2: Write the relevant equation

$$\frac{\lambda - \lambda_0}{\lambda_0} = \frac{\Delta\lambda}{\lambda_0} = \frac{v}{c}$$

Step 3: Rearrange equation for the velocity of the galaxy, *v*

$$v = \frac{c \times \Delta\lambda}{\lambda_0}$$

Step 4: Calculate the change in wavelength, Δλ

$$\Delta\lambda = \lambda - \lambda_0$$

$$\Delta\lambda = (485 \times 10^{-9}) - (435 \times 10^{-9}) = 5 \times 10^{-8} \text{ m}$$

Step 5: Substitute values into the velocity equation

$$v = \frac{(3 \times 10^{8}) \times (5 \times 10^{-8})}{(435 \times 10^{-9})} = 3.4 \times 10^{7} \text{ m/s}$$