2.6 Transformations of Graphs

Question Paper

Course	DP IB Maths
Section	2. Functions
Topic	2.6 Transformations of Graphs
Difficulty	Hard

Time allowed: 120

Score: /93

Percentage: /100

Question 1a

Let $f(x) = 2(x+4)^3$ and $g(x) = x^3$, for $x \in \mathbb{R}$.

(a) Give a full geometric description of the two individual transformations that can be combined to obtain the graph of f from the graph of g.

[2 marks]

Question 1b

The graph of f is translated by the vector $\begin{pmatrix} 2 \\ -5 \end{pmatrix}$ to give the graph of h.

Now consider the graph of h as a transformation of the graph of g. The point A on the graph of h corresponds to the point $(2, 8)$ on the graph of g.

(b) Find the coordinates of A.

[4 marks]

Question 2a

Let f and g be functions such that $g(x) = 2f(x-1) + 2$, for $x \in \mathbb{R}$.

The transformation that maps the graph of f onto the graph of g may be represented as a combination of two simpler transformations:

a vertical stretch by a factor of v,

followed by

a translation by the vector $\begin{pmatrix} a \\ b \end{pmatrix}$.

(a) Write down the values of

(i) v

(ii) a

(iii) b.

[3 marks]

Question 2b

The point A$(3, 4)$ on the graph of f is mapped to point B on the graph of g.

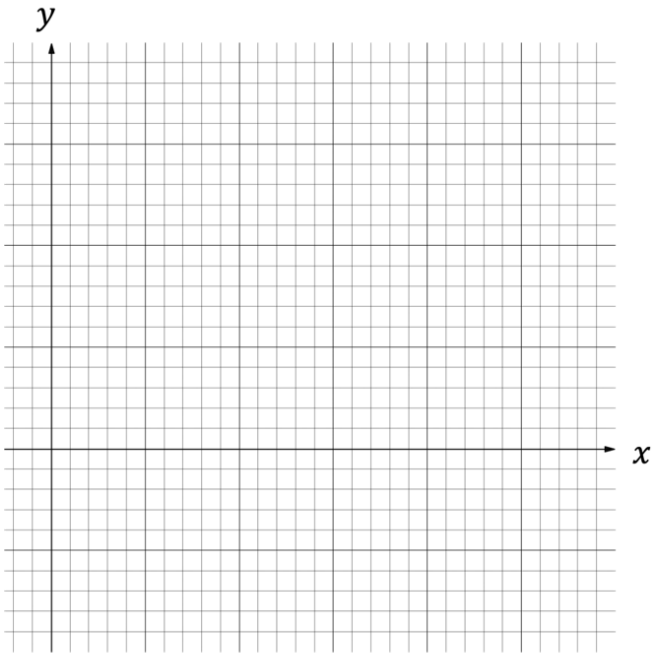
(b) Find the coordinates of B.

[3 marks]

Question 3a

Let $f(x) = x^2 - 9$, for $x \in \mathbb{R}$.

(a) Sketch the graph of $y = f(x)$ on the following grid in the interval $0 \leq x \leq 5$. Use an appropriate scale and clearly label any intersections the graph makes with the coordinate axes.



[3 marks]

Question 3b

(b) Find $(f \circ f)(2)$.

[2 marks]

Question 3c

The function g is obtained when the graph of f is translated by the vector $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$.

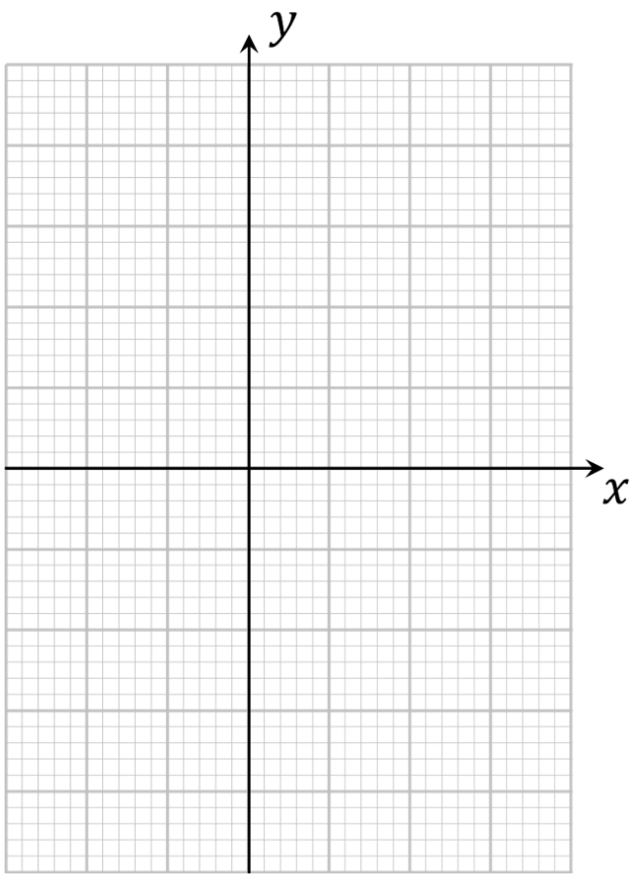
(c) Sketch the graph of $g(x)$ on the same grid above, also for the interval $0 \leq x \leq 5$. Clearly label any intersections the graph makes with the coordinate axes and label the graph in the form $g(x) = ax^2 + bx + c$ where a, b and c are constants to be determined.

[5 marks]

Question 4a

Let $f(x) = 1.1e^{x-1} - 4$, for $-2 \leq x \leq 3$.

(a) Sketch the graph of $y = f(x)$ on the grid below, clearly labelling any intersections the graph makes with the coordinate axes.



[4 marks]

Question 4b

The graph of f is reflected in the x-axis and then translated by the vector $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ to obtain the graph of $y = g(x)$.

(b) Find an expression for $g(x)$.

[2 marks]

Question 5

The function f is defined by

$$f(x) = \begin{cases} 1 + 2x & \text{if} \quad x \leq 2 \\ x^2 - 2x + 5 & \text{if} \quad x > 2 \end{cases}$$

The graph of the function g is obtained by applying the following transformations to the graph of f:

a translation by the vector $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$,

followed by

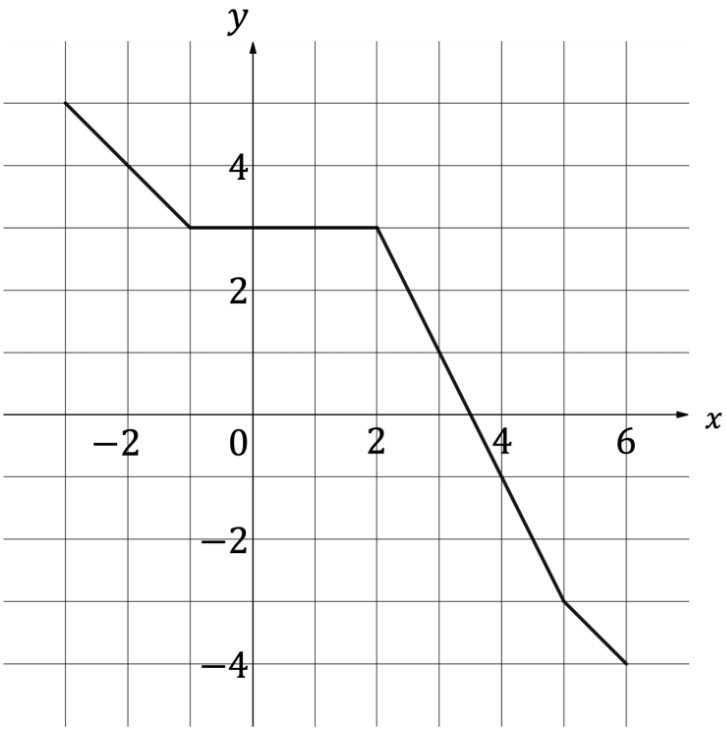
a reflection in the x-axis.

Find an expression for $g(x)$.

[4 marks]

Question 6a

The following diagram shows the graph of $y = f(x)$, $-3 \leq x \leq 6$.



(a) Write down the value of

(i) $f(-2)$

(ii) $f^{-1}(1)$.

[2 marks]

Question 6b

(b) Find the value of $(f \circ f)(0)$.

[1 mark]

Question 6c

(c) Given that $g(x) = f(x+5) - 5$, find the domain and range of g.

[2 marks]

Question 7a

Let $v(t) = 4t^2 + 64$, where $t > 0$.

The graph of a function g is obtained when the graph of v is transformed by

a vertical stretch by a factor of $\frac{1}{8}$,

followed by

a translation by the vector $\begin{pmatrix} 8 \\ 3 \end{pmatrix}$.

(a) Find $g(t)$, giving your answer in the form $at^2 + bt + c$.

[4 marks]

Question 7b

A particle moves along a straight line so that its velocity in ms^{-1}, at time t seconds, is given by $g(t)$.

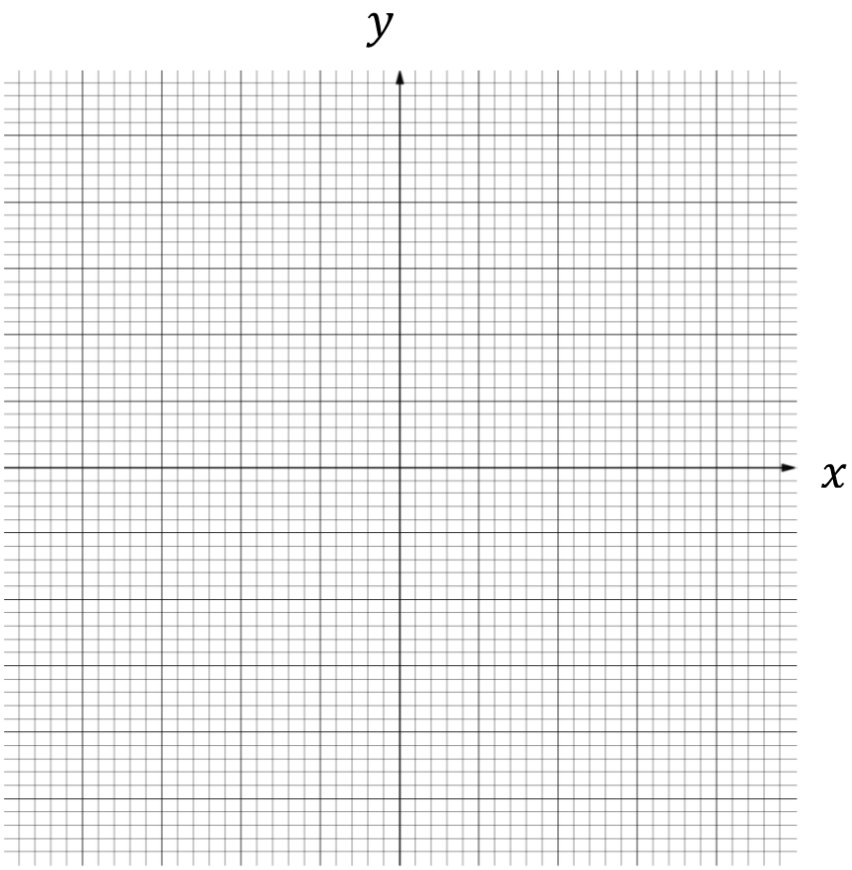
(b) Find the value of t when the particle's velocity is $11\ \text{ms}^{-1}$.

[2 marks]

Question 8a

Let $f(x) = 2x^2 - 6x$, for $x \in \mathbb{R}$.

(a) Sketch the graph of $y = f(x)$ on the grid below, clearly labelling the vertex as well as any intersections the graph makes with the coordinate axes.



[4 marks]

Question 8b

The graph of a function g is obtained from the graph of f by a reflection in the y-axis, followed by a horizontal stretch with scale factor $\frac{1}{2}$.

(b) Find an expression for $g(x)$, giving your answer in the form $g(x) = a(x-h)^2 + k$.

[5 marks]

Question 9a

Let $f(x) = 2x^2 + bx + 8$, for $x \in \mathbb{R}$, where $b \in \mathbb{Z}$.

Given that the equation $f(x) = 0$ has two equal roots, and that $b < 0$,

(a) find the value of b.

[3 marks]

Question 9b

(b) Find the coordinates of the vertex of the graph of f.

[2 marks]

Question 9c

The graph of a function g is obtained from the graph of f by a reflection in the y-axis, followed by a horizontal stretch with scale factor 2.

(c) Find an expression for $g(x)$ and state the coordinates of the y-intercept of the graph of g.

[3 marks]

Question 10a

Let $f(x) = 2x^2 - 12x + 10$

(a) For the graph of f, find

(i) the x-intercepts

(ii) the y-intercept

(iii) the coordinates of the vertex.

[5 marks]

Question 10b

The graph of a function g is obtained from the graph of f by a reflection in the x-axis followed by a translation by the vector $\begin{pmatrix}1\\6\end{pmatrix}$.

(b) Find $g(x)$, giving your answer in the form $g(x) = a(x-h)^2 + k$.

[3 marks]

Question 11a

Consider the functions f and g defined by $f(x) = \ln x$ and $g(x) = \ln(2x + 5)$, where each function has the largest possible valid domain.

(a) Write down the domain of g.

[1 mark]

Question 11b

The graph of f can be transformed onto the graph of g by a single translation and a single stretch, both of which are parallel to one of the coordinate axes.

(b) Describe the sequence of transformations in the case where:

(i) the translation occurs first.

(ii) the stretch occurs first.

[4 marks]

Question 11c

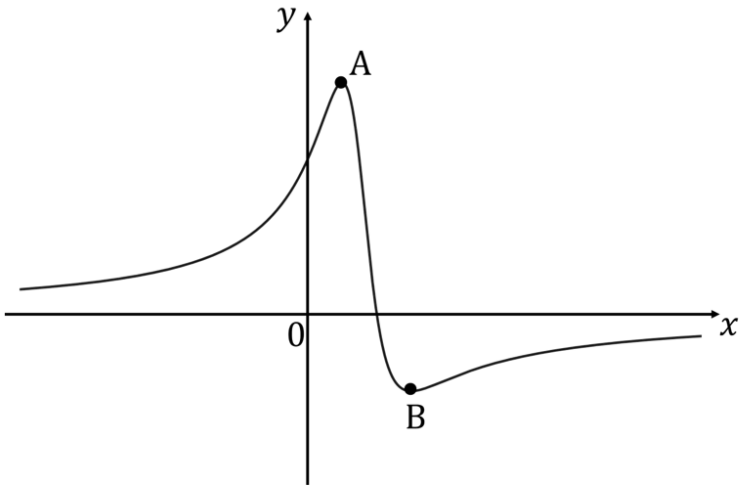
The graph of f can be also transformed onto the graph of g by a single translation using the vector $\begin{pmatrix} a \\ b \end{pmatrix}$.

(c) Find the exact values of a and b.

[3 marks]

Question 12a

The graph of a function f is shown below. The points $A(1, 6)$ and $B(3, -3)$ lie on the graph and are a local maximum and a local minimum respectively. The x-axis is an asymptote to the graph.



(a) On separate sets of axes, sketch the graphs of:

(i) $y = 2(f(x) + 3)$.

(ii) $y = f(2x + 3)$.

In each case give the coordinates of the points onto which A and B are mapped, and state the equation of the asymptote.

[6 marks]

Question 12b

The graph of $y = f(x)$ is stretched horizontally by a scale factor of k then translated by the vector $\begin{pmatrix} a \\ b \end{pmatrix}$ to map it onto the graph of $y = f(5x + 10) + 4$.

(b) (i) Find the values of a, b and k.

(ii) Find the coordinates of the points onto which A and B are mapped.

[5 marks]

Question 13a

Consider the function f defined by $f(x) = 0.4e^{x+1} - 3,\ -6 \leq x \leq 3$.

(a) Find the coordinates of

(i) the x-intercept

(ii) the y-intercept

of the graph of $y = f(x)$.

[2 marks]

Question 13b

(b) Sketch the graph of $y = f(x)$.

[2 marks]

Question 13c

The graph of f is first reflected in the y-axis and then translated by the vector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ to obtain the graph of a function g.

(c) Find an expression for $g(x)$.

[2 marks]