2.5 Transformations of Graphs

Question Paper

Course	DP IB Maths
Section	2. Functions
Topic	2.5 Transformations of Graphs
Difficulty	Medium

Time allowed: 110

Score: /91

Percentage: /100

Question 1

The point $P(-1, 4)$ lies on the curve with equation $y = f(x)$.

State the coordinates of the image of point P on the curves with the following equations:

(i) $y = f(x) + 3$

(ii) $y = f(x + 3)$

(iii) $y = 3f(x)$

(iv) $y = f(3x)$

[4 marks]

Question 2

The point $P(-3, -4)$ lies on the curve with equation $y = f(x)$.

State the coordinates of the image of point P on the curves with the following equations:

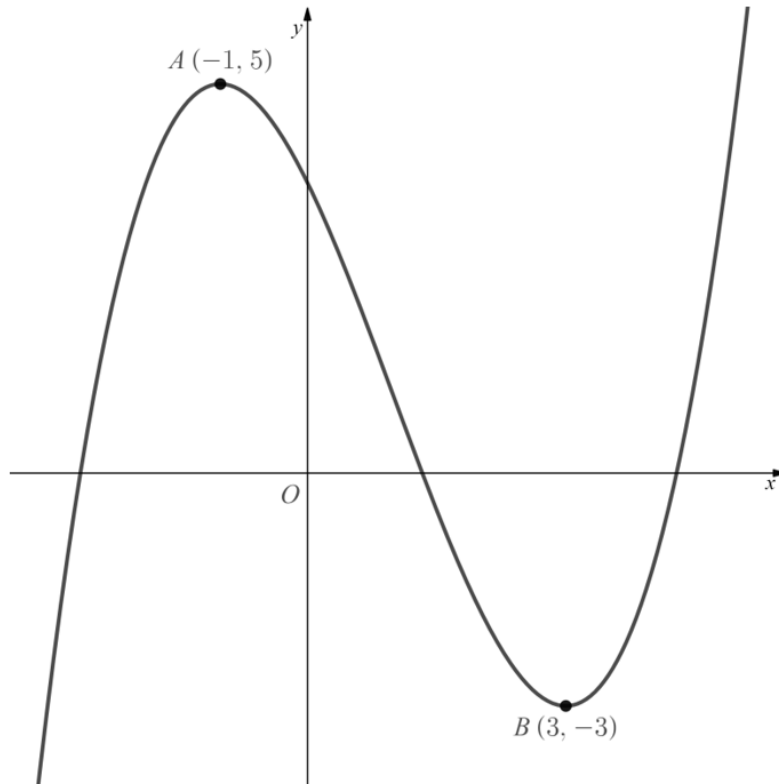(i) $y = f(-x)$

(ii) $y = -f(x)$

[2 marks]

Question 3

The point $P(3, 2)$ lies on the curve with equation $y = f(x)$.

(i) On the graph of $y = f(x) + a$, where a is a constant, the point P is mapped to the point $(3, -5)$. Determine the value of a.

(ii) On the graph of $y = f(x + b)$, where b is a constant, the point P is mapped to the point $(-1, 2)$. Determine the value of b.

(iii) On the graph of $y = cf(x)$, where c is a constant, the point P is mapped to the point $(3, 1)$. Determine the value of c.

(iv) On the graph of $y = f(dx)$, where d is a constant, the point P is mapped to the point $(1, 2)$. Determine the value of d.

[4 marks]

Question 4a

The diagram below shows the graph of $y = f(x)$. The two marked points $A(-1, 5)$ and $B(3, -3)$ lie on the graph.

(a) In separate diagrams, sketch the curves with equation

(i) $y = f(x - 1)$

(ii) $y = f(x) + 3$

On each diagram, give the coordinates of the images of points A and B under the given transformation.
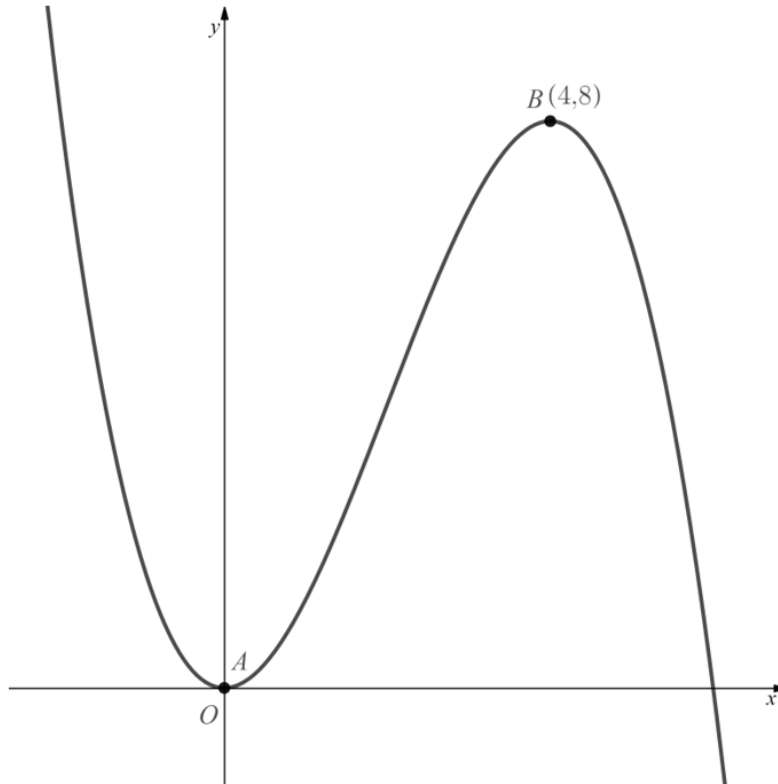
[4 marks]

Question 4b

(b) On the graph of $y = \mathrm{f}(x + a)$ the image of one of the two marked points has an x coordinate of 2. Find the two possible values of a.

[2 marks]

Question 5a

The diagram below shows the graph of $y = f(x)$. The marked point $B(4, 8)$ lies on the graph, and the graph meets the origin at the marked point A.

(a) In separate diagrams, sketch the curves with equation

    (i) $y = -f(x)$

    (ii) $y = f(4x)$

On each diagram, give the coordinates of the images of points A and B under the given transformation.
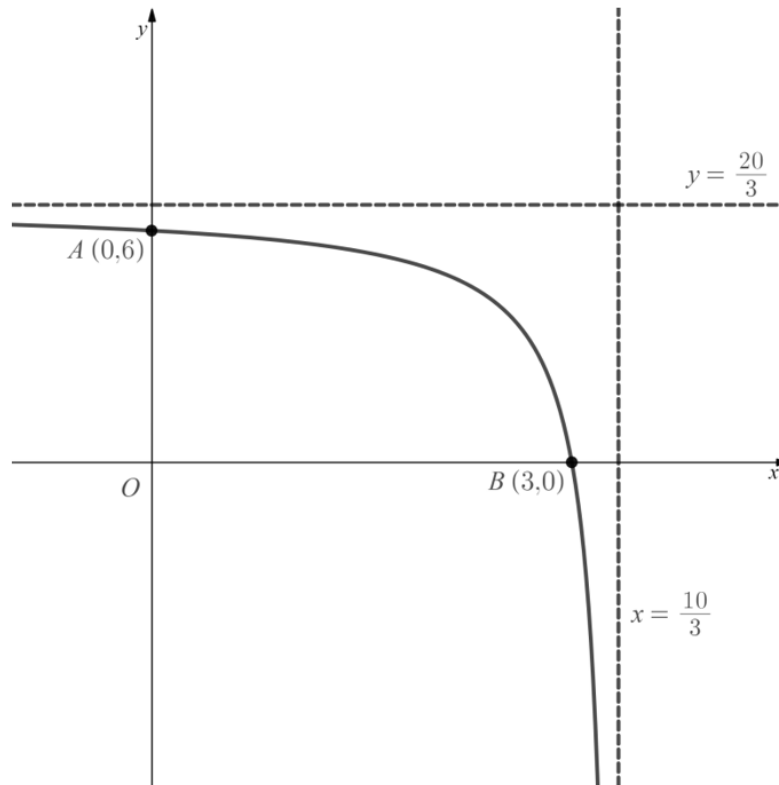
[4 marks]

Question 5b

(b) On the graph of $y = a\mathrm{f}(x)$ the image of one of the two marked points has a y coordinate of 4. Find the value of a.

[2 marks]

Question 6a

The diagram below shows the graph of $y = f(x)$. The graph intersects the coordinate axes at the two marked points $A(0, 6)$ and $B(3, 0)$. The graph has two asymptotes as shown, with equations $y = \dfrac{20}{3}$ and $x = \dfrac{10}{3}$.

(a) In separate diagrams, sketch the curves with equation

    (i) $y = f(x) - 6$

    (ii) $y = f(-x)$

On each diagram, give the coordinates of the images of points A and B under the given transformation, as well as stating the equations of the transformed asymptotes.

[6 marks]

Question 6b

(b) The graph of $y = \mathrm{f}(x + a)$ has an asymptote at one of the coordinate axes. Find the value of a.

[2 marks]

Question 7

Describe, in order, a sequence of transformations that maps the graph of $y = \mathrm{f}(x)$ onto the following graphs:

(i) $y = 3\mathrm{f}(x + 2)$,
(ii) $y = \mathrm{f}(-x) - 1$.

[3 marks]

Question 8

Given that $f(x) = 3x^2 - 2x$ find an expression for $g(x)$, where $g(x)$ is obtained by applying the following sequence of transformations to $f(x)$.

1. Translation by $\binom{2}{0}$
2. Vertical stretch of scale factor 4
3. Translation by $\binom{0}{-3}$

[4 marks]

Question 9a

(a) (i) Sketch the graph of $y = p(x)$, where $p(x) = 3x - 4$.
   (ii) On the same set of axes, sketch the graph of $y = p^{-1}(x)$.
      Label the coordinates of the points where each graph crosses the coordinate axes.
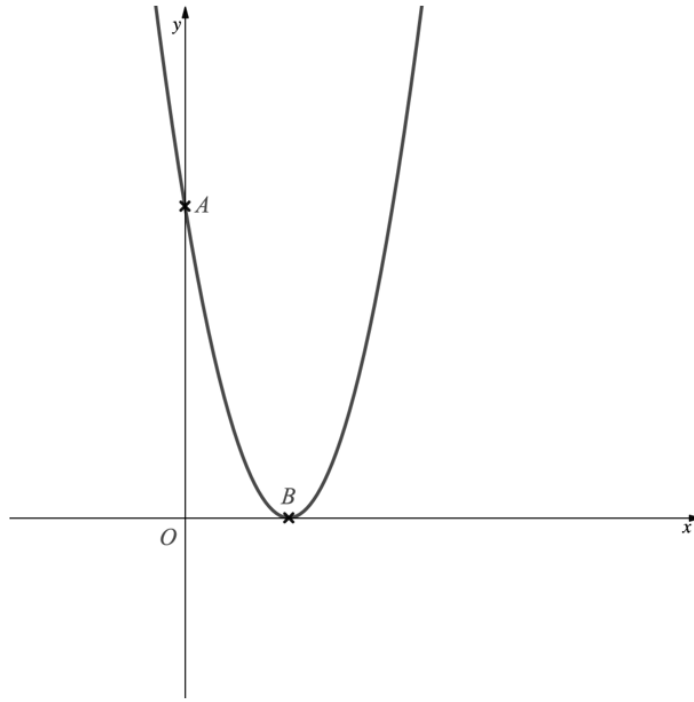
[4 marks]

Question 9b

(b) (i) Find an expression for $p^{-1}(x)$.

(ii) Find an expression for $\frac{1}{9}[p(x) + 16]$.

(iii) What can you deduce about the sequence of transformations given
by $\frac{1}{9}[p(x) + 16]$?

[4 marks]

Question 10a

The equation $y = f(x)$, where $f(x) = (x - a)^2$, with $a > 1$, is shown below.

The points A and B are the points where the graph intercepts the coordinate axes.

(a) Write down, in terms of a, the coordinates of A and B.

[2 marks]

Question 10b

(b) Sketch the graph of $y = -f(-x)$, labelling the images of the points A and B and stating their coordinates in terms of a.

[3 marks]

Question 10c

(c) Write down the value of a such that the point A is three times as far from the origin as the point B.

[1 mark]

Question 11

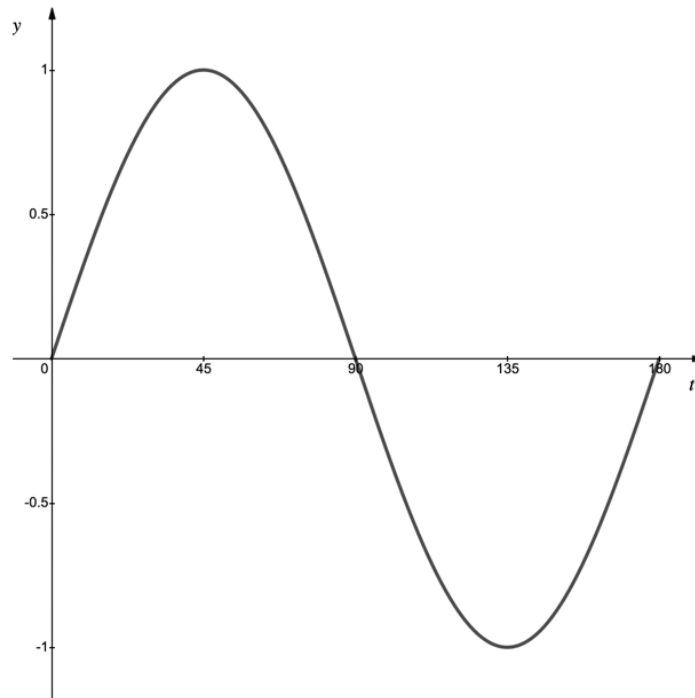The function $f(x)$ is to be transformed by a sequence of functions, in the order detailed below:

1. A horizontal stretch by scale factor 2
2. A reflection in the x-axis
3. A translation by $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$

Write down an expression for the combined transformation in terms of $f(x)$.

[3 marks]

Question 12a

The diagram shows the graph of $y = f(t)$, where $f(t) = \sin 2t, \; 0° \leq x \leq 180°$.

(a) (i) Write down the maximum value of y when $y = 3f(t)$.

   (ii) Write down the first value of t for which this maximum occurs.

[2 marks]

Question 12b

(b) (i) Write down the minimum value of y when $y = 5f(t + 30°)$.

   (ii) Write down the first value of t for which this minimum occurs.

[2 marks]

Question 12c

(c) Find, in terms of $f(t)$, the combination of transformations that would map the graph of $y = f(t)$ onto the graph of $y = 2 + \sin t, \ 0° \leq x \leq 180°$.

[2 marks]

Question 13a

Let $f(x) = 3x^2 + 18x + 27$.

(a) Write down the value of $f(-3)$.

[1 mark]

Question 13b

The function f can be written in the form of $f(x) = a(x - h)^2 + k$.

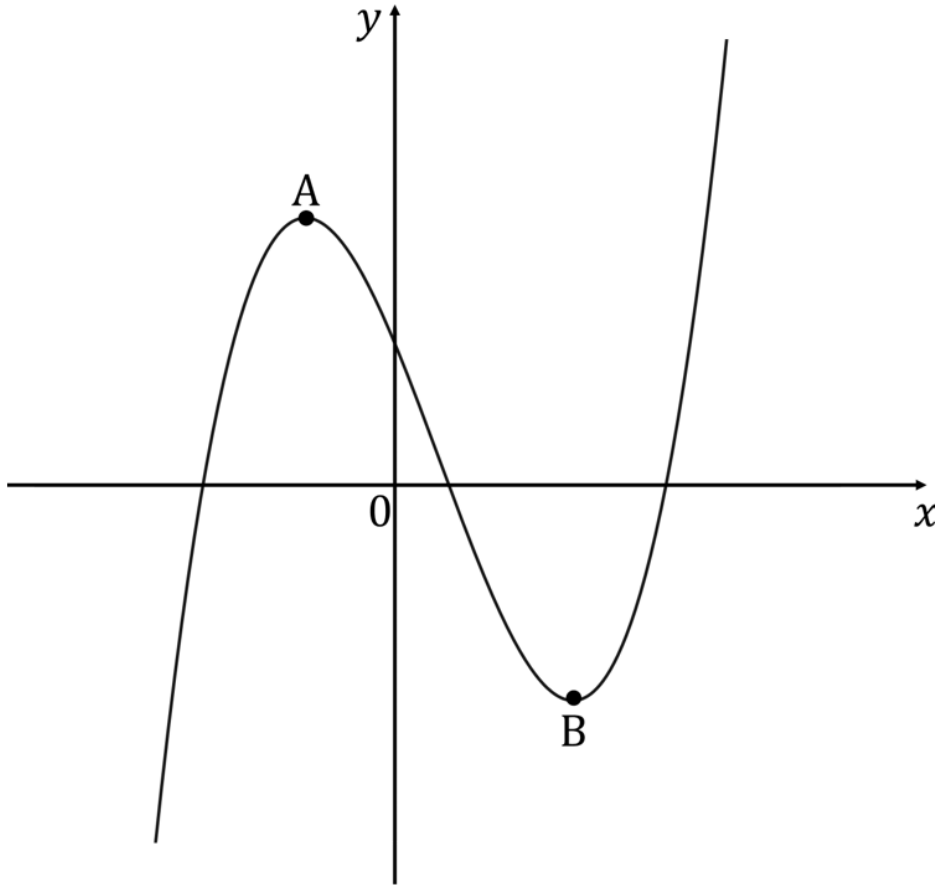(b) Find the values of a, h and k.

[3 marks]

Question 13c

The graph of g is obtained from the graph of f by a reflection in the x-axis followed by a translation by the vector $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

(c) Find $g(x)$, giving your answer in the form of $g(x) = rx^2 + sx + t$.

[4 marks]

Question 14

The graph of f is shown below. The points $A(-2, 10)$ and $B(4, -10)$ lie on the curve.

Sketch the graph of:

(i) $y = f(2x - 1)$,

(ii) $y = f(4 - x)$,

Clearly indicate the new coordinates of the images of the points A and B.

[7 marks]

Question 15

Describe a sequence of transformations that map the graph of $y = \ln x$ onto the graph of $y = 5 + \ln\left(\frac{1}{2}x + 4\right)$.

[4 marks]

Question 16a

The function f is defined by

$$f(x) = \begin{cases} ax + 1 & \text{if } x \leq 7, \\ x^2 - 2x + 1 & \text{if } x > 7. \end{cases}$$

(a) Find the value of a such that the graph of f is continuous at $x = 7$.

[3 marks]

Question 16b

The graph of the function g is obtained by translating the graph of f by the vector $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$, followed by a reflection in the x-axis.

(b) Find $g(x)$.

[4 marks]

Question 17

Let $f(x) = \dfrac{1}{x}$ and $g(x) = \dfrac{x+1}{x-2}$.

Explain fully the transformations of the graph of f to obtain the graph of g.

[5 marks]