

# 5.1 Differentiation

## Question Paper

Course	DPIB Maths
Section	5. Calculus
Topic	5.1 Differentiation
Difficulty	Hard

**Time allowed:** 100  
**Score:** /82  
**Percentage:** /100

**Question 1a**

The equation of a curve is  $y = x - \frac{9}{x} + 8$  for  $x > 0$ .

(a) Find  $\frac{dy}{dx}$ .

[2 marks]

**Question 1b**

The gradient of the tangent to the curve at point A is 2.

(b) Find the coordinates of point A.

[3 marks]

**Question 1c**

(c) Find the equation of the normal to the curve at point A. Give your answer in the form  $ax + by + d = 0$ .

[3 marks]

**Question 2a**

The volume of a sphere of radius  $r$  is given by the formula  $V = \frac{4}{3}\pi r^3$ .

(a) Find  $\frac{dV}{dr}$ .

[1 mark]

**Question 2b**

(b) Find the rate of change of the volume with respect to the radius when  $r = 5$ .  
Give your answer in terms of  $\pi$ .

[2 marks]

**Question 2c**

(c) Show that  $\frac{dV}{dr}$  is an increasing function for all relevant values of  $r$ .

[3 marks]

**Question 3a**

A curve has the equation

$$f(x) = \frac{1}{3}x^3 - 2x^2 - 4x + \frac{31}{3}$$

Points A and B are the two points on the curve where the gradient is equal to 1, and the  $x$ -coordinate of A is less than zero.

(a) Find the coordinates of points A and B.

[3 marks]

**Question 3b**

(b) Find the equations of

- (i) the tangent to the curve at point A
- (ii) the normal to the curve at point B.

[5 marks]

**Question 3c**

Point C is the point of intersection of the two lines found in part (b).

(c) Find the coordinates of point C.

[2 marks]

**Question 4**

The gradient of the tangent to the curve with equation  $f(x) = ax^2 + 2x + 9$  at the point  $(-2, b)$  is 14.

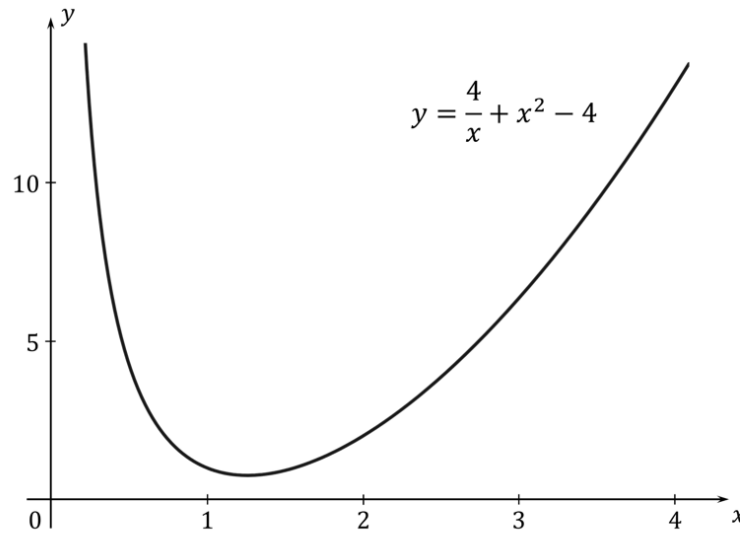
Find the values of  $a$  and  $b$ .

[5 marks]

**Question 5a**

The diagram below shows a part of the graph of the function  $y = f(x)$ , where

$$f(x) = \frac{4}{x} + x^2 - 4, \quad x > 0$$



(a) Calculate the instantaneous rate of change of  $f(x)$  when  $x = 2$ .

[2 marks]

**Question 5b**

(b) Calculate the average rate of change of  $f(x)$  between  $x = 2$  and

- (i)  $x = 3$
- (ii)  $x = 2.5$
- (iii)  $x = 2.25$

[4 marks]

**Question 5c**

(c) Explain what would happen if you continued to calculate the average rates of change in part (b), moving the second  $x$  value closer and closer to 2 each time.

[2 marks]

**Question 6a**

The equation of a curve is  $y = x^3 + 9x^2 + 24x + 17$ .

(a) Show that the curve has exactly two stationary points. Determine the coordinates and nature of each point.

[6 marks]

**Question 6b**

(b) Show that the curve has exactly one point of inflection and determine its coordinates.

[3 marks]

**Question 6c**

(c) Give an example of a curve with equation  $y = ax^3 + bx^2 + cx + d$ , where  $a, b, c$  and  $d$  are real numbers and  $a \neq 0$ , for which there is a point of inflection that is also a stationary point. Be sure to justify your answer.

[3 marks]



**Question 7a**

A function  $f$  is defined by

$$f(x) = \frac{1}{5}x^5 + \frac{2}{3}x^3 - 3x + 3$$

for all real numbers  $x$ .

- (a) (i) Show that the  $x$ -coordinates of the stationary points on the curve  $y = f(x)$  must satisfy the equation

$$(x^2 - 1)(x^2 + 3) = 0$$

- (ii) Hence determine the coordinates of the stationary points on the curve.

[6 marks]

**Question 7b**

(b) Determine the ranges of values of  $x$  for which  $f(x)$  is

- (i) increasing
- (ii) decreasing.

[3 marks]

**Question 7c**

(c) Sketch the curve of  $y = f(x)$ , showing the coordinates of any minimum and maximum points, as well as the point where the curve crosses the  $y$ -axis.

[3 marks]

**Question 8a**

A function  $f$  is defined for all for  $x > 0$ . The derivative of  $f$  is given by

$$f'(x) = 3x^2 + \frac{2}{x^3} - 25$$

(a) Find  $f''(x)$ .

[2 marks]

**Question 8b**

The graph of  $f$  is concave up when  $x > n$ , where  $n$  is the least possible number that makes that inequality true.

(b) Find the value of  $n$ .

[4 marks]

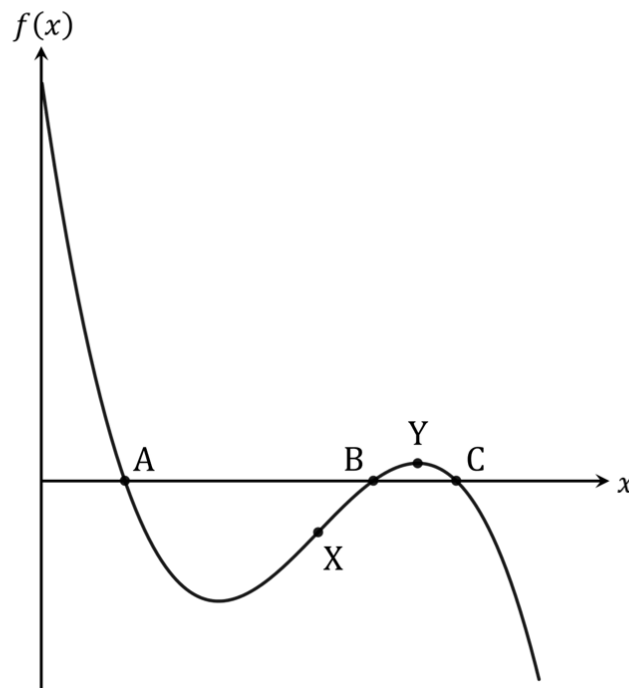
**Question 8c**

(c) Show that the curve  $y = f(x)$  has only one point of inflection, and find the gradient of the curve at that point of inflection.

[3 marks]

**Question 9a**

Let  $f$  be a function defined by  $f(x) = -3x^3 + 30x^2 - 87x + 60$  for all  $x$  in the interval  $0 \leq x \leq 6$ . The following diagram shows the graph of  $f$ :



The curve intercepts the  $x$ -axis at points  $A(a, 0)$ ,  $B(b, 0)$  and  $C(c, 0)$ . There is a point of inflection at point  $X$  and a local maximum at point  $Y$ .

(a) Find the values of  $a$ ,  $b$  and  $c$ .

[2 marks]

**Question 9b**

- (b) Find  $f'(x)$ , and hence determine the  $x$ -coordinate of the local maximum at point Y.  
You should give your answer as an exact value.

[4 marks]

**Question 9c**

- (c) Find the equation of the tangent at point X. Give your answer in the form  $px + qy + r = 0$ , where  $p, q$  and  $r$  are integers.

[6 marks]

